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A NUMERICAL METHOD FOR CALCULATING NEAR-FIELD SONIC-BOOM PRESSURE SIGNATURES

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SUMMARY

A numerical method, based on the modified linear-theory analysis of G. B. Whitham, is presented for calculating the complete pressure field of supersonic projectiles or equivalent bodies representing airplane configurations. The method does not involve graphical solutions and is easily adaptable to digital-computer techniques. A description of the method is given and its application is illustrated by use of examples.

INTRODUCTION

G. B. Whitham, in his paper on the flow pattern of a supersonic projectile (ref. 1), developed a method for calculating the complete pressure field of the projectile based on a modified linear-theory analysis. The method proceeded from the assumption that linear theory satisfactorily approximates the magnitude of disturbances contributing to the projectile pressure field but fails to locate these disturbances properly and does not account for the convergence of weak disturbances into shocks of finite strength. Whitham devised a correction to the linearized theory which takes into account variations in local flow directions and velocities and provides a more exact network of flow characteristics to describe the entire flow field, including shocks.

The method of reference 1 for describing the pressure distribution and shock location in the flow field of the projectile is applicable to any slender, axisymmetric body-wake combination. With the addition of a suitable term to account for the effect of lift, the method has been extended to include the analysis of complete aircraft configurations through a consideration of the equivalent slender body (ref. 2). Such an equivalent body generates a very complex pressure field because of inputs from the many component parts of the aircraft. A graphical construction of the pressure signature in the flow field near the aircraft, following the method of reference 1, therefore becomes quite laborious.

This report presents a numerical method, as an alternate to the graphical method, of implementing the theory described in reference 1. The method and its application to high-speed digital-computer programing is described and examples of its use are given.

SYMBOLS

$F(y)$ effective-area-distribution function (see eq. (1))

$F(y_t)$ transposed-effective-area-distribution function

$I(y_t)$ integral of $F(y_t)$, $\int_0^{y_t} F(y_t)\,dy_t$

$k = \frac{(\gamma + 1)M^4}{\sqrt{2}\,\beta^{3/2}}$

K_r reflection factor

M free-stream Mach number

n integer

p reference pressure

Δp incremental pressure due to flow field of slender supersonic body

Δx position of field point relative to undisturbed characteristic from body nose, $x - \beta r$

r perpendicular distance from body to measurement point

S body cross-sectional area

t distance measured along longitudinal axis from body nose, dummy variable of integration for y

x distance from body nose to point in flow field, same origin and units as y

y distance measured along longitudinal axis from body nose

y_t transposed position of $F(y)$, $y - k\sqrt{r}\,F(y)$

y_o value of y giving largest positive value of $I(y)$

$\beta = \sqrt{M^2 - 1}$

γ ratio of specific heats (1.4 for air)

A prime is used to indicate a first derivative with respect to distance and a double prime is used to indicate a second derivative.

DISCUSSION

In reference 1, a method is developed for calculating the pressure field, including the strength and location of the shock waves, produced by a slender, axisymmetric body-wake combination. Fundamental to the method is a basic mathematical function F(y), which is related to the body cross-sectional area development by the following expression:

$$F(y) = \frac{1}{2\pi}\int_0^y \frac{S''(t)\,dt}{\sqrt{y - t}} \tag{1}$$

While equation (1) is strictly applicable only to "smooth" body shapes, it illustrates the general form of the F(y) function. A more complicated expression for F(y), applicable to nonsmooth bodies, is also presented in reference 1. A numerical method of integrating equation (1) to obtain F(y) for smooth bodies of nonanalytic shape is presented elsewhere (e.g., ref. 2), so the calculation of the F(y) function will not be discussed here.

The purpose of the present report is to present a method, easily adaptable to digital-computing techniques, for calculating the pressure signature corresponding to a given F(y) function. To do so, a brief summary of the techniques derived in reference 1 is required, although the reader is referred to reference 1 for a complete description of the theory.

The coordinate system used in the description of the body flow field is illustrated in figure 1. The field point at which it is desired to evaluate the flow-field pressure is referenced to the undisturbed characteristic from the body nose ($\Delta x = x - \beta r$).

