# Computer Program To Obtain Ordinates for NACA Airfoils 

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#### Abstract

Computer programs to produce the ordinates for airfoils of any thickness, thickness distribution, or camber in the NACA airfoil series were developed in the early 1970's and are published as NASA TM X-3069 and TM X-3284. For analytic airfoils, the ordinates are exact. For the 6 -series and all but the leading edge of the $6 A$-series airfoils, agreement between the ordinates obtained from the program and previously published ordinates is generally within $5 \times 10^{-5}$ chord. Since the publication of these programs, the use of personal computers and individual workstations has proliferated. This report describes a computer program that combines the capabilities of the previously published versions. This program is written in ANSI FORTRAN 77 and can be compiled to run on DOS, UNIX, and VMS based personal computers and workstations as well as mainframes. An effort was made to make all inputs to the program as simple as possible to use and to lead the user through the process by means of a тепи.


## Introduction

Although modern high-speed aircraft generally make use of advanced NASA supercritical airfoil sections, there is still a demand for information on the NACA series of airfoil sections, which were developed over 50 years ago. Computer programs were developed in the early 1970's to produce the ordinates for airfoils of any thickness, thickness distribution, or camber in the NACA airfoil series. These programs are published in references 1 and 2. These programs, however, were written in the Langley Research Center version of FORTRAN IV and are not easily portable to other computers. The purpose of this paper is to describe an updated version of these programs. The goal was to combine both programs into a single program that could be executed on a wide variety of personal computers and workstations as well as mainframes. The analytical design equations for both symmetrical and cambered airfoils in the NACA 4-digitseries, 4-digit-modified-series, 5-digit-series, 5-digit-modified-series, 16 -series, 6 -series, and 6A-series airfoil families have been implemented. The camber-line designations available are the 2-digit, 3-digit, 3-digit-reflex, 6 -series, and 6A-series. The program achieves portability by limiting machine-specific code. An effort was made to make all inputs to the program as simple as possible to use and to lead the user through the process by means of a menu.

## Symbols

The symbols in parentheses are the ones used in the computer program and in the computer-generated listings (.rpt file).

A
camber-line designation, fraction of chord from leading edge over which design load is uniform
$a_{i} \quad$ constants in airfoil equation, $i=0,1, \ldots, 4$

| $b_{i}$ | constants in camber-line equation, $i=0$, 1,2 |
| :---: | :---: |
| $c(\mathrm{C}, \mathrm{CHD})$ | airfoil chord |
| $c_{l, i}(\mathrm{CLI})$ | design section lift coefficient |
| $d_{i}$ | constants in airfoil equation, $i=0,1,2,3$ |
| $d x$ | derivative of $x$; also basic selectable interval in profile generation |
| $d(x / c), d y, d \delta$ | derivatives of $x / c, y$, and $\delta$ |
| I | leading-edge radius index number |
| $k_{1}, k_{2}$ | constants |
| $m$ | chordwise location for maximum ordinate of airfoil or camber line |
| $p$ | maximum ordinate of 2-digit camber line |
| $R$ | radius of curvature |
| $R_{l e}(\mathrm{RLE})$ | leading-edge radius |
| $r$ | chordwise location for zero value of second derivative of 3-digit or 3-digit-reflex camber-line equation |
| $t$ | thickness |
| $x(\mathrm{X})$ | distance along chord |
| $y(\mathrm{Y})$ | airfoil ordinate normal to chord, positive above chord |
| $z$ | complex variable in circle plane |
| $z^{\prime}$ | complex variable in near-circle plane |
| $\delta$ | local inclination of camber line |
| $\varepsilon$ | airfoil parameter, $\phi-\theta$ |
| $\zeta$ | complex variable in airfoil plane |
| $\theta$ | angular coordinate of $z^{\prime}$ |
| $\phi$ | angular coordinate of $z$ |
| $\psi$ | airfoil parameter determining radial coordinate of $z^{\prime}$ |

$\psi_{0} \quad$ average value of $\psi, \frac{1}{2 \pi} \int_{0}^{2 \pi} \psi d \phi$
Subscripts:

| cam | cambered |
| :--- | :--- |
| $l(\mathrm{~L})$ | lower surface |
| $N$ | forward portion of camber line |
| $T$ | aft portion of camber line |
| t | thickness |
| $u(\mathrm{U})$ | upper surface |
| $x$ | derivative with respect to $x$ |

## Computer Listing Symbols

For reasons having to do with code portability, the computer-generated listing (.rpt file) will always have the alphabetic characters in upper case. The following list is intended to eliminate any confusion.

| A | camber-line designation, fraction of chord <br> from leading edge over which design load <br> is uniform |
| :--- | :--- |
| A0,...,A4 | constants in airfoil equation |
| CHD, C | airfoil chord |
| CLI | design section lift coefficient |
| CMB | maximum camber in chord length, |
|  | $p=\frac{y(m)}{c}$ |

CMBNMR number of camber lines to be combined in 6 - and 6A-series multiple camber-line option
CMY $\quad m$, location of maximum camber
CRAT cumulative scaling of EPS, PSI, $\Pi$ RAT(I), $\mathrm{I}=1 \rightarrow$ IT

