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A MODIFIED THEODORSEN ε-FUNCTION AIRFOIL DESIGN PROCEDURE

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16. Abstract

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A MODIFIED THEODORSEN ϵ-FUNCTION AIRFOIL DESIGN PROCEDURE

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SUMMARY

The Theodorsen theory of airfoil design for incompressible flow can be used with the modifications proposed in this paper to design airfoils that satisfy a much wider variety of pressure variations than are permitted by the original Theodorsen procedure. Several examples illustrating this method are computed and discussed.

INTRODUCTION

A number of methods are available for the design of airfoils for low speed application. (See refs. 1 to 3 and the references therein.) The ϵ-function method of Theodorsen (ref. 4) has a number of desirable features, not all of which are shared by other methods. With this method it is possible to specify not only the form of the pressure distribution, but also various combinations of characteristic parameters such as the design lift coefficient, the ideal angle of attack, the angle of zero lift, the location of the aerodynamic center, and the pitching moment about the aerodynamic center. Since these parameters are expressed simply in terms of ϵ and its conjugate function ψ, the designer can control them in the design process.

First, an airfoil having a pressure distribution that roughly approximates the one desired is selected; then a modification to the pressure distribution is prescribed and the corresponding change in airfoil coordinates is computed. The problem that arises in applying the Theodorsen method is to find a pressure distribution modification that approximates the one desired and also satisfies two required mathematical constraints. In the present paper this problem is handled by a device that allows small modifications to be made in the pressure distribution almost arbitrarily, but there is a small perturbation in the pressure distribution on other regions of the airfoil. The Theodorsen design analysis is simple and direct in application when this procedure is used.

SYMBOLS

A proportionality factor (see eq. (8))

a arbitrary scale parameter

c	airfoil chord
c_l	airfoil lift coefficient
c_p	airfoil pressure coefficient
$P_s = \frac{1}{2}\left(\frac{v}{V}\right)^2$	
R, φ	polar coordinates of exact-circle transformation of airfoil
r, θ	polar coordinates of near-circle transformation of airfoil
V	undisturbed free-stream velocity
v	local velocity
$X = x - x_o$	
x, y	rectangular coordinates of airfoil in physical plane
x_o	value of x at leading edge of airfoil
α	angle of attack
Δ	increment
E	discontinuity in ϵ-function due to failure to satisfy constraint (see eq. (11))
ϵ	function relating angular coordinates of near-circle and exact-circle airfoil transformations
ϵ_A	error in integral of variation of ϵ-function due to failure to satisfy constraint
$\epsilon_a, \epsilon_{aa}$	ϵ-functions adjusted to satisfy constraints
ϵ_N	value of ϵ at airfoil leading edge
$\epsilon_o(\theta)$	computed approximation to desired ϵ-function before adjustments are made to satisfy constraints

ϵ_T value of ϵ at airfoil trailing edge

ψ function relating radial coordinates of near-circle and exact-circle airfoil transformations

ψ_0 average value of ψ

A prime with a symbol indicates derivative with respect to θ.

An asterisk with a symbol denotes dummy integration variable.

THEORETICAL CONSIDERATIONS

The Theodorsen airfoil theory (ref. 3) involves a conformal transformation of the airfoil (defined by rectangular coordinates x,y) into a shape approximating a circle (see fig. 1). In the transformed plane this near circle is described by polar coordinates r,θ. The approximate circle is then transformed into an exact circle having coordinates R,φ. The function ψ and the constant ψ_0 are defined by $r = ae^{\psi}$ and $R = ae^{\psi_0}$, respectively. Theodorsen shows that ψ_0 is the average value of ψ, that is,

$$\psi_0 = \frac{1}{2\pi} \int_0^{2\pi} \psi \, d\varphi$$

and that the functions $\psi - \psi_0$ and $\epsilon \equiv \varphi - \theta$ are related by the equations

$$\epsilon = -\frac{1}{2\pi} \int_0^{2\pi} \psi \cot \frac{\varphi^* - \varphi}{2} \, d\varphi^* \tag{1}$$

$$\psi - \psi_0 = \frac{1}{2\pi} \int_0^{2\pi} \epsilon \cot \frac{\varphi^* - \varphi}{2} \, d\varphi^* \tag{2}$$

The function ψ is directly related to the airfoil coordinates in the physical plane by

$$x = 2a \cosh \psi \cos \theta \tag{3a}$$

$$y = 2a \sinh \psi \sin \theta \tag{3b}$$

With the use of these equations, ψ can be written

$$\psi = \ln\left(\frac{x}{2a \cos \theta} + \frac{y}{2a \sin \theta}\right) \tag{4}$$

The relation between θ and the airfoil coordinates (see ref. 5) is given by

$$2 \sin^2 \theta = k + \sqrt{k^2 + \left(\frac{y}{a}\right)^2} \tag{5a}$$

where

$$k = 1 - \left(\frac{x}{2a}\right)^2 - \left(\frac{y}{2a}\right)^2 \tag{5b}$$

Expressions (4) and (5) are sufficient to compute ψ as a function of θ. Then $\epsilon(\theta)$ is obtained from $\psi(\theta)$ by replacing φ^* and φ in equation (1) with θ^* and θ, respectively. Henceforth in this paper, ϵ will refer to $\epsilon(\theta)$ which, according to reference 6, is a close approximation to the exact function defined by equation (1). This approximation yields a very slight error in the velocity equation; that is,

$$\frac{v}{V} = \frac{\left[\sin(\alpha + \epsilon + \theta) + \sin(\alpha + \epsilon_T)\right]\left(1 + \epsilon'\right)e^{\psi_o}}{\sqrt{\left(\sinh^2 \psi + \sin^2 \theta\right)\left(1 + \psi^2\right)}} \tag{6}$$