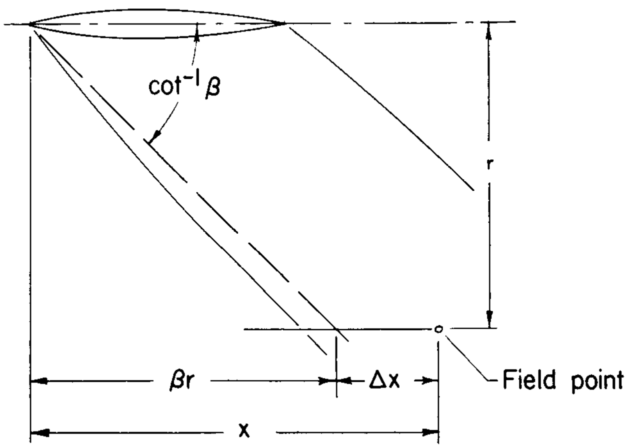

Figure 1.- Flow-field coordinate system.

Pressures within the disturbance field of a body are proportional to its F(y) function, and the location and magnitude of pressure jumps at shock waves within the body flow field are determined from an "area-balancing" technique applied to F(y). Physically, the "area-balancing" situation is required by the condition that (to first order), if two regions of supersonic flow are separated by a shock, the direction of the shock bisects the Mach line directions of the two regions of the flow. Geometrically, the "area-balancing" consists of passing lines of slope $\frac{1}{k\sqrt{r}}$ through the F(y) curve so that the lobes cut off on either side of the curve by the line are equal in area.

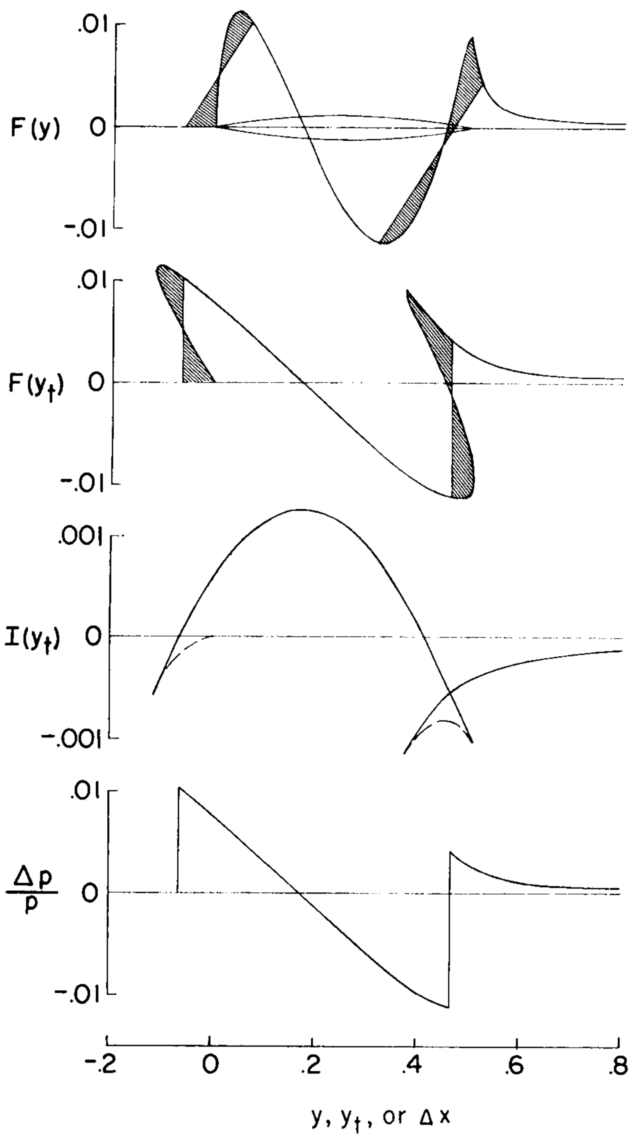

Figure 2.- Development of pressure signature for parabolic body of fineness ratio = 10. M = 1.414; r = 8 body lengths.

The development of a pressure signature from the F(y) curve is illustrated in figure 2 for a parabolic body of revolution of a fineness ratio of 10. The $\frac{1}{k\sqrt{r}}$ line shown in the upper part of the figure corresponds to a Mach number of 1.414 and a radial distance to the signature survey line of 8 body lengths.

The F(y) curve for the parabolic body indicates the presence of two compressive shock waves (denoted by $F'(y) > 0$). The Δx positions of these two shock waves are located by the y value of the intercept of the $\frac{1}{k\sqrt{r}}$ lines defining the shaded (balanced) lobe areas with the y-axis. The corresponding pressures on either side of the shock waves are proportional to the values of F(y) at the extremities of the area-balancing line; the jump in F(y) is proportional to the discontinuous pressure rise across the shock wave. Note that the bow shock is located at a negative value of y which indicates that, for the chosen flow-field survey position, the finite nose disturbance will have advanced that distance ahead of the undisturbed characteristic or Mach line from the nose.