D0,...,D3 constants in airfoil equation
DX basic selectable interval in profile generation
DY/DX first derivative of $y$ with respect to $x, \frac{d y}{d x}$
D2Y/DX2 second derivative of $y$ with respect to $x$, $\frac{d^{2} y}{d x^{2}}$
EPS airfoil parameter, $\varepsilon=\phi-\theta$
IT number of iterations to converge 6-series profile
K1, K2 3 -digit-reflex camber parameters, $k_{1}$ and $k_{2}$
PHI $\quad \phi$, angular coordinate of $z$
PSI $\quad \psi$, airfoil parameter determining radial coordinate of $z^{\prime}$

| RAT(I) | $i$ th iterative scaling of $\varepsilon, \psi$ |
| :---: | :---: |
| RC | radius of curvature at maximum thickness for 4-digit modified profile |
| RK2OK1 | 3-digit reflex camber parameter ratio, $\frac{k_{2}}{k_{1}}$ |
| RLE | leading-edge radius |
| RNP | radius of curvature at origin |
| SF | ratio of input $t / c$ to converged $t / c$ after scaling |
| TOC | thickness-chord ratio |
| X | distance along chord |
| XMT | $m$, location of maximum thickness for 4-digit modified profile |
| $\begin{aligned} & \mathrm{XT}(12), \\ & \mathrm{YT}(12), \\ & \text { YTP(12) } \end{aligned}$ | location and slope of ellipse nose fairing for 6 - and 6A-series thickness profiles |
| XU, XL | upper and lower surface locations of $x$ |
| XTP | $x / c$ location of slope sign change for 6 - and 6A-series thickness profile |
| XYM | $m$, location of maximum thickness for 6 - and 6A-series profiles or chordwise location for maximum ordinate of airfoil or camber line |
| Y | airfoil ordinate normal to chord, positive above chord |
| YM | $y / c$ location of slope sign change for $6-$ and 6A-series thickness profile |
| YMAX | $y(m)$, maximum ordinate of thickness distribution |
| YU, YL | upper and lower surface $y$ ordinate |

## Analysis

## Thickness Distribution Equations for Analytic Airfoils

The design equations for the analytic NACA airfoils and camber lines have been presented in references 3 to 7. They are repeated herein to provide a better understanding of the computer program and indicate the use of different design variables. A summary of some of the design equations and ordinates for many airfoils from these families is also presented in references 8 to 10 .

The traditional NACA airfoil designations are shorthand codes representing the essential elements (such as thickness-chord ratio, camber, design lift coefficient) controlling the shape of a profile generated within a given airfoil type. Thus, for example the NACA 4-digitseries airfoil is specified by a 4-digit code of the form $p m \mathrm{xx}$, where $p$ and $m$ represent positions reserved for
specification of the camber and xx allows for specification of the thickness-chord ratio as a percentage, that is, "pm12" designates a 12-percent-thick $(t / c=0.12) 4$-digit airfoil.

NACA 4-digit-series airfoils. Symmetric airfoils in the 4-digit-series family are designated by a 4-digit number of the form NACA 00xx. The first two digits indicate a symmetric airfoil; the second two, the thickness-chord ratio. Ordinates for the NACA 4-digit airfoil family (ref. 2) are described by an equation of the form:

$$
\frac{y}{c}=a_{0}\left(\frac{x}{c}\right)^{1 / 2}+a_{1}\left(\frac{x}{c}\right)+a_{2}\left(\frac{x}{c}\right)^{2}+a_{3}\left(\frac{x}{c}\right)^{3}+a_{4}\left(\frac{x}{c}\right)^{4}
$$

The constants in the equation (for $t / c=0.20$ ) were determined from the following boundary conditions:

Maximum ordinate:

$$
\frac{x}{c}=0.30 \quad \frac{y}{c}=0.10 \quad \frac{d y}{d x}=0
$$

Ordinate at trailing edge:

$$
\frac{x}{c}=1.0 \quad \frac{y}{c}=0.002
$$

Magnitude of trailing-edge angle:

$$
\frac{x}{c}=1.0 \quad\left|\frac{d y}{d x}\right|=0.234
$$

Nose shape:

$$
\frac{x}{c}=0.10 \quad \frac{y}{c}=0.078
$$

The following coefficients were determined to meet these constraints very closely:

$$
\begin{array}{ll}
a_{0}=0.2969 & a_{1}=-0.1260 \\
a_{2}=-0.3516 & a_{3}=0.2843 \\
a_{4}=-0.1015 &
\end{array}
$$

To obtain ordinates for airfoils in the family with a thickness other than 20 percent, the ordinates for the model with a thickness-chord ratio of 0.20 are multiplied by the ratio $(t / c) / 0.20$. The leading-edge radius of this family is defined as the radius of curvature of the basic equation evaluated at $x / c=0$. Because of the term $a_{0}(x / c)^{1 / 2}$ in the equation, the radius of curvature is finite at $x / c=0$ and can be shown to be (see appendix)

$$
R_{l e}=\frac{a_{0}^{2}}{2}\left(\frac{t / c}{0.20}\right)^{2}
$$

by taking the limit as $x$ approaches zero of the standard expression for radius of curvature:

$$
R=\frac{\left[1+(d y / d x)^{2}\right]^{3 / 2}}{d^{2} y / d x^{2}}
$$

To define an airfoil in this family, the only input necessary to the computer program is the desired thicknesschord ratio.

One might expect that this leading-edge radius $R(0)$, found in the limit as $x \rightarrow 0$ to depend only on the $a_{0}$ term of the defining equation, would also be the minimum radius on the profile curve. This is not true in general; for the NACA 0020 airfoil, for example, a slightly smaller radius ( $R=0.0435$ as compared to $R(0)=$ 0.044075 ) is found in the vicinity of $x=0.00025$.

NACA 4-digit-modified-series airfoils. The 4-digit-modified-series airfoils are designated by a 4-digit number followed by a dash and a 2-digit number (such as NACA 0012-63). The first two digits are zero for a symmetrical airfoil and the second two digits indicate the thickness-chord ratio. The first digit after the dash is a leading-edge-radius index number, and the second is the location of maximum thickness in tenths of chord aft of the leading edge.

