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Panel Methods—An Introduction

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SUMMARY

Panel methods are numerical schemes for solving (the Prandtl-Glauert equation) for linear, inviscid, irrotational flow about aircraft flying at subsonic or supersonic speeds. The tools at the panel-method user's disposal are (1) surface panels of source-doublet-vorticity distributions that can represent nearly arbitrary geometry, and (2) extremely versatile boundary condition capabilities that can frequently be used for creative modeling. This report discusses panel-method capabilities and limitations, basic concepts common to all panel-method codes, different choices that have been made in the implementation of these concepts into working computer programs, and various modeling techniques involving boundary conditions, jump properties, and trailing wakes. An approach for extending the method to nonlinear transonic flow is also presented.

Three appendixes supplement the main text. In appendix A, additional detail is provided on how the basic concepts are implemented into a specific computer program (PAN AIR). In appendix B, we show how to evaluate analytically the fundamental surface integral that arises in the expressions for influence-coefficients, and evaluate its jump property. In appendix C a simple example is used to illustrate the so-called finite part of improper integrals.

1. INTRODUCTION

Panel methods are numerical schemes for solving (the Prandtl-Glauert equation) for linear, inviscid, irrotational flow at subsonic or supersonic free-stream Mach numbers. Currently, panel-method codes are the only codes commonly in use that are sufficiently developed for routinely analyzing the complex geometries of realistic aircraft. The objective of this report is to give the reader some idea of what panel methods can and cannot do, to describe their common roots, to describe the differences between various specific implementations, and to show some example applications. In addition, recent progress in solving nonlinear transonic flow problems by combining portions of panel-method technology with other numerical techniques, is described. This material is followed by three appendixes that contain additional details: appendix A describes how the basic ideas common to all panel methods are actually implemented in a specific code; appendix B shows how to evaluate some of the integrals that arise in the influence-coefficient equations; and appendix C discusses the so-called finite part of improper integrals.

There are fundamental analytic solutions to the Prandtl-Glauert equation known as source, doublet, and vorticity singularities. Panel methods are based on the principle of superimposing surface distributions of these singularities over small quadrilateral portions, called panels, of the aircraft surface, or to some approximation to the aircraft surface. The resulting distribution of superimposed singularities automatically satisfies the Prandtl-Glauert equation. To make the solution correspond to the desired geometry, boundary conditions are imposed at discrete points of the panels. (Mathematicians refer to these discrete points as collocation points; panel-method users refer to them as control points.)

Panel codes are often described as being lower-order or higher-order. The term lower-order refers to the use of constant-strength singularity distributions over each panel, and the panels are usually flat. Higher-order codes use something of higher order than constant, for example, a linear or quadratic singularity distribution, and sometimes curved panels.

Panel methods were initially developed as lower-order methods for incompressible and subsonic flows (e.g., refs. 1, 2; see ref. 3 for a review of panel methods existing through about 1976). The first successful panel method for supersonic flow became available in the mid-1960s (refs. 4, 5). This was also a lower-order method, and is variously referred to as the constant-pressure panel method, or the Woodward-Carmichael method.

The panel methods for three-dimensional subsonic flow allowed the actual vehicle surface to be paneled, whereas the Woodward-Carmichael method was more severely restricted in the placement of the panels. For example, the wing was a planar array of panels, the body (fuselage) volume was modeled with a line distribution of source and doublet singularities (resulting in a body-of-revolution) and the body boundary conditions were imposed with a cylindrical "interference" shell of wing-type panels. This representation was later extended to include multiple wing-body components (ref. 6), but was still restricted to the planar panel representation. These two extremes of actual-surface models and mean-surface models (Woodward-Carmichael) are illustrated in figure 1.

The mean-surface model used in the Woodward-Carmichael panel method was a consequence of numerical stability problems that arose in supersonic flow. The constant-strength, elementary horseshoe vortex singularity distribution (producing a constant pressure over each panel) often produced unstable numerical behavior (the solutions would "blow-up") when a panel was inclined to a supersonic flow. The method worked only when all the panels were kept parallel to the free-stream flow. This required that angle of attack, wing thickness, camber, and twist be simulated through the boundary conditions; that is, it was necessary to have the panels generate flow perpendicular to themselves and thereby turn the flow through the desired angles, as is done in classic thin-wing theory. As a consequence of this restricted geometric model, several new approaches to the supersonic problem were pursued in the 1970s.