Actually, the shock-wave positions shown are but special cases of the general expression relating the position of the pressure disturbance emanating from a point on the body, at y, to its corresponding position in the flow field x:

$$x = y + \beta r - k\sqrt{r}\, F(y)$$

The value of $k\sqrt{r}\, F(y)$ is thus seen to be the displacement of the pressure value at x from the undisturbed characteristic projected from y (which has the Mach line equation

$x = y + \beta r$); this displacement is the "correction" in disturbance positioning which forms the basis for the theory of reference 1.

The complete $F(y)$ function transposed to its relative position along the y_t-axis by use of the equation $y_t = y - k\sqrt{r}\,F(y)$ is shown in the $F(y_t)$ plot of figure 2. This transposition results in a tilted $F(y)$ function, or $F(y_t)$. In $F(y_t)$, the $\frac{1}{k\sqrt{r}}$ lines are rotated to a vertical attitude, with the two area-balancing lines of the $F(y)$ function now defining the limit, or cutoff, lines of the pressure signature which correspond to the two compressive shock waves of the solution. Again the shaded areas, which now define portions of the $F(y_t)$ function which are discarded in favor of the limit lines, are shown for illustrative purposes. The limit lines thus negate the possibility of a multiple-valued solution.

With the introduction of a suitable constant to convert $F(y)$ to pressure units, the transposed $F(y)$ function becomes the pressure signature for the parabolic body at the selected Mach number and radius. That pressure function is

$$\frac{\Delta p}{p} = \frac{K_r \gamma M^2 F(y)}{\sqrt{2\beta r}} \tag{2}$$

where p is a reference pressure for the atmosphere through which the pressures are propagating, and K_r is a reflection factor (1.0 until the signature comes into contact with a reflective surface). The pressure signature for the parabolic body is shown in the bottom part of figure 2.

The pressure-signature solution for a given $F(y)$ function at a given $\frac{1}{k\sqrt{r}}$ value is, then, primarily a problem of locating balanced lobe areas. A graphical solution is one approach. However, the technique of solving for the "balance-points" of the $F(y)$ function may be greatly facilitated through a consideration of the area under the curve of the transposed $F(y)$ function. With y_t taken as the variable of integration,

$$\left.\begin{aligned}
I(y_t) &= \int_0^{y_t} F(y_t)\,dy_t && (y_t \text{ between } 0 \text{ and } y_{t,1}) \\
I(y_t) &= I(y_{t,1}) + \int_{y_{t,1}}^{y_t} F(y_t)\,dy_t && (y_t \text{ between } y_{t,1} \text{ and } y_{t,2}) \\
I(y_t) &= I(y_{t,2}) + \int_{y_{t,2}}^{y_t} F(y_t)\,dy_t && (y_t \text{ between } y_{t,2} \text{ and } y_{t,3}) \\
I(y_t) &= I(y_{t,n}) + \int_{y_{t,n}}^{y_t} F(y_t)\,dy_t && (y_t \text{ between } y_{t,n} \text{ and } y_{t,n+1})
\end{aligned}\right\} \tag{3}$$

where values of $y_{t,n}$ are successive maximum and minimum values of y_t for individual loops of the $F(y_t)$ curve.

The $I(y_t)$ function for the parabolic body is shown directly under the $F(y_t)$ function. Using $I(y_t)$, the area-balancing process and the location of the shock waves and definition of the $F(y_t)$ cutoff values become remarkably simple. Values of y_t corresponding to a balanced-area situation occur where the $I(y_t)$ curve recrosses itself. It is necessary to consider only right-running legs of $I(y_t)$, that is, portions of the function in which y_t is increasing positively. The significance of the crossovers in $I(y_t)$ is that equal-area lobes are thus isolated, in the same fashion as a planimeter reading returns to its initial value after traversing a figure eight having equal-area lobes. That consideration of the right-running legs only is required may be observed from an inspection of the $F(y_t)$ function. For a shock wave to begin or terminate at a left-running leg would result in an inadmissible multiple-valued pressure signature, which results from the doubling back of the preceding or following lobe.

The bow shock wave is located by the special case of the $I(y_t)$ curve crossing the zero axis, since $I(y_t)$ is necessarily zero ahead of the body zone of influence. This crossover must occur at a negative y_t value for a finite bow shock, since the flow deflection caused by the body requires the bow shock to stand ahead of a Mach line, or zero-disturbance wave, from the body nose.

For the parabolic body, the left-running legs of the $I(y_t)$ function are dashed in for illustrative purposes. With the $I(y_t)$ crossovers used to define the shock-wave locations, the resulting pressure signature may be constructed as outlined previously.