In reference 4 Theodorsen argues that the quantity

$$P_s = \frac{1}{2}\left(\frac{v}{V}\right)^2 = \frac{1}{2}\left(1 - c_p\right) \tag{7}$$

can be written

$$P_s = A(1 + \epsilon')^2 \approx A(1 + 2\epsilon') \tag{8}$$

where A is a function of position only. Then

$$\Delta P_s = -\frac{1}{2} \Delta c_p \approx 2A \, \Delta\epsilon' = 2A \frac{d \, \Delta\epsilon}{d\theta} \tag{9a}$$

and

$$\frac{\Delta P_s}{P_s} = \frac{2A \frac{d\,\Delta\epsilon}{d\theta}}{A(1 + 2\epsilon')} \approx 2\,\frac{d\,\Delta\epsilon}{d\theta} \tag{9b}$$

Solving for $d\,\Delta\epsilon$ and integrating gives the variation in ϵ as

$$\Delta\epsilon(\theta) = \frac{1}{2}\int_{\theta_1}^{\theta} \frac{\Delta P_s}{P_s}\,d\theta \tag{10}$$

where θ_1 is the initial angular coordinate of the region of change. The error in this approximation arises primarily in equation (9), in which the quantity A does not remain constant when ϵ is varied, as assumed. However, A is not as sensitive to variations in ϵ as is ϵ' and, consequently, such an approximation appears to be a good one.

The problem that arises in the use of equation (9) is that the desired variations in pressure ΔP_s cannot be arbitrarily prescribed, but are subject to two strict constraints. First, the ϵ-function is to remain unchanged outside the prescribed region of change. Furthermore, within the region of change,

$$\int_{\theta_1}^{\theta_2} \frac{\Delta P_s}{P_s}\,d\theta = 0$$

where the integration is over the entire region of change. The second constraint is that

$$\int_{\theta_1}^{\theta_2} \Delta\epsilon\,d\theta = 0$$

where again the integral is over the region of change.

In general, desired variations in the pressure distribution do not satisfy these constraints and a straight-forward trial-and-error process to find an approximation to the desired variation that satisfies the constraints can lead to many time-consuming iterations. Therefore, the possibility of violating the constraints needs to be explored.

These constraints arise, in the first place, from the fact that the ϵ-function of any airfoil can be expressed as a Fourier series without a constant term. As a result, the ϵ-function must satisfy two conditions:

(1) It must be periodic and consequently $\epsilon(2\pi) = \epsilon(0)$.

(2) Its integral over $(0, 2\pi)$ must be zero.

Inasmuch as the original ϵ-function satisfies these two conditions, any variation in ϵ must satisfy the constraints stated by Theodorsen in order that the revised ϵ-function correspond to an actual airfoil.

Now, assume that an arbitrary small variation ΔP_s is prescribed such that both constraints are violated; then the integral

$$E = \int_{\theta_1}^{\theta_2} \frac{\Delta P_s}{P_s}\, d\theta = \int_0^{2\pi} \frac{\Delta P_s}{P_s}\, d\theta \neq 0$$

Consequently,

$$\epsilon(2\pi) - \epsilon(0) = E \qquad (11)$$

A simple way to eliminate this discontinuity is to alter the entire revised ϵ-function with a linear adjustment expressed as

$$\epsilon_a(\theta) = \epsilon_o(\theta) - \frac{E\theta}{2\pi}$$

where $\epsilon_o(\theta)$ is the originally computed ϵ-function which violates the constraints.

Now the first constraint is satisfied by the function $\epsilon_a(\theta)$, but the velocity has been altered slightly outside the originally prescribed region of change. However, small changes in ϵ influence the velocity primarily through changes in ϵ', which for this variation equals $-E/2\pi$. Because this quantity is quite small and constant, the alteration in the velocity outside the prescribed region of change should be slight and smooth.

At this point the second constraint is still violated; that is,

$$\epsilon_A = \int_{\theta_1}^{\theta_2} \Delta\epsilon\, d\theta = \int_0^{2\pi} \Delta\epsilon\, d\theta \neq 0 \qquad (12)$$

where now

$$\Delta\epsilon = \epsilon_a(\theta) - \epsilon_o(\theta)$$

The simplest way to restore this constraint is to subtract a constant from ϵ_a:

$$\epsilon_{aa}(\theta) = \epsilon_a(\theta) - \frac{\epsilon_A}{2\pi}$$

This adjustment has an insignificant effect on the basic shape of the pressure distribution since $\epsilon'_{aa} = \epsilon'_a$. It is interesting to note that this adjustment also alters the angle of attack zero lift, which is $-\epsilon_T$, and the ideal angle of attack $\frac{\epsilon_N + \epsilon_T}{2}$ but not the design lift coefficient which depends on $\epsilon_T - \epsilon_N$.

Thus, for arbitrary small prescribed changes in P_s, the constraints on ϵ can be satisfied with simple adjustments that result in smooth slight variations in the pressure distribution outside the region of change.

The $\psi - \psi_o$ function corresponding to the adjusted ϵ-function ϵ_{aa} is determined from equation (2), and then the coordinates of the revised airfoil are given by equations (3).

CALCULATION PROCEDURE

The design calculation proceeds according to the following steps:

1. Compute distribution of c_p and $\epsilon(\theta)$ for the original airfoil.
2. Prescribe desired pressure distribution and compute the difference Δc_p between the desired and original distributions.
3. From c_p and Δc_p, compute $\Delta P_s/P_s$ with the use of equations (7) and (9).
4. Compute $\Delta\epsilon(\theta)$ from equation (10) and add this function to the original ϵ-function. to obtain the revised $\epsilon(\theta)$.
5. Adjust this function to satisfy the required contraints.
6. Compute the revised $\psi(\theta)$ according to equation (2).
7. Compute the new airfoil coordinates by means of equations (3).
8. Return to step 1 and iterate the calculation.