The first of these was also due to Woodward; it evolved into the series of computer programs known as USSAERO (ref. 7). For fuselage panels, USSAERO uses constant-strength source singularities. Wing panels use elementary horseshoe-vortex singularities whose strength distribution varies linearly in the chordwise direction and is constant in the spanwise direction. Although this representation gave an improved modeling capability, numerical problems would still often occur when the wing panels were inclined to a supersonic free-stream flow.

Another approach, developed by Morino and his associates, uses a superposition of constant-strength sources and doublets on hyperboloidal panels. The constant strength doublets produce a velocity field that is identical to that produced by line-vortex elements, having the same strength as the doublet panel, placed head to tail around the panel perimeter (a so-called ring-vortex panel). This method is available in the computer program called SOUSSA; it too is unable to handle the steady supersonic case (ref. 8, pp. 2-20; private communication, L. Morino, Feb. 1981).

The key to eliminating the numerical stability problems associated with supersonic flow, was to use doublet distributions that were continuous over the entire surface of the aircraft. This approach, using quadratic doublet distributions (equivalent to linear vorticity distributions) was first used in the PAN AIR code (refs. 9-14) and its pilot code predecessor (ref. 15). It has since been implemented in the European version of PAN AIR, called HISSS (ref. 16). The continuous-doublet distribution eliminates the appearance of spurious line-vortex terms at the panel edges, which was the cause of the numerical stability problems in the earlier approaches.

Within the limitations of the Prandtl-Glauert equation, the higher-order singularity distributions used in the PAN AIR and HISSS codes allowed the actual-surface paneling models, long in use for subsonic flow, to also be used for supersonic flow. It also had a very beneficial side effect: the numerical solutions turned out to be much less sensitive to the size, shape, and arrangement of the paneling than in earlier methods, including the subsonic-only methods. Partly for this reason, continuous quadratic doublets were incorporated into the subsonic-only MCAERO code (ref. 17). These advantages did not come free however. The higher-order distributions require much more analytic work to derive the influence-coefficient equations, and demand many more arithmetic operations than the simpler lower-order (constant-strength) methods, which results in significantly higher run costs.

It was subsequently discovered that for subsonic flow, setting the perturbation potential to zero at the interior side of panels, in conjunction with the original lower-order singularity distributions, also reduced the solution sensitivity to variations in panel layout. This led to a renewed interest in the lower-order methods, resulting in the VSAERO (refs. 18, 19) and QUADPAN (refs. 20, 21) codes. QUADPAN was later revised to handle the supersonic case by changing its constant-strength doublets to continuous linear doublets.

2. WHAT PANEL METHODS CAN AND CANNOT DO

Panel-method-based computer programs are currently the workhorse codes for predicting the aerodynamics of complete configurations. Representative aircraft examples that have been analyzed with panel method codes are shown in figure 2. Although such codes are routinely used to analyze very complicated geometries, they do so at the expense of ignoring much fluid physics. The equation that panel codes solve is the Prandtl-Glauert equation. For steady subsonic flow this equation is usually written as

$$\tilde{\nabla}^2\phi = \left(1 - M_\infty^2\right)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \qquad (1)$$

and for supersonic flow it is sometimes multiplied by −1,

$$-\tilde{\nabla}^2\phi = \left(M_\infty^2 - 1\right)\phi_{xx} - \phi_{yy} - \phi_{zz} = 0 \qquad (2)$$

where M_∞ is the free-stream Mach number and ϕ is the perturbation velocity potential.

For subsonic free-stream flow, equations (1) and (2) are elliptic, being similar to Laplace's equation. Such equation types have the property that any disturbance at some point is felt everywhere in the flow field (although the effect usually dies out rapidly with distance). For supersonic free-stream flow the equations are hyperbolic, with the x-derivative term behaving like time in the wave equation. Solutions for the supersonic case are fundamentally different, disturbances having restricted zones of influence (or in Von Karman's words, zones of "silence," or "forbidden signals"; ref. 22). The disturbances only propagate downstream, along rays defined by the Mach cones (characteristic surfaces), reflecting off downstream geometry and interfering in a wave-like manner with other disturbances.