By using standard digital-computing techniques, the construction of the $I(y_t)$ function and search for the crossover points of the right-running legs is easily accomplished. However, the parabolic-body example does not illustrate the shock-wave solution "choice" required by complex $F(y)$ functions normally encountered in the pressure-signature solutions for typical supersonic aircraft. The "choice" comes from multiple shock-wave combinations as the solution is obtained for successively greater radial distances; the various component shock waves combine to produce eventually the characteristic "N-wave" of the asymptotic, or "far-field" type of pressure signature. The crossover "choice" technique is illustrated by a second example.

In figure 3, a rather complex pressure-signature solution is shown for an arbitrarily chosen $F(y)$ function. In this example, five compressive shock waves are indicated (five regions of $F'(y) > 0$). However, two of these shock waves combine with succeeding waves to form a three-shock-wave system for the selected $\frac{1}{k\sqrt{r}}$ value.

The $I(y_t)$ function in figure 3 shows only the right-running legs of the transposed function, the left-running legs having been deleted for clarity. The different crossover points (designated a, b, c, d, and e) on the $I(y_t)$ curve show the existence of five balanced lobe conditions. In the case of the bow-shock solution, point a locates an area balance for the first lobe of the $F(y)$ function; point b locates an area balance for the first two lobes of $F(y)$. Since point b is more negative in y_t than point a, the shock wave corresponding to the first lobe is seen to have combined with a stronger shock wave corresponding to the second lobe. Therefore, the pertinent solution for the bow-shock wave is that of point b, and this combined solution is shown by the shaded areas at the left of the figure. A similar situation is seen to exist for the tail shock, that is, a weaker shock wave is overtaken by a stronger one, as indicated by the value of y_t for point e which is less than that of point d. The solution also includes the fairly weak shock wave located by point c. Thus it may be seen that the condition of shock-wave combination is indicated by the relative positioning of the y_t value for crossover points. For any given right-running leg, the solution must seek out and compare the y_t values corresponding to $I(y_t)$ crossovers with all successive right-running legs. The pertinent solution is the one having the smallest value of y_t, and, as shown in figures 2 and 3, the overall jump in $F(y_t)$ between the extremities of the balance line is proportional to the pressure jump associated with the combined shock wave.

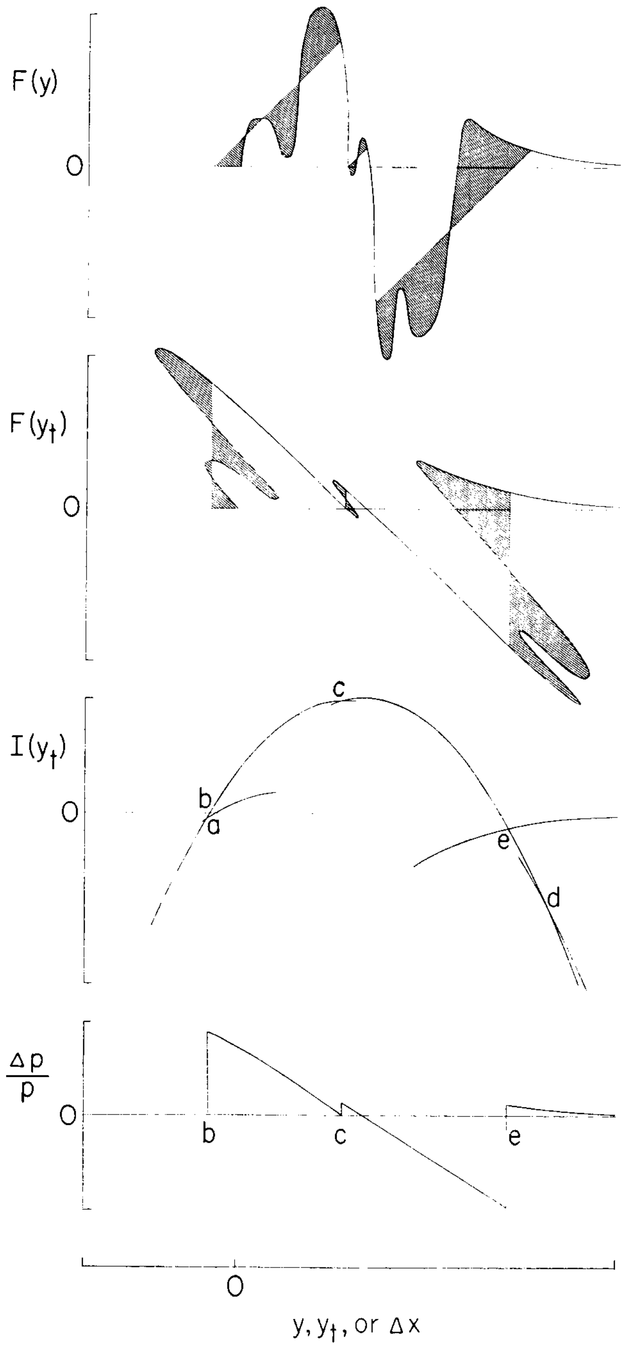

Figure 3.- Development of pressure signature for arbitrary F(y) function.