EXAMPLES

In the first example (see fig. 2), the original airfoil was 12 percent thick with maximum thickness at 0.40 chord, leading-edge radius equal to 0.055, design $c_l = 0.2$, and maximum camber line ordinate at 0.35 chord. The variation in the pressure distribution on the lower surface shown in figure 2(a) was prescribed to reduce the suction near the leading edge, and within the prescribed region of change, the desired distribution is very nearly attained (see fig. 2(b)). Outside the prescribed region of change, the new pressure distribution deviates somewhat from the original distribution because the prescribed change did not satisfy the constraints. Nevertheless, the pressure distribution obtained has the desired form, and the pressure near the leading edge on the lower surface is now positive. Figure 2(c) gives a comparison of the pressure distributions of the original and modified airfoils computed by the method discussed in reference 7. This method includes the boundary-layer calculation. Figure 2(c) also depicts the original and modified airfoil sections.

The results presented in figure 3 demonstrate that the method is also applicable at a high angle of attack. A modification was prescribed in the pressure distribution at $\alpha = 10^{\circ}$. Again the prescribed distribution is not obtained exactly, but a reasonable approximation is obtained. The capability to design for performance at a large angle of attack is a feature which is potentially useful in the design of airfoils for high maximum lift values.

A somewhat different type of problem is illustrated by the example shown in figure 4. Here the goal was to modify the airfoil in such a way as to keep the basic form of the pressure distribution and to maintain the same maximum thickness but to increase the lift by 20 percent. This purpose could be accomplished by using the method described in reference 8. However, the procedure to be used here, which is based on the Theodorsen airfoil modification analysis, is applicable to larger variations than the method of reference 8. A linear variation $0.2\epsilon_T\left(\frac{2\theta}{\pi} - 1\right)$, where $0 < \theta < \pi$, is added to the ϵ-function for the upper surface, and a similar linear function $0.2\epsilon_T\left(3 - \frac{2\theta}{\pi}\right)$, where $\pi < \theta < 2\pi$, is added to that part of the ϵ-function corresponding to the lower surface. This variation satisfies the conditions that it is continuously periodic and that its integral over $(0, 2\pi)$ is zero. Furthermore, Theodorsen's argument that local variations in pressure are proportional to local variations in ϵ' indicates that the variation in c_p should be nearly constant where c_p varies gradually. The results shown in figure 4 indicate that such is the case except for the regions very near the leading and trailing edges. Thus, the basic character of the pressure distribution is retained, while the lift is increased. A comparison of the airfoil profiles is given in figure 4(a), and the ϵ and ψ functions for the original and modified airfoils are shown in figures 4(b) and 4(c), respectively.

The effect of this modification is to decrease the angle of zero lift $-\epsilon_T$ by 20 percent and thus increase the lift at zero angle of attack by 20 percent. The design lift coefficient is increased by 20 percent, but the ideal angle of attack is unchanged.

CONCLUDING REMARKS

The Theodorsen theory of airfoil design for incompressible flow can be used, with modifications proposed herein, to design airfoils that satisfy a much wider variety of pressure variations than permitted by the original Theodorsen procedure. This method starts with an airfoil whose pressure distribution roughly approximates that desired and revises the airfoil shape so that the resulting pressure distribution is a better approximation to the desired distribution. The four examples which have been computed and discussed illustrate this method.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., August 5, 1974.