The Prandtl-Glauert equation is the simplest form of the fluid-flow equations that contain compressibility effects (i.e., the effect of Mach number on fluid density). It is obtained from the more general Navier-Stokes equation by (1) neglecting all the viscous and heat-transfer terms; (2) assuming that the flow is irrotational, thereby admitting the introduction of a velocity potential; and (3) discarding all nonlinear terms. This restricts the flow to be inviscid, irrotational, and linear. Often, the flow is also assumed to be steady. Physically, these restrictions mean that important flow behavior such as separation, skin-friction drag, and transonic shocks are not predicted with panel methods. Items that are predicted include drag-due-to-lift (often called induced drag for subsonic flow, and vortex drag for supersonic flow), and wave drag.

Wave drag is predicted because the Prandtl-Glauert equation admits solutions that approximate the weak-shock solutions of shock-expansion theory (ref. 23, pp. 215, 216). A simple example is the supersonic flow over a thin wedge (fig. 3(b)). For small wedge (deflection) angles, the shock is attached at the wedge leading edge, forms at an angle very nearly to that of the Mach angle, and the flow remains supersonic on the downstream side of the shock. The limiting case for these weak shocks, in which the shocks form at exactly the Mach angle, is predicted by the Prandtl-Glauert equation.

The absence of any explicit viscous effects causes subsonic flow solutions to be non-unique unless a Kutta condition at sharp trailing edges is somehow imposed (ref. 24, pp. 80, 81). This is done with the addition of some type of wake panels that trail downstream from lifting-surface trailing edges (fig. 3(a)), causing the flow to separate smoothly from these edges and allowing the potential to jump (be discontinuous) across the wake. Most panel methods require the user to assume the shape and position of the wakes. For a simple wing body this poses no difficulty, the wake position being relatively unimportant. However, for multiple-lifting-surface configurations, the wake placement is important since it affects the flow experienced by downstream geometry. A few codes iteratively solve for the wake shape and location.

Because panel methods are able to treat complete configurations, they have often been used in combination with other methods to approximately account for additional physics neglected by the Prandtl-Glauert equation. One fairly common practice is to include the presence of the wing boundary layer (ref. 25). The basic idea is to use the pressure distribution from the panel-code solution as input to a boundary-layer code and compute the displacement thickness. This incremental thickness is then represented in a second run of the panel code. This is usually done in one of two ways, as illustrated in figure 4 (ref. 25). The first is to actually recompute the wing surface coordinates and the new wing-body intersection by adding the displacement thickness to the actual wing geometry. An alternative approach is to use "blowing," in which the source strengths of the wing panels are adjusted such that each panel ejects (or sucks) enough fluid to cause the resultant flow field to be approximately displaced by the displacement thickness. For either approach, the resulting change in actual or apparent wing shape has two effects: it reduces the effective camber of a cambered wing and it increases the wing thickness. For a specified angle of attack, the primary aerodynamic effect of these changes is a reduced lift owing to the reduced camber. The second, but usually less important effect, is a slight increase in lift owing to the increased wing thickness.

Another example is the coupling of panel codes to propulsion codes. In reference 26, the PAN AIR code is coupled to a parabolized Navier-Stokes propulsion code. The purpose was to account for the viscous, high-energy, exhaust-flow effect on the aerodynamic flow about the complete aircraft.

Panel-method codes have also been built to model the flow separation that occurs off highly swept wings with sharp leading edges (refs. 27, 28). In these codes, wake panels emanate from the wing leading edge, as well as from the trailing edge (fig. 5). Iterative techniques are used to solve for the correct shape and position of the leading-edge wake panels. The criteria to be satisfied are (1) that the Kutta condition be enforced and (2) that the entire wake surface be a stream surface (i.e., no flow crosses it, and it supports no pressure jump).

3. COMMON ROOTS OF PANEL METHODS

As indicated in section 2, panel methods rely on surface distributions of sources, doublets, and vorticity. We will see later that doublet and vortex distributions are related. Since surface vorticity is a vector and a doublet is a scalar, it is often easier to work with doublets than with vorticity, and then compute the vorticity from the doublet-strength distribution. Most higher-order panel-method codes take this approach.

It can be verified by direct substitution, that the following expressions, called unit point sources and doublets, respectively, satisfy the Prandtl-Glauert equation (eqs. (1) or (2)).