The design equation for the 4-digit-series airfoil family was modified (ref. 4) so that the same basic shape was retained but variations in leading-edge radius and chordwise location of maximum thickness could be made. Ordinates for these airfoils are determined from the following equations:

$$
\frac{y}{c}=a_{0}\left(\frac{x}{c}\right)^{1 / 2}+a_{1}\left(\frac{x}{c}\right)+a_{2}\left(\frac{x}{c}\right)^{2}+a_{3}\left(\frac{x}{c}\right)^{3}
$$

from leading edge to maximum thickness, and

$$
\frac{y}{c}=d_{0}+d_{1}\left(1-\frac{x}{c}\right)+d_{2}\left(1-\frac{x}{c}\right)^{2}+d_{3}\left(1-\frac{x}{c}\right)^{3}
$$

from maximum thickness to trailing edge.
The constants in these equations (for $t / c=0.20$ ) can be determined from the following boundary conditions:

Maximum ordinate:

$$
\frac{x}{c}=m \quad \frac{y}{c}=0.10 \quad \frac{d y}{d x}=0
$$

Leading-edge radius:

$$
\frac{x}{c}=0 \quad R=\frac{a_{0}^{2}}{2}
$$

Radius of curvature at maximum thickness:

$$
\frac{x}{c}=m \quad R=\frac{(1-m)^{2}}{2 d_{1}(1-m)-0.588}
$$

Ordinate at trailing edge:

$$
\frac{x}{c}=1.0 \quad \frac{y}{c}=d_{0}=0.002
$$

Magnitude of trailing-edge angle:

$$
\frac{x}{c}=1.0 \quad \frac{d y}{d x}=d_{1}=f(m)
$$

Thus, the maximum ordinate, slope, and radius of curvature of the two portions of the surface match at $x / c=m$. The values of $d_{1}$ were chosen, as stated in reference 4 , to avoid reversals of curvature and are given in the following table:

| $m$ | $d_{1}$ |
| :---: | :---: |
| 0.2 | 0.200 |
| 0.3 | 0.234 |
| 0.4 | 0.315 |
| 0.5 | 0.465 |
| 0.6 | 0.700 |

By use of these constraints, equations were written for each of the constants (except $a_{0}$ and $d_{1}$ ) in the equation for the airfoil family and are included in the computer program. As in the 4-digit-series airfoil family, ordinates vary linearly with variations in thickness-chord ratio and any desired thickness shape can be obtained by scaling the design ordinates by the ratio of the desired thickness-chord ratio to the design thickness-chord ratio.

The leading-edge index is an arbitrary number assigned to the leading-edge radius in reference 4 and is proportional to $a_{0}$. The relationship between leadingedge radius $R_{l e}$ and index number $I$ is as follows:

$$
R_{l e}=0.5\left(0.2969 \frac{t / c}{0.2} \frac{I}{6}\right)^{2}
$$

Thus, an index of 0 indicates a sharp leading edge (radius of zero) and an index of 6 corresponds to $a_{0}=0.2969$, the normal design value for the 20-percentthick 4-digit airfoil. A value of leading-edge index of 9 for a three times normal leading-edge radius was arbitrarily assigned in reference 4 , but $I=9$ cannot be used in this equation. In reference 4 , the index $I$ is not used in computation. Instead, the index $I=9$ was assigned to an airfoil where $a_{0}=0.2969 \sqrt{3}=0.514246$, which (because $R_{l e}=a_{0}^{2} / 2$ ) thus has a three times normal leading-edge radius. The computer program is written so that the desired value of leading-edge radius or the index
$I$ is the input parameter. The value of $a_{0}$ is then computed in the program.

NACA 16-series airfoils. The NACA 16-series airfoil family is described in references 6 and 7 . From the equation for the ordinates in reference 7, this series is a special case of the 4-digit-modified family although this is not directly stated in the references. The 16 -series airfoils are thus defined as having a leading-edge index of 4 and a location of maximum thickness at 0.50 chord. The designation NACA 16-012 airfoil is equivalent to an NACA 0012-45.

## Thickness Distribution Equations for Derived Airfoils

NACA 6-series airfoils. As described in references 9 and 10 , the basic symmetrical NACA 6 -series airfoils were developed by means of conformal transformations. The use of these transformations to relate the flow about an arbitrary airfoil to that of a near circle and then to a circle had been developed earlier, and the results are presented in reference 11 . The basic airfoil parameters $\psi$ and $\varepsilon$ are derived as a function of $\phi$, where $\theta-\phi$ is defined as $-\varepsilon$. Figure 1, taken from reference 10, shows the relationship between these variables in the complex plane. These parameters are used to compute both the airfoil ordinates and the potential flow velocity distribution around the airfoil. For the NACA 6-series airfoils, the shape of the velocity distribution and the longitudinal location of maximum velocity (or minimum pressure) were prescribed. The airfoil parameters $\psi$ and $\varepsilon$ which give the desired velocity distribution were obtained through an iterative process. Then the airfoil ordinates can be calculated from these parameters by use of the equations presented in references 10 and 11. Thus, for each prescribed velocity distribution, a set of basic airfoil parameters is obtained. However, as stated in reference 10, it is possible to define a set of basic parameters $\psi$ and $\varepsilon$ which can be multiplied by a constant factor to obtain airfoils of various thickness-chord ratios while maintaining the minimum pressure at the same chordwise location. Thus, for each NACA 6-series airfoil family (i.e., 63-, 64-, or 65-series), there is one basic set of values for $\psi$ and $\varepsilon$.