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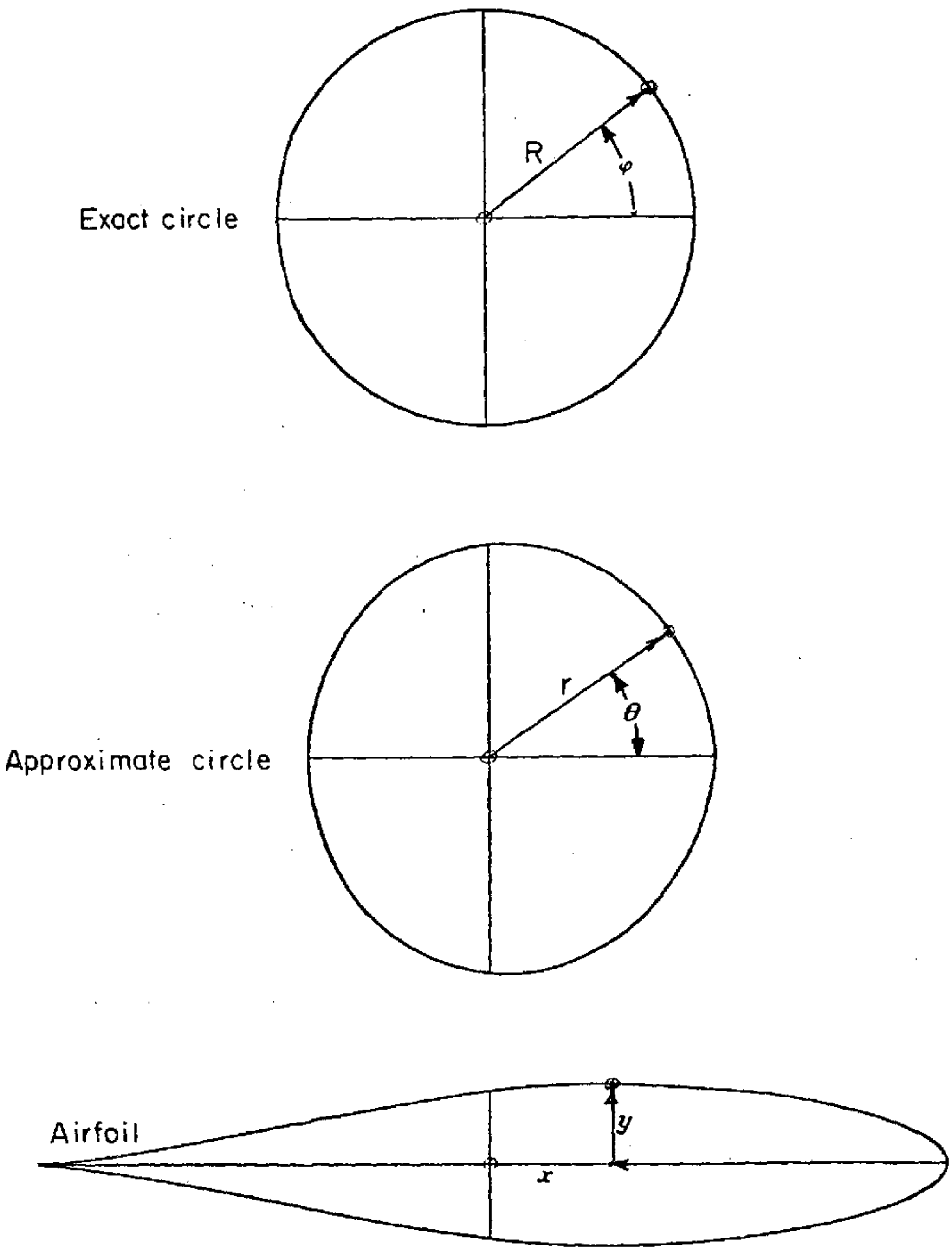