Point source:

$$\phi_P^S(\vec{x}_Q) = \frac{-1}{R(\vec{x}_P, \vec{x}_Q)} \tag{3}$$

Point doublet:

$$\phi_P^D(\vec{x}_Q) = \hat{n} \cdot \tilde{\nabla}_Q \frac{1}{R(\vec{x}_P, \vec{x}_Q)} = \hat{n} \cdot \frac{-\vec{R}(\vec{x}_P, \vec{x}_Q)}{R^3} \tag{4}$$

where the so-called hyperbolic distance R is given by

$$R = \sqrt{(x_Q - x_P)^2 + \beta^2\left[(y_Q - y_P)^2 + (z_Q - z_P)^2\right]} \tag{5a}$$

where

$$\beta^2 = 1 - M_\infty^2 \tag{5b}$$

In these expressions, point P is the influenced point in space having coordinates $\vec{x}_P = (x_P, y_P, z_P)$ and point Q is the influencing point $\vec{x}_Q = (x_Q, y_Q, z_Q)$ at'which the unit point source or doublet is located (see fig. 6). There is an elemental area dS_Q associated with the doublet, and the doublet axis is normal to this area. (Recall from elementary fluid mechanics that a doublet can be thought of as a source-sink pair approaching each other along an axis. This definition of a doublet produces the same result as eq. (4).) The subscript Q on the scaled gradient operator means that the derivatives are to be taken with respect to the coordinates of point Q, not point P.

For incompressible flow, R becomes simply the geometric distance between the two points P and Q. Equation (3) then tells us that a point source at Q produces a disturbance at P that diminishes inversely as the distance between the two points. The meaning of equation (4) is not so obvious until one works out the expression indicated by the dot product. If one chooses the xyz coordinate system at point Q as shown in figure 6, then the unit normal $\hat{n}$ equals the unit vector $\hat{k}$ and equation (4) becomes simply

$$\phi_P^D = \frac{-\sin\theta}{R^2} \tag{6}$$

This form clearly shows the directional properties of a point doublet and reveals that a doublet disturbance dies off at least as rapidly as the inverse of the distance squared.

Since the Prandtl-Glauert equation is a linear partial differential equation, sums of the source and doublet solutions are also solutions. Thus, panel methods are usually thought of as superposition methods, and, hence, are restricted to linear problems. There is a more general approach, however, that, while containing superposition as a special case, can also be used to solve nonlinear problems. In section 6 we will take a look at how panel-method technology can be combined with other techniques to solve the nonlinear full-potential equation, so it is advantageous to look at this more general approach, known as Green's third theorem (ref. 29, p. 21, eq. (7)).

In reference 29, the derivation corresponds to incompressible potential flow; in reference 30, this is generalized to the compressible case. The result is the following identity:

$$\phi_P \equiv \iint_S \left[\sigma(\vec{x}_Q) K_\sigma(\vec{x}_Q, \vec{x}_P) + \mu(\vec{x}_Q) K_\mu(\vec{x}_Q, \vec{x}_P)\right] dS_Q + \iiint_V (\tilde{\nabla}^2 \phi) K_\sigma(\vec{x}, \vec{x}_P) dV \tag{7}$$

where

$$K_\sigma(\vec{x}_Q, \vec{x}_P) = \frac{1}{k} \phi_P^S(\vec{x}_Q)$$

$$K_\mu(\vec{x}_Q, \vec{x}_P) = \frac{1}{k} \phi_P^D(\vec{x}_Q)$$

$$K_\sigma(\vec{x}, \vec{x}_P) = \frac{1}{k} \phi_P^S(\vec{x})$$

$$dV = dx\, dy\, dz$$

$$\sigma = \Delta(\hat{n} \cdot \vec{w}) \tag{8}$$

$$\mu = \Delta\phi$$

$$\vec{w} = (\beta^2 u, v, w) = (\beta^2 \phi_x, \phi_y, \phi_z) = \tilde{\nabla}\phi$$

In the above equations σ is the source strength and μ is the doublet strength at any point Q, on the surface S, which in our case will be all (for subsonic flow) or part (for supersonic flow) of the aircraft surface and wakes. These strengths are equal to jumps (discontinuities) across the panels of certain flow properties. The source strength equals the jump in the normal component of the mass-flux vector $\vec{w}$. The doublet strength equals the jump in potential, and the gradient of the doublet strength equals the jump in tangential component of velocity. The values of these strengths are the (as yet unknown) amplitudes of the source and doublet singularity solutions appearing in equations (3) and (4). Here, these source and doublet solutions, when multiplied by a constant k^{-1}, are denoted as K_σ and K_μ, respectively (K is used to denote that the singularities are called kernels). For $M_\infty < 1$, $k = 4\pi$, and S is the entire surface of the aircraft and wake(s). For $M_\infty > 1$, $k = 2\pi$, and S is that portion of the aircraft surface and wake(s) that lies in the upstream Mach cone emanating from the influenced point P.