NACA 6A-series airfoils. The NACA 6-series airfoil sections were developed in the early 1940's. As aircraft speeds increased, more attention was focused on the thinner airfoils of this series. However, difficulties were encountered in the structural design and fabrication of these thinner sections because of the very thin trailing edges. As a result, the NACA 6A-series airfoil sections were developed, and details of these have been published in reference 12 . Essentially, the modification consisted of


Figure 1. Illustration of transformations used to derive airfoils and calculate pressure distribution. From reference $1 ; a$ is basic length, usually considered unity.
a near-constant slope from about the 80-percent-chord station to the trailing edge and an increase in the trailingedge thickness from zero to a finite value.

Calculation procedure. For each NACA 6-series airfoil family, a unique curve of $\psi$ and $\varepsilon$ exists as a function of $\phi$. This curve can be scaled by a constant factor to provide airfoils of different thickness within this family. A computer program could therefore be developed to calculate the airfoil ordinates for given values of $\psi$ and $\varepsilon$. Although the values of these basic airfoil parameters were not published, tabulated values existed in files or could be computed by the method of reference 11 from published airfoil ordinates. To provide more values of $\psi$ and $\varepsilon$ for storage in a computer subroutine, a fit to the original values was made with a parametric-linked cubic-spline-fit program, and nine additional values were obtained between each of the original values. This process was carried out for each airfoil series, and the results were stored in the computer program as two arrays for each airfoil family (one for $\varepsilon$ as a function of $\phi$ and one for $\psi$ as a function of $\phi$ ). For the new program, the
enrichment was completely redone and the $\phi$ array is stored in the program only once for all 6- and 6A-series profiles.

To calculate the ordinates for an arbitrary airfoil, the program first determines which airfoil series is desired and calls for the subroutine for this series. The airfoil represented by the stored values of $\psi$ and $\varepsilon$ is calculated and its maximum thickness-chord ratio is determined. The ratio of the desired value to that obtained in this determination is calculated. Then, $\psi$ and $\varepsilon$ and are multiplied by this ratio to arrive at a new airfoil thicknesschord ratio. The iteration is repeated until the computed thickness-chord ratio is within 0.01 percent of the desired value or until 10 iterations have been performed. Usually convergence occurs within four iterations. After the iterative process has converged within the limit established, any residual difference between the computed thicknesschord ratio and that desired is eliminated by linearly scaling the $y$ ordinate and its first and second derivatives by the appropriate scale factor. The first and second derivatives of the airfoil ordinates as a function of chord are computed by a subroutine labeled "DERV" in the program. Although these ordinates and slopes are calculated at more than 200 internally controlled chord stations, a subroutine is used to interpolate between these points (by use of a piecewise cubic curve fit labeled "NTRP") so that the output will be in specified increments of chord stations.

Calculation of leading-edge radius. The values of leading-edge radius of the derived airfoils, published in references 9 to 12 , were initially determined by plotting the ordinates to a large scale and fairing in the best circle fit by hand. Values of the tangency point between the circle and airfoil surface obtained in this manner were not published. To provide smooth analytic ordinates around the leading edge for the computer program, a tilted ellipse has been used. (See fig. 2.) This tilted ellipse is described by the basic ellipse function plus an additive term, linear in $x$, which vanishes at the origin. The tilted ellipse expression thus has three arbitrary constants, which are determined in the procedure. The resulting fit to the airfoil ordinates is exact for the ordinate itself and the first derivative and quite close for the second derivative, though examination of the second derivative in the region of tangency generally reveals a small discrepancy (even smaller in the current version than in the original computer program (ref. 1)). The ellipse is defined so that it has the same ordinate and slope as the airfoil surface at the 12th (of 201) tabulated value of $\phi$ in the airfoil parameter subroutine. (The 11th stored point, which is actually the second point of the original tabulated values, was used in ref. 1.) This tangency point is usually located at about 0.005 chord but varies with airfoil thickness and


Figure 2. Leading-edge fairing for 6 -series and 6A-series profiles.
series. By use of this method, a smooth transition between airfoil and ellipse is produced, the tangency point is known, and there is a continuous variation of leading-edge shape with thickness-chord ratio. The nondimensional radius of curvature of the ellipse at the airfoil origin is also calculated in the program, and its value is in close agreement with the published values of the leading-edge radius for known airfoils.

## Camber-Line Equations

2-digit. The 2-digit camber line is designated by a 2-digit number and, when used with a 4-digit airfoil, would have the form NACA $p m x x$ where $p$ is the maximum camber in percent chord, $m$ is the chordwise location of maximum camber in tenths of chord, and $x x$ is the airfoil thickness in percent chord. The NACA 2-digit camber line is described in reference 3. This camber line is formed by two parabolic segments which have a general equation of the form:

$$
\frac{y}{c}=b_{0}+b_{1}\left(\frac{x}{c}\right)+b_{2}\left(\frac{x}{c}\right)^{2}
$$

The constants for the two equations are determined from the following boundary conditions:

Camber-line extremities:

$$
\frac{x}{c}=0 \quad \frac{y}{c}=0 \quad \frac{x}{c}=1.0 \quad \frac{y}{c}=0
$$

Maximum ordinate:

$$
\frac{x}{c}=m \quad \frac{y}{c}=p \quad \frac{d y}{d x}=0
$$

From these conditions, the camber-line equations then become

$$
\frac{y}{c}=\frac{p}{m^{2}}\left(2 m \frac{x}{c}\right)-\left(\frac{x}{c}\right)^{2}
$$

forward of the maximum ordinate and

$$
\frac{y}{c}=\frac{p}{(1-m)^{2}}\left[(1-2 m)+2 m \frac{x}{c}-\left(\frac{x}{c}\right)^{2}\right]
$$

aft of the maximum ordinate. Both the ordinate and slope of the two parabolic segments match at $x / c=m$. Tables of ordinates for some of these camber lines are tabulated in references 9 and 10 . The ordinates are linear with amount of camber $p$ and these can be scaled up or down as desired.