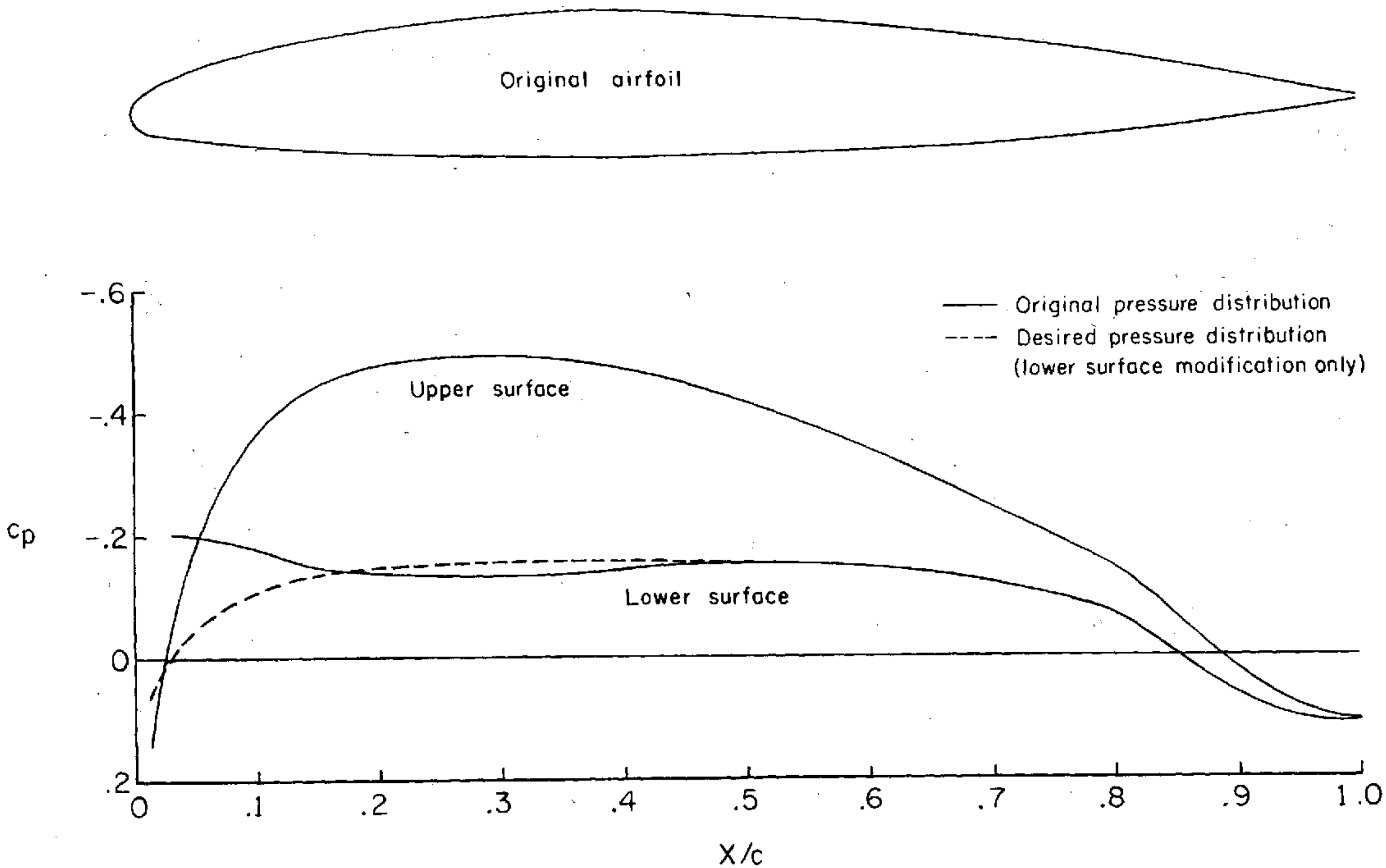
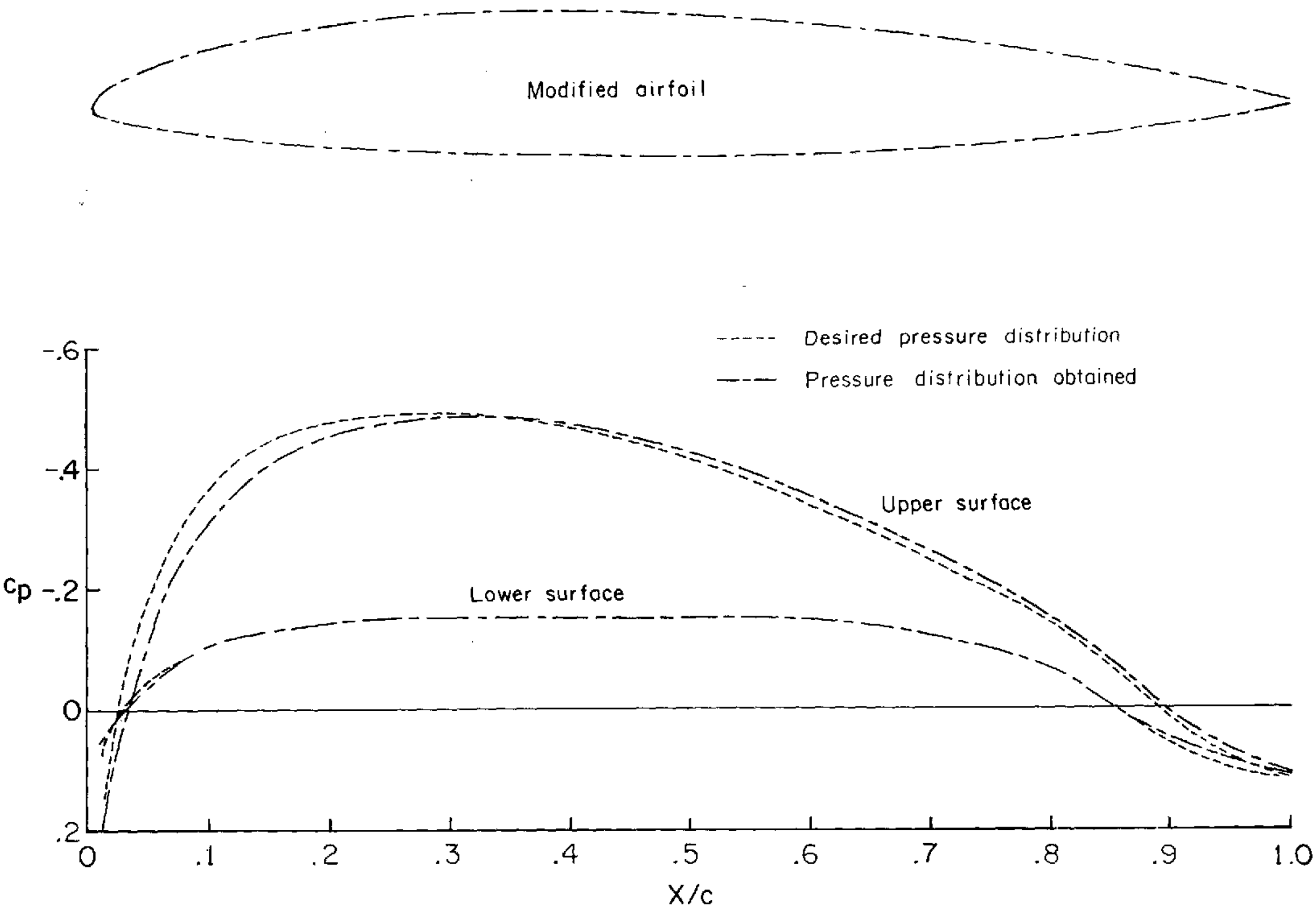