Equation (7) shows us that the velocity potential at point P is related to the source and doublet distributions on S, and to the spatial distribution of $\tilde{\nabla}^2\phi$ in the volume V bounded (wetted by) both sides of S. If ϕ_P is constructed according to the surface integral terms in equation (7), that is,

$$\phi_P = \iint_S \left[\sigma K_\sigma + \mu K_\mu\right] dS_Q \tag{9}$$

then, because equation (7) is an identity, it follows that

$$\iiint_V \left(\tilde{\nabla}^2\phi\right) K_\sigma \, dV = 0 \tag{10}$$

Since K_σ is a function of the arbitrary point P, $\tilde{\nabla}^2\phi$ must be zero. Thus, construction of ϕ_P according to equation (9) implies that equations (1) and (2), the Prandtl-Glauert equation, has been satisfied throughout V.

Equation (9) is the basic starting formula for panel methods using sources and doublets. If the source and doublet strength distribution is known (we will see how this is done later), then the velocity at point P is obtained from equation (9) by differentiating with respect to the coordinates at P, that is

$$\vec{v}_P = \vec{\nabla}_P \phi_P = \iint_S \left[\sigma \vec{\nabla}_P K_\sigma + \mu \vec{\nabla}_P K_\mu\right] dS_Q \tag{11}$$

Equations (9) and (11) are used to generate influence-coefficient equations that relate source and doublet strengths at particular points Q on the surface S to the potential and velocity at field points P. The basic idea is to break S into a collection of panels Σ and to assume a functional form for σ and μ over each panel. For example, a constant-strength source-doublet panel with index j is given simply by

$$\sigma\left(\vec{x}_j\right) = \sigma_j(\xi,\eta) = \lambda_j^S \tag{12a}$$

$$\mu\left(\vec{x}_j\right) = \mu_j(\xi,\eta) = \lambda_j^D \tag{12b}$$

where the unknown constants λ_j^S and λ_j^D are called source and doublet singularity parameters, respectively, for panel j, and (ξ,η) are local surface coordinates associated with the panel.

Once the functional form for $\sigma_j(\xi,\eta)$ and $\mu_j(\xi,\eta)$ are specified, equations (9) and (11) can be integrated over each panel (a nontrivial task) so that ϕ_P and $\vec{v}_P$ are expressions involving only the unknown singularity parameters. If P is made a control point (a panel point at which a boundary condition will be imposed) with index i, equations (9) and (11) give the potential and velocity at that point in terms of (as influenced by) the source and doublet distributions of the single panel j (see fig. 7). Note that the fixed point P and the variable point Q of the analytic formulation correspond to control point i and panel j, respectively, in the discretized implementation. Summing the effects from all the panels on the aircraft surface gives the potential and velocity at control point i in terms of the total number (N) of singularity parameters.

If $\vec{v}_{ij}$ denotes the velocity at control point i, owing to the source-doublet distributions at panel j, then the velocity at point i owing to all N panels is

$$\vec{V}_i = \vec{V}_\infty + \sum_{j=1}^{N} \vec{v}_{ij} \tag{13}$$

If the panel associated with control point i is to be a solid (impermeable) panel represented by a zero normal component of the total velocity, then the boundary condition is

$$\vec{V}_i \cdot \hat{n}_i = \left(\vec{V}_\infty + \sum_{j=1}^{N} \vec{v}_{ij}\right) \cdot \hat{n}_i = 0 \tag{14}$$

Thus, for control point i, we have

$$\sum_{j=1}^{N} \vec{v}_{ij} \cdot \hat{n}_i = -\vec{V}_\infty \cdot \hat{n}_i \tag{15}$$

and since the $\vec{v}_{ij}$ are known (from eq. (11)) in terms of the N singularity parameters, equation (15) can ultimately be expressed as the single equation