With respect to the asymptotic, or "far-field" pressure signature, this situation is seen to be reached when the $F(y)$ lobe area to be balanced for the bow-shock solution is that corresponding to the maximum positive area of the $F(y)$ function. The value of y associated with the maximum value of the integral of $F(y)$ is termed y_o. In the transposed-function solution, the far-field situation is approached when the most negative bow-shock crossover point in $I(y_t)$ involves the right-running leg which includes y_o in the original $F(y)$ function (or, physically interpreted, the far-field situation is approached when all bow shocks have combined). The far-field condition is actually achieved when the value of $F(y)$ corresponding to the bow-shock solution is defined by the maximum positive area under the $F(y)$ curve:

$$F(y)_{far\text{-}field} = \sqrt{\frac{2\int_0^{y_o} F(y)\,dy}{k\sqrt{r}}} \tag{4}$$

or, introducing equation (2) and the definition of k,

$$\left(\frac{\Delta p}{p}\right)_{far\text{-}field} = \frac{1.075\beta^{1/4}K_r\sqrt{\int_0^{y_o} F(y)\,dy}}{r^{3/4}} \tag{5}$$

Summarizing, the digital-computer technique of solving for the pressure signature for a given $F(y)$ function (given as a series of discrete points) and $\frac{1}{k\sqrt{r}}$ line may be stated as follows:

(1) Solve for y_t corresponding to each value of $F(y)$, with $y_t = y - k\sqrt{r}\,F(y)$. Tabulate $F(y_t)$ as a function of y_t.

(2) Using equation (3) and standard machine techniques, solve for and tabulate the running integral of $I(y_t)$ for all values of y_t.

(3) Isolate the successive right-running legs of $F(y_t)$ and $I(y_t)$, that is, sequences in $F(y_t)$ in which $y_{t,n}$ is greater than $y_{t,n-1}$.

(4) Using standard interpolation techniques, obtain the bow-shock solution by finding the most negative value of y_t for which $I(y_t) = 0$. This value of y_t locates the shock wave; the corresponding value of $F(y_t)$ substituted in equation (2) defines the pressure jump across the shock wave and the corresponding value of $y = y_b$ serves as a starting point for the search for other shock-wave solutions, where

$$y_b = y_{t_{bow\ shock}} + k\sqrt{r}\,F(y)_{bow\ shock}$$

(5) Selecting the right-running leg containing y_b in the original $F(y)$ function as a base leg, solve for all $I(y_t)$ crossovers between the base leg and all succeeding legs $\left(\text{legs for which } y > y_b\right)$. The pertinent solution thus obtained is that having the smallest crossover value of y_t. The shock wave is thus located at this smallest y_t, and the pressure ahead of the shock is proportional to the value of $F(y)$ for the base leg and the pressure behind the shock is proportional to the value of $F(y)$ for the selected succeeding leg.

(6) Repeat step (5), working with the right-running leg at which the solution of step (5) terminates. If crossovers are found, the shock wave is located as it was in step (5). This process must be continued until the last right-running leg is selected as a base leg.

(7) The shock-wave signature is defined by discarding the cutoff lobes of the $F(y_t)$ function, inserting the corresponding limit-line solutions, and converting $F(y_t)$ to pressure units by using equation (2). Intermediate points in $F(y_t)$ between shock-wave solutions are added to complete the pressure signature. Here it is useful to recall the correspondence between y and y_t in discarding or retaining $F(y)$ points; points are discarded if they lie in a range of y "jumped" by a y_t solution.

CONCLUDING REMARKS

A numerical method, based on the modified linear-theory analysis of G. B. Whitham, has been presented for calculating the complete pressure field of a supersonic projectile or an equivalent body representing an airplane configuration. The solution based on a transposed effective-area-distribution function avoids graphical steps and is thus easily adaptable to digital-computer techniques. Application of the method has been illustrated by use of examples.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., July 29, 1965.

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