Figure 1.- Transformed planes used to derive airfoils and calculate pressure distributions.



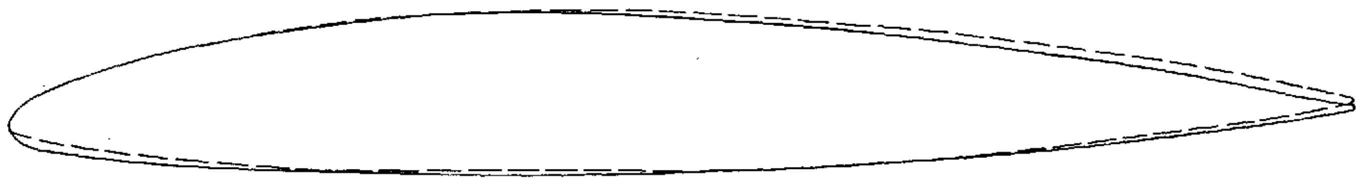
(a) Original airfoil section, original pressure distribution, and desired modification.

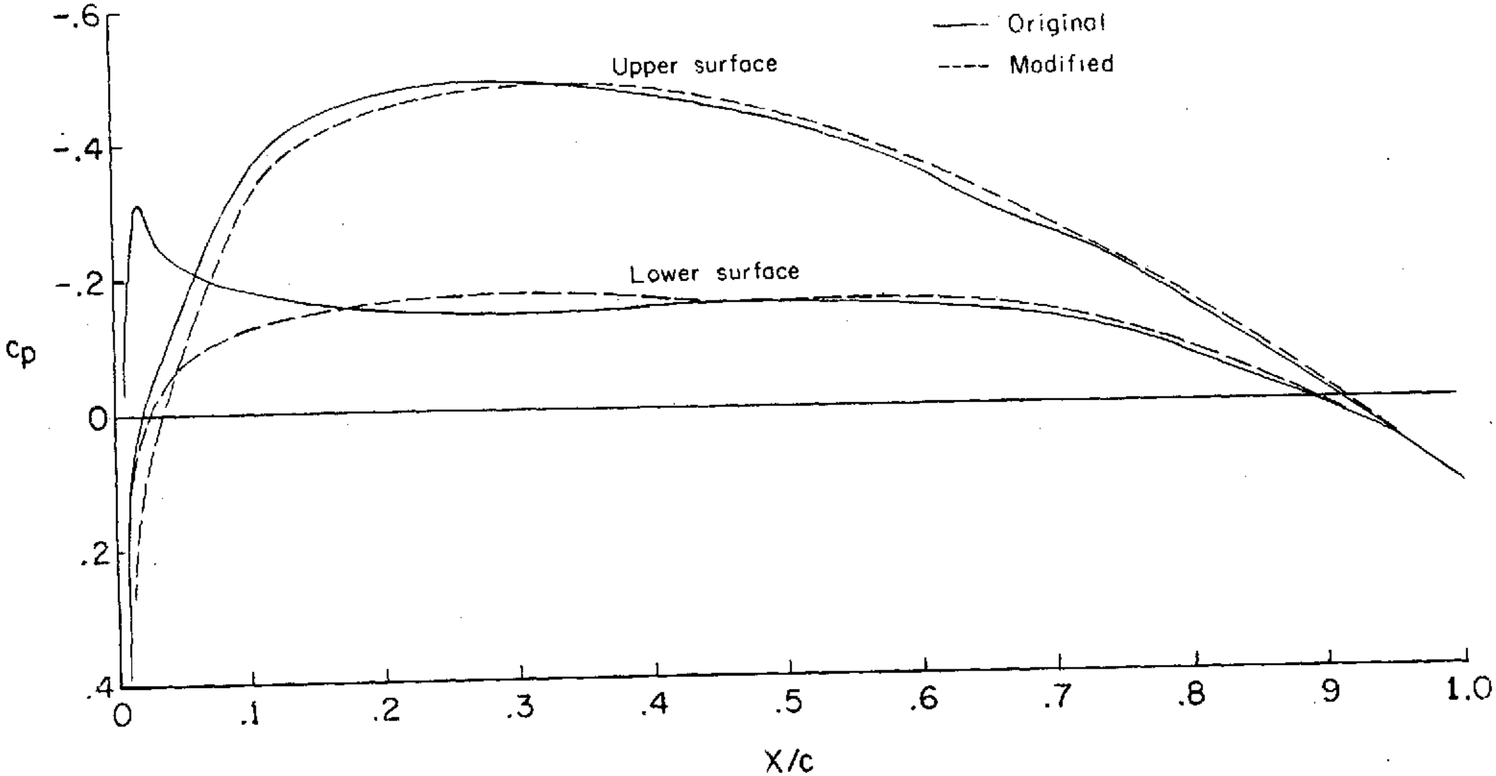
Figure 2.- Illustration of application of the method at zero angle of attack. Thickness ratio = 0.12.



(b) Modified airfoil and comparison of desired pressure distribution with that obtained.

Figure 2.- Continued.





(c) Comparison of original and modified airfoils and their pressure distributions as evaluated by the viscous flow method of reference 7.

Figure 2.- Concluded.