3-digit. To provide a camber line with a very far forward location of the maximum camber, the 3-digit camber line was developed and presented in reference 5. The first digit of the 3-digit camber-line designation is defined as two thirds of the design lift coefficient (in tenths, i.e., 2 denotes $c_{l, i}=0.3$ ); the second digit, as twice the longitudinal location of maximum camber in tenths of chord; and the third digit of zero indicates a nonreflexed trailing edge.

This camber line is also made up of two equations so that the second derivative decreases to zero at a point $r$ aft of the maximum ordinate and remains zero from this point to the trailing edge. The equations for these conditions are as follows:

$$
\frac{d^{2} y}{d x^{2}}=k_{1}\left(\frac{x}{c}-r\right)
$$

from $x / c=0$ to $x / c=r$, and

$$
\frac{d^{2} y}{d x^{2}}=0
$$

from $x / c=\mathrm{r}$ to $x / c=1.0$. The boundary conditions are as follows:

Camber-line extremities:

$$
\begin{array}{llll}
\frac{x}{c}=0 & \frac{y}{c}=0 & \frac{x}{c}=1.0 & \frac{y}{c}=0
\end{array}
$$

At the junction point:

$$
\frac{x}{c}=r \quad\left(\frac{y}{c}\right)_{N}=\left(\frac{y}{c}\right)_{T} \quad\left(\frac{d y}{d x}\right)_{N}=\left(\frac{d y}{d x}\right)_{T}
$$

The equation for the camber line then becomes

$$
\frac{y}{c}=\frac{k_{1}}{6}\left[\left(\frac{x}{c}\right)^{3}-3 r\left(\frac{x}{c}\right)^{2}+r^{2}(3-r) \frac{x}{c}\right]
$$

from $x / c=0$ to $x / c=\mathrm{r}$, and

$$
\frac{y}{c}=\frac{k_{1} r^{3}}{6}\left(1-\frac{x}{c}\right)
$$

from $x / c=r$ to $x / c=1.0$. These equations were then solved for values of $r$ which would give longitudinal locations of the maximum ordinate of $5,10,15,20$, and 25 percent chord. The value of $k_{1}$ was adjusted so that a theoretical design lift coefficient of 0.3 was obtained at the ideal angle of attack. The value of $k_{1}$ can be linearly scaled to give any desired design lift coefficient. Values
of $k_{1}$ and $r$ and the camber-line designation were taken from reference 5 and are presented in the following table:

| Camber-line <br> designation | $m$ | $r$ | $k_{1}$ |
| :---: | :---: | :---: | ---: |
| 210 | 0.05 | 0.0580 | 361.400 |
| 220 | 0.10 | 0.1260 | 51.640 |
| 230 | 0.15 | 0.2025 | 15.957 |
| 240 | 0.20 | 0.2900 | 6.643 |
| 250 | 0.25 | 0.3910 | 3.230 |

3-digit reflex. The camber-line designation for the 3-digit-reflex camber line is the same as that for the 3-digit camber line except that the last digit is changed from 0 to 1 to indicate the reflex characteristic, which is the normally negative camber-line curvature, becomes positive in the aft segment.

For some applications, for example, rotorcraft main rotors, to produce an airfoil with a quarter-chord pitching-moment coefficient of zero may be desirable. The 3-digit-reflexed camber line was thus designed to have a theoretical zero pitching moment as described in reference 5 . The forward part of the camber line is identical to the 3-digit camber line but the aft portion was changed from a zero curvature segment to a segment with positive curvature. The equation for the aft portion of the camber line is expressed by

$$
\frac{d^{2} y}{d x^{2}}=k_{2}\left(\frac{x}{c}-r\right)>0 \quad\left(\frac{x}{c}>r\right)
$$

By using the same boundary conditions as were used for the 3-digit camber line, the equations for the ordinates are

$$
\frac{y}{c}=\frac{k_{1}}{6}\left[\left(\frac{x}{c}-r\right)^{3}-\frac{k_{2}}{k_{1}}(1-r)^{3} \frac{x}{c}-r^{3} \frac{x}{c}+r^{3}\right]
$$

from $x / c=0$ to $x / c=r$ and

$$
\frac{y}{c}=\frac{k_{1}}{6}\left[\frac{k_{2}}{k_{1}}\left(\frac{x}{c}-r\right)^{3}-\frac{k_{2}}{k_{1}}(1-r)^{3} \frac{x}{c}-r^{3} \frac{x}{c}+r^{3}\right]
$$

for $x / c=r$ to $x / c=1.0$. The ratio $k_{2} / k_{1}$ is expressed as

$$
\frac{k_{2}}{k_{1}}=\frac{3}{1-r}(r-m)^{2}-r^{3}
$$

Values of $k_{1}, k_{2} / k_{1}$, and $m$ for several camber-line designations from reference 5 are presented in the following table:

| Camber-line <br> designation | $m$ | $r$ | $k_{1}$ | $k_{2} / k_{1}$ |
| :---: | :---: | :---: | :---: | :--- |
| 221 | 0.10 | 0.1300 | 51.990 | 0.000764 |
| 231 | 0.15 | 0.2170 | 15.793 | 0.00677 |
| 241 | 0.20 | 0.3180 | 6.520 | 0.0303 |
| 251 | 0.25 | 0.4410 | 3.191 | 0.1355 |