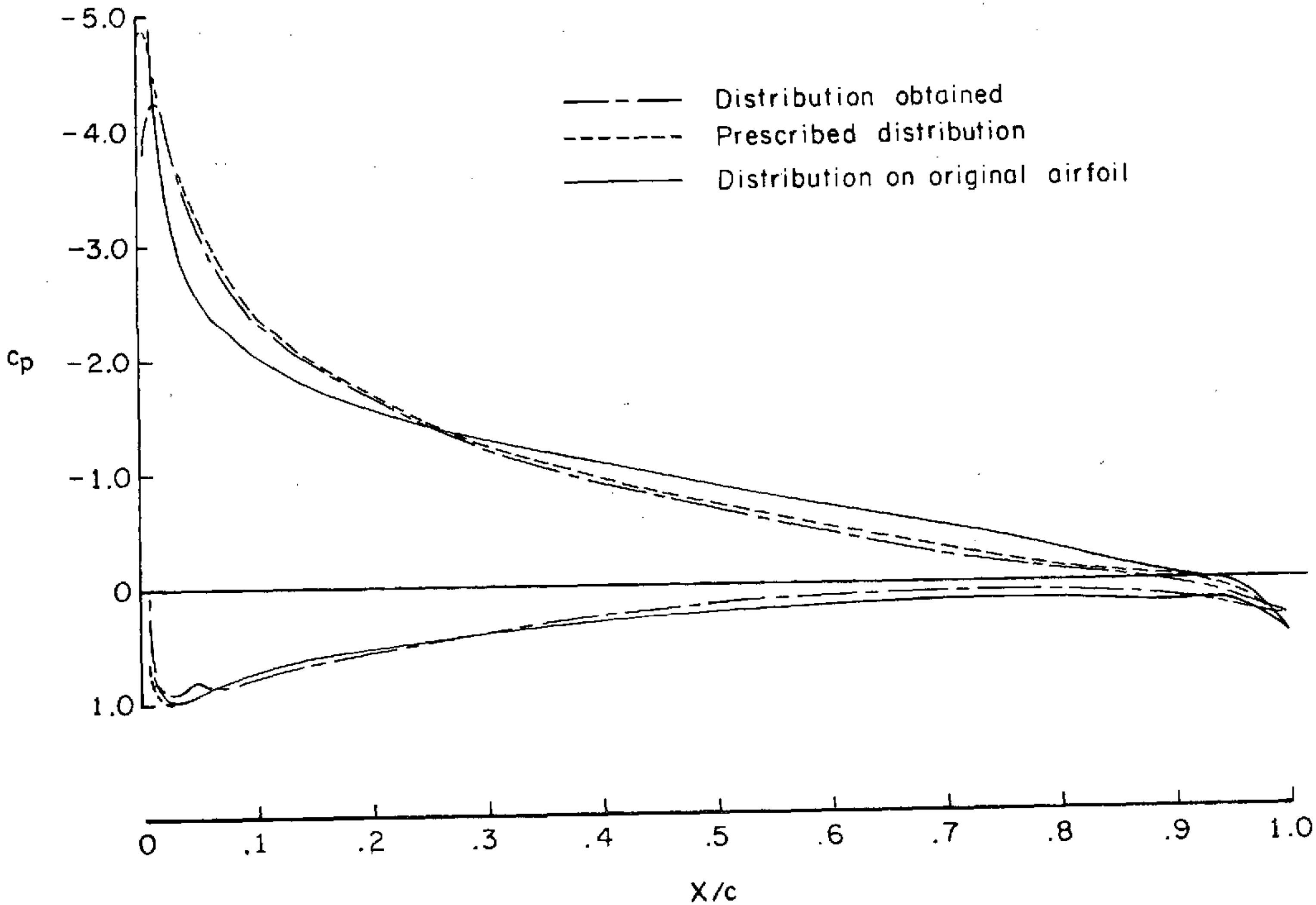
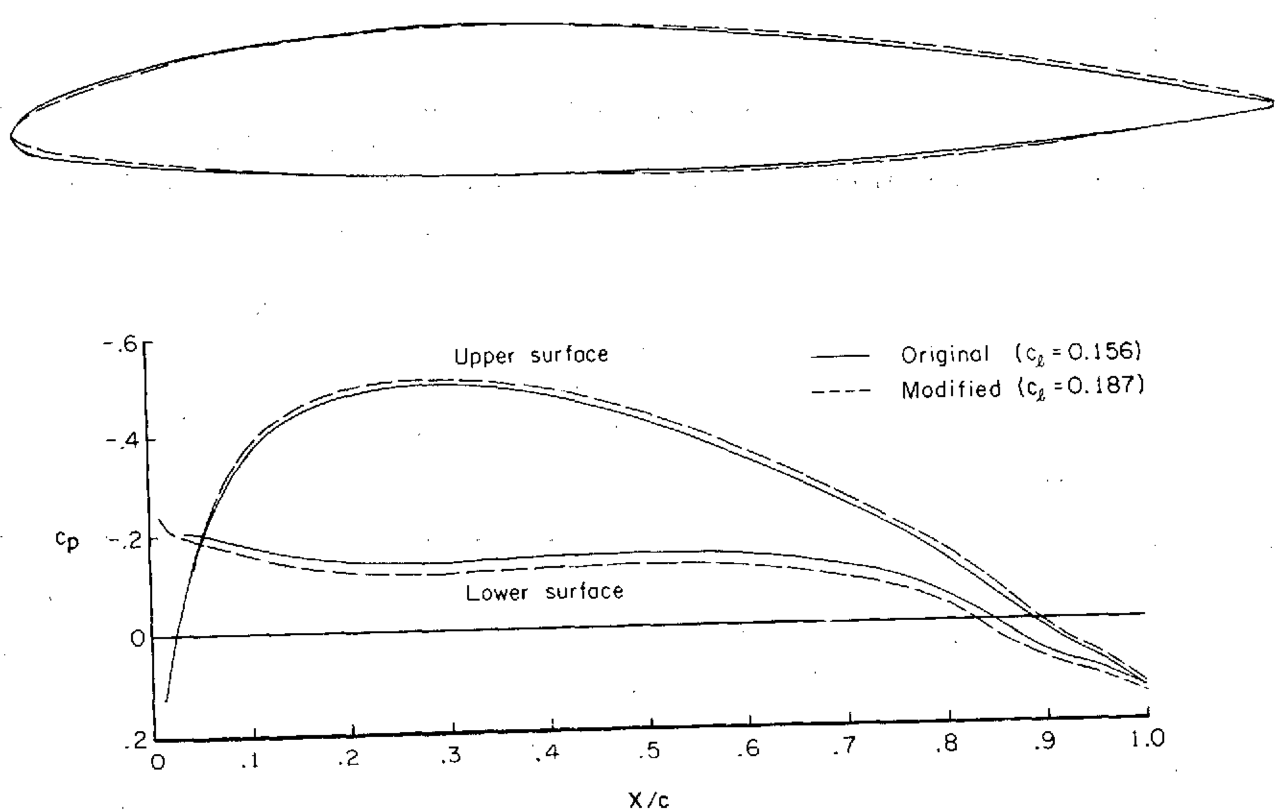
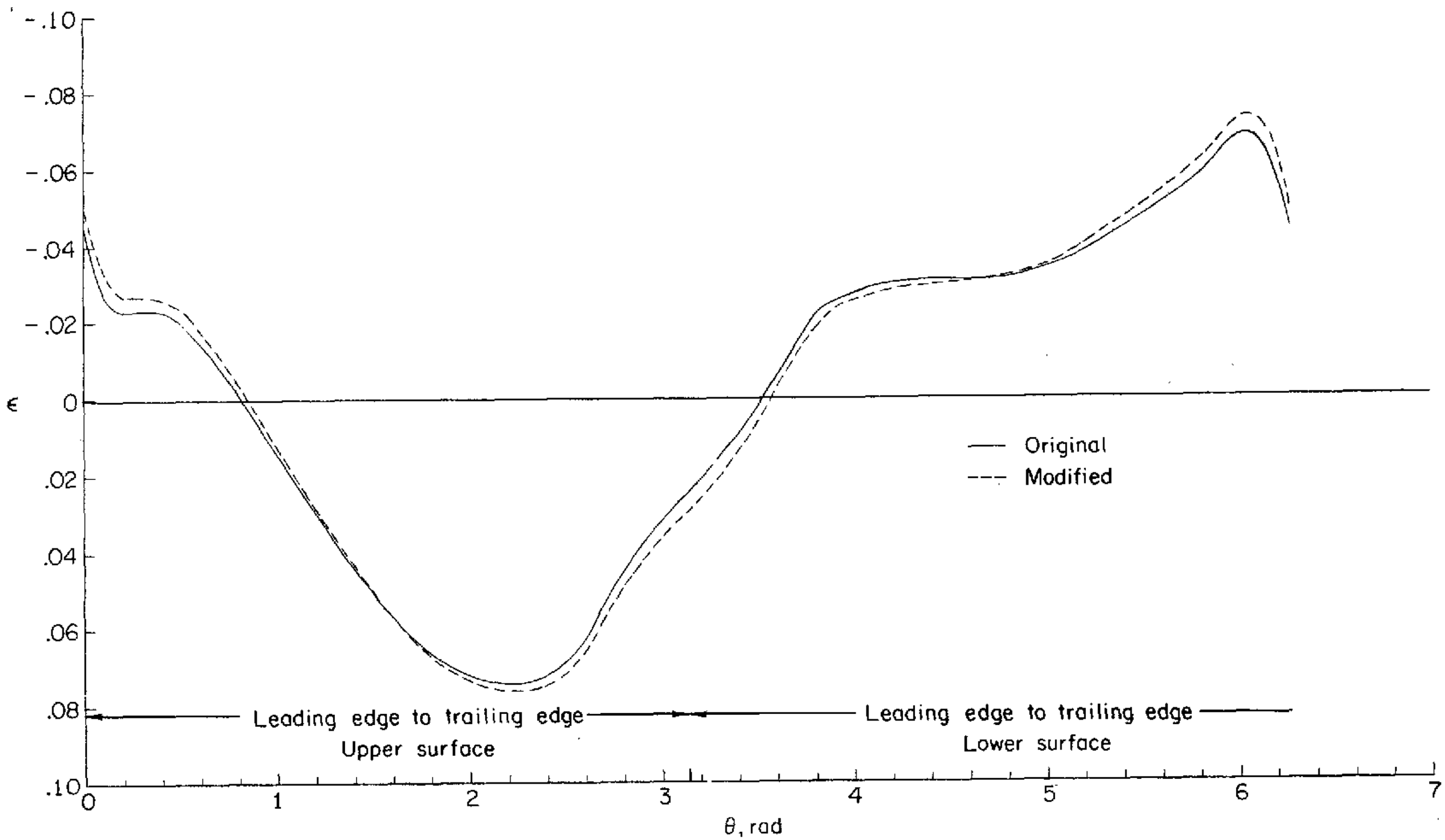