6 -series. The equations for the 6 -series camber lines are presented in reference 9 . The camber lines are a function of the design lift coefficient $c_{l, i}$ and the chordwise extent of uniform loading $A$. The equation for these camber lines is as follows:

$$
\begin{aligned}
\frac{y}{c}= & \frac{c_{l, i}}{2 \pi(A+1)}\left\{\frac { 1 } { 1 - A } \left[\frac{1}{2}\left(A-\frac{x}{c}\right)^{2} \log _{2}\left(A-\frac{x}{c}\right)\right.\right. \\
& -\frac{1}{2}\left(1-\frac{x}{c}\right)^{2} \log _{e}\left(1-\frac{x}{c}\right)+\frac{1}{4}\left(1-\frac{x}{c}\right)^{2} \\
& \left.\left.-\frac{1}{4}\left(A-\frac{x}{c}\right)^{2}\right]-\frac{x}{c} \log _{e} \frac{x}{c}+g-h \frac{x}{c}\right\}
\end{aligned}
$$

where

$$
\begin{gathered}
g=-\frac{1}{1-A}\left[A^{2}\left(\frac{1}{2} \log _{e} A-\frac{1}{4}\right)+\frac{1}{4}\right] \\
h=\frac{1}{1-A}\left[\frac{1}{2}(1-A)^{2} \log _{e}(1-A)-\frac{1}{4}(1-A)^{2}\right]+g
\end{gathered}
$$

As was true in reference 1 , the program is capable of combining (by cumulative addition of $y / c$ ) up to 10 camber lines of this series to provide many types of loading.

16-series. The 16 -series cambered airfoils, as described in reference 6 , are derived by using the 6 -series camber-line equation with the mean-line loading $A$ set equal to 1.0 , which is

$$
\frac{y}{c}=\frac{c_{l, i}}{4 \pi}\left(1-\frac{x}{c}\right)^{2} \log _{e}\left(1-\frac{x}{c}\right)
$$

This equation is the one for the standard mean line for this series.

6A-series. The 6A-series cambered airfoils, as described in reference 12 , are derived by using a special form of the 6 -series camber-line equation. This special form is designated as "the $A=0.8$ modified mean line." The modification basically consists of holding the slope of the mean line constant from about the 85 -percentchord station to the trailing edge. As the reference indicates, this mean-line loading should always be used for the 6A-series airfoils. This camber-line equation is given
as one of the options in selection of mean lines in the program.

## Calculation of Cambered Airfoils

To calculate ordinates for a cambered airfoil (including the limiting cases of zero camber $\left(y_{\mathrm{cam}}(x)=0\right)$ and zero thickness $\left(y_{t}(x)=0\right)$ ), the desired mean (camber) line is first computed and then the ordinates of the symmetrical airfoil are measured normal to the mean line at the same chord station. This procedure leads to a set of parametric equations where $(y / c)_{t},(y / c)_{\text {cam }}$, and $\delta$ are all functions of the original independent variable $x / c$. The upper surface ordinates on the cambered airfoil $(x / c)_{u}$ and $(y / c)_{u}$ are given by

$$
\begin{gathered}
\left(\frac{x}{c}\right)_{u}=\left(\frac{x}{c}\right)-\left(\frac{y}{c}\right)_{t} \sin \delta \\
\left(\frac{y}{c}\right)_{u}=\left(\frac{y}{c}\right)_{c a m}+\left(\frac{y}{c}\right)_{t} \cos \delta
\end{gathered}
$$

where $\delta$ is the local inclination of the camber line and $(y / c)_{t}$ is assumed to be negative to obtain the lower surface ordinates $(x / c)_{l}$ and $(y / c)_{l}$. This procedure is also described in reference 10 .

The local slopes of the cambered airfoil are

$$
\left(\frac{d y}{d x}\right)_{u}=\frac{\tan \delta \sec \delta+\left(\frac{d y}{d x}\right)_{t}-\left(\frac{y}{c}\right)_{t}\left[\frac{d \delta}{d}\left(\frac{x}{c}\right)\right] \tan \delta}{\sec \delta-\left(\frac{d y}{d x}\right)_{t} \tan \delta-\left(\frac{y}{c}\right)_{t}\left[\frac{d \delta}{d}\left(\frac{x}{c}\right)\right]}
$$

and

$$
\left(\frac{d y}{d x}\right)_{l}=\frac{\tan \delta \sec \delta-\left(\frac{d y}{d x}\right)_{t}+\left(\frac{y}{c}\right)_{t}\left[\frac{d \delta}{d}\left(\frac{x}{c}\right)\right] \tan \delta}{\sec \delta+\left(\frac{d y}{d x}\right)_{t} \tan \delta+\left(\frac{y}{c}\right)_{t}\left[\frac{d \delta}{d}\left(\frac{x}{c}\right)\right]}
$$

by parametric differentiation of $(x / c)_{u, l}$ and $(y / c)_{u, l}$ with respect to the original $x / c$ and use of the relationship

$$
\left(\frac{d y}{d x}\right)_{u}=\frac{d(y / c)_{u} / d(x / c)}{d(x / c)_{u} / d(x / c)}
$$

Although specific camber lines are generally used with specific thickness distributions, this program has been written in a general format. As a result, any camber line can be used with either type thickness distribution so that any shape desired can be generated.

The user should understand that these equations for combining a thickness distribution with a mean line, while mathematically stable, can produce profiles that
are geometrically discontinuous and aerodynamically useless. An NACA 2224 airfoil, for example, will have a small region on the lower surface where $(y / c)_{l}$ is triple valued.

The density of the final output of the procedure (as in tables I and II) is determined by the user-supplied value of the basic $x / c$ interval $d x$. As the leading edge is approached, the increments become smaller to provide good definition of the leading edge of a blunt profile. Ordinates are printed at increments of $d x / 40$ chord from the leading edge to $x / c=0.01250$, at increments of $d x / 4$ chord from $x / c=0.01250$ to 0.1000 , and at increments of $d x$ chord from $x / c=0.1000$ to the trailing edge. The user is offered the option of setting $d x$ and the smaller increments for $x / c$ less than 0.1 will be reset in the same ratio. This procedure is devised to provide rational $x$ values for $d x=0.01,0.05$, and 0.10 .

## Results and Discussion

## Program Capabilities

AIRFOLS is the computer program which was developed to provide the airfoil shapes described in the section "Analysis." AIRFOLS, the NACA Airfoil Ordinate Generator, is the result of merging two previous efforts reported in references 1 and 2. This program calculates and then reports the ordinates and surface slope for airfoils of any thickness, thickness distribution, or camber in the designated NACA series. AIRFOLS is a portable, ANSI FORTRAN 77 code with limited platform dependencies. Parameters describing the desired airfoil are entered into the software with menu and prompt driven input. Output consists of a report file and a data file.

Provisions have been made in the AIRFOLS program to combine basic airfoil shapes and camber lines from different series so that nonstandard as well as standard airfoils can be generated. The analytical design equations for both symmetrical and cambered airfoils in the NACA 4-digit-series, 4-digit-series modified, 5-digitseries, 5 -digit-series modified, 16 -series, 6 -series, and 6A-series airfoil families have been implemented. The camber-line designations available are the 2 -digit, 3-digit, 3-digit-reflex, 6-series, and 6A-series. The program achieves portability by limiting machine-specific code. The two exceptions are a compiler-specific format edit descriptor and a terminal-specific (display device) set of screen control codes. The user's targets, selected from the available platform configuration and terminal device choices, are communicated to the software enabling the machine-specific setup to be performed internally with coded logic. The program attempts to be completely self-guiding and to reduce required input to
the minimum necessary for the requested airfoil section and mean line. The convention used for the two output file names consist of combining a user-supplied file prefix respectively suffixed with ".rpt" and ".dta." The output report (.rpt) is an ASCII file containing the tabulated nondimensional and dimensional airfoil ordinates as well as other input and output parameters. The output data (.dta) is an ASCII file of X-Y pairs representing the dimensional airfoil ordinates.

To execute AIRFOLS, enter the appropriate command then follow the program prompts. A message printed by the program at the beginning of execution should reiterate the platform, operating system, and FORTRAN compiler to which the present executable is targeted. Examples of the supported platform configuration options are

## CONVEX C2 using CONVEX FORTRAN compiler

IBM compatible PC using IBM FORTRAN/2 compiler

Sun-3 using Sun FORTRAN compiler
VAX 780 using VAX FORTRAN compiler
IRIS-4D using Silicon Graphics FORTRAN compiler

Sun-4 using Sun FORTRAN compiler
Gateway 2000 PC using Lahey FORTRAN compiler
Generic, use only if other devices unavailable
The program's initial request is for identification of the display device being used. Examples of the supported display device options are

ANSI terminal or emulator
IBM personal computer or compatible
TEKTRONIX 4107
Sun Microsystems workstation console

## DEC VT terminal 50

DEC VT terminal 100
DEC VT terminal 200
DEC VT terminal 240 with REGIS
IBM personal computer VTERM package
There are two forms of input for controlling the AIRFOLS program. The program prompts the user to enter textual information or select a choice from a menu. The primary technical input requests that a user sees are

## Choice of standard or nonstandard airfoil

Selection from airfoil section options

Choice of airfoil with camber or no camber
Selection from mean-line options
Prompt for airfoil title
Prompt for output file prefix
Prompts for specific airfoil and mean-line parameters

The program also makes available a nontechnical tutorial in the form of a generic menu example to show the use of menu control. To view the tutorial reply "F" (false) to the menu query: "USER IS EXPERIENCED WITH AIRFOLS." Replying to any menu query with "/EX" (respond with example) will also execute the tutorial. The tutorial interrupts the user's interactive control to execute the example menu type query. The desired replies are entered by the tutorial, alleviating requirements for user interaction. Each reply produces a program response and a tutorial explanation of that response. Program control is returned to the user when the example terminates. The tutorial example will display the program's response to a number of correct and incorrect entries. The responses to the program control characters printed with each menu are covered as well. The program control characters and the typical response to expect are
cr (or Enter) Clears screen then redisplays menu

| / | Responds with system prompt options <br> menu |
| :--- | :--- |
| $?$ | Provides additional details on prompted <br> request |
| E (or e) | Exits from submenu to previous menu <br> level |
| Q (or q) | Terminates program execution |

AIRFOLS was written with emphasis on user friendliness, self-direction, and informative error processing with the goal of producing a valuable software tool for the researcher.

## Accuracy of Results

Analytical airfoils. All the airfoils and camber lines generated by this program are defined by closed analytical expressions and no approximations have been made in the program. Thus, all results are exact. Many cases have been run and compared with previously published results to check the procedure, and for all cases the comparisons were exact except for occasional differences beyond the fifth digit caused by rounding differences.

Derived airfoils. In reference 1 , about 25 cases, including several from each airfoil family, were computed for thickness-chord ratios from 0.06 to 0.15 , and the results were compared with the values published in references 9,10 , and 12 . For the NACA 6 -series airfoils, the agreement was generally within $5 \times 10^{-5}$ chord. The NACA 6A-series airfoils show differences of as much as $9 \times 10^{-5}$ chord near the leading edge, but from about $x / c=0.10$ to 0.95 the accuracy is about the same as for the 6 -series. The equations for the 6 -series airfoil geometry dictate that the trailing-edge thickness be zero; however, the 6A-series airfoils have a finite trailing-edge thickness. For the 6A-series airfoils, the profiles obtained from the program are not useful past $x / c=0.95$. For construction or grid generation, the user would probably modify the last 5 percent of the chord by using the ordinate and slope at $x / c=0.95$ and extrapolating to the trailing edge.