Figure 3.- Illustration of application of the method to pressure distribution modification at $\alpha = 10^{0}$.



(a) Comparison of airfoil sections and of pressure distributions.

Figure 4.- Illustration of application of the method to obtain an increase in lift without an increase in thickness.



(b) Comparison of ϵ functions.

Figure 4.- Continued.

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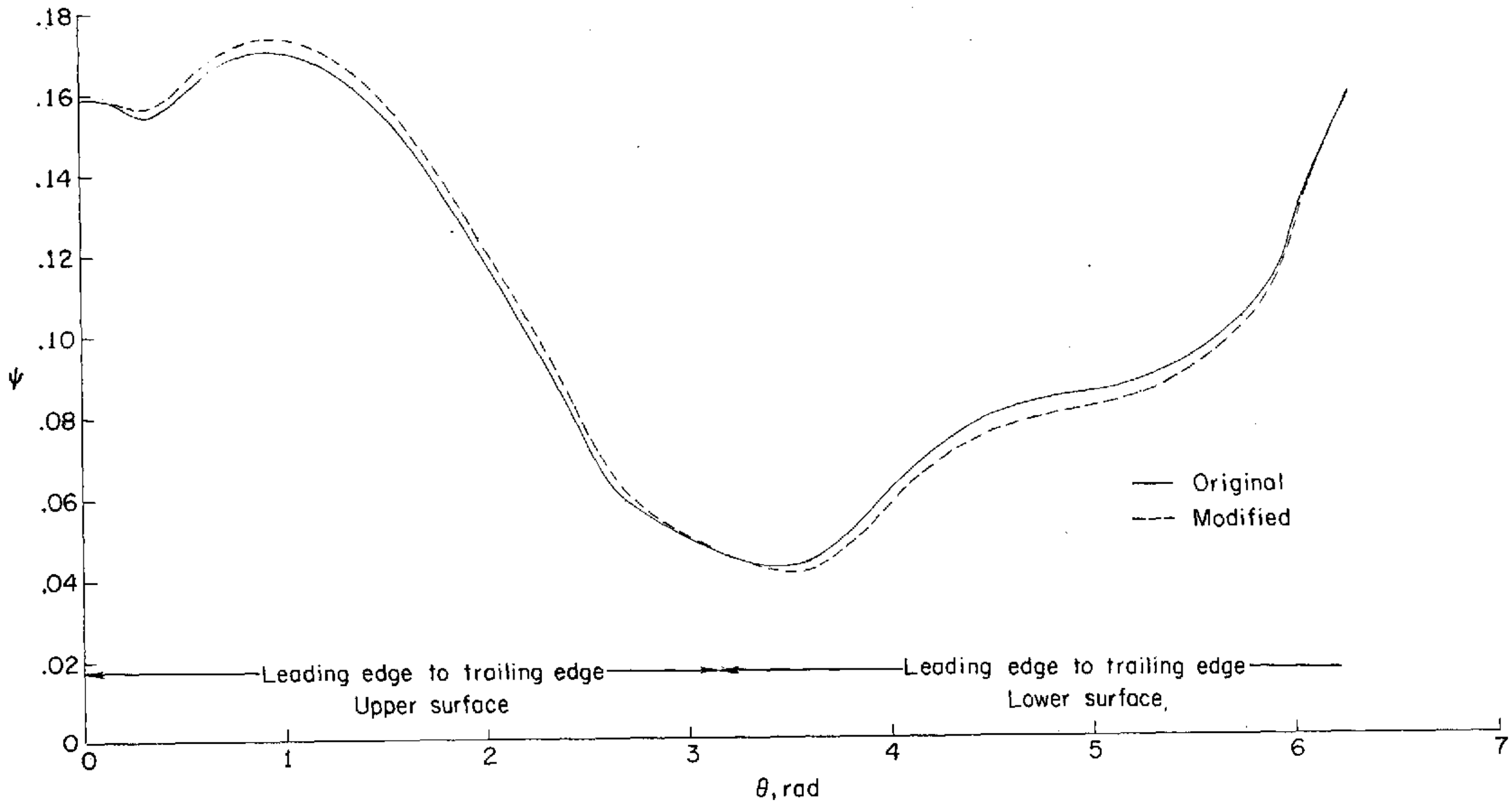

(c) Comparison of ψ functions.

Figure 4.- Concluded.