## Early Release Version

A version of this program (0.A, March 31, 1992) already in use produces the same results for actual construction of an airfoil from the output coordinates. However, this early version does display small discrepancies in the second derivative of the thickness distribution upstream of 0.5 percent chord. In the current version, this problem has been corrected by allowing the tilted ellipse fit loop to go one step further and by replacing the interpolation subroutine NTRP with an improved procedure. Users concerned about very small perturbations, as in grid generation, may want to be sure they have the version documented in this report (1.01, April 1996).

## Sample Output Tabulations

Sample computed ordinates for both a symmetric and a cambered airfoil are presented in tables I and II,
respectively. Printed at the top of the first page for each table is the airfoil and camber-line family selected, the airfoil designation, and a list of the input parameters for both airfoil shape and camber line. Both nondimensional and dimensional ordinates are listed. The dimensional quantities have the same units as the input value of the chord, which is also listed at the top of the page. First and second derivatives of the surface ordinates are also presented for symmetrical airfoils, but only first derivatives are tabulated for the cambered airfoils.

## Concluding Remarks

Computer programs to produce the ordinates for airfoils of any thickness, thickness distribution, or camber in the NACA airfoil series were developed in the early 1970's and are published as NASA TM X-3069 and TM X-3284. For analytic airfoils, the ordinates are exact. For the 6 -series and all but the leading edge of the 6Aseries airfoils, agreement between the ordinates obtained from the program and previously published ordinates is generally within $5 \times 10^{-5}$ chord. Since the publication of these programs, the use of personal computers and individual workstations has proliferated. This report describes a computer program that combines the capabilities of the previously published versions. This program is written in ANSI FORTRAN 77 and can be compiled to run on DOS, UNIX, and VMS based personal computers and workstations as well as mainframes. An effort was made to make all inputs to the program as simple as possible to use and to lead the user through the process by means of a menu.

NASA Langley Research Center Hampton, VA 23681-0001
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## Appendix

## Convergence of Leading-Edge Radius of NACA 4-Digit and 4-Digit-Modified Series <br> Airfoils

The demonstration follows that in the limit as $x \rightarrow 0$, the radius of curvature at the nose of the NACA 4-digit-series airfoil profile (or the 4-digit-modified, which has the same equation at the leading edge) is finite and equal to $a_{0}^{2} / 2$, where $a_{0}$ is the coefficient of the $x^{1 / 2}$ term in the defining equation:

$$
\begin{aligned}
y & =a_{0} x^{1 / 2}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+a_{4} x^{4} \\
y_{x} & =\frac{1}{2} a_{0} x^{-1 / 2}+a_{1}+2 a_{2} x+3 a_{3} x^{2}+4 a_{4} x^{3} \\
y_{x x} & =-\frac{1}{4} a_{0} x^{-3 / 2}+2 a_{2}+6 a_{3} x+12 a_{4} x^{2}
\end{aligned}
$$

The leading-edge radius is the radius of curvature at $x=0\left(R_{l e}=R(0)\right):$

$$
\begin{aligned}
R(x) & =\frac{\left(1+y_{x}^{2}\right)^{3 / 2}}{y_{x x}} \\
R(x)^{2} & =\left.\frac{\left(1+y_{x}^{2}\right)^{3}}{y_{x x}^{2}}\right|_{x=0}
\end{aligned}
$$

This expression for the radius of curvature cannot be evaluated at $x=0$ by direct substitution. Evaluating the numerator and denominator separately in the limit as $x \rightarrow 0$, where all terms in $x^{e>0}$ must vanish and all constant terms become negligible compared with the terms in $x^{e<0}$, gives

For the numerator,

$$
\begin{aligned}
1+y_{x}^{2}= & \frac{1}{4} a_{0}^{2} x^{-1}+a_{1} a_{0} x^{-1 / 2} \\
\left(1+y_{x}^{2}\right)^{3}= & \left(\frac{1}{4}\right)^{3} a_{0}^{6} x^{-3}+3\left(\frac{1}{4}\right)^{2} a_{1} a_{0}^{5} x^{-5 / 2} \\
& +3\left(\frac{1}{4}\right) a_{1}^{2} a_{0}^{4} x^{-2}+a_{1}^{3} a_{0}^{3} x^{-3 / 2}
\end{aligned}
$$

For the denominator,

$$
\begin{aligned}
y_{x x}^{2}= & \left(\frac{1}{4}\right)^{2} a_{0}^{2} x^{-3}-2\left(\frac{1}{2}\right) a_{2} a_{0} x^{-3 / 2} \\
& -2\left(\frac{3}{2}\right) a_{3} a_{0} x^{-1 / 2}
\end{aligned}
$$

Multiplying both numerator and denominator by $x^{3}$ gives:

$$
R(x)^{2}=\frac{(1 / 4)^{3} a_{0}^{6}+3(1 / 4)^{2} a_{1} a_{0}^{5} x^{1 / 2} \cdots}{(1 / 4)^{2} a_{0}^{2} \cdots}
$$

where the ellipses represent vanishing terms in $x^{e>0}$. Thus,

$$
\begin{aligned}
& R(0)^{2}=\frac{1}{4} a_{0}^{4} \\
& R(0)=\frac{1}{2} a_{0}^{2}
\end{aligned}
$$

This result clearly depends on the $a_{0}$ term in the original equation with its fractional power of the independent variable. This term prevents the first and second derivatives from vanishing at the origin. Instead, both the first and second derivatives increase without limit at the origin but in such a way that the radius of curvature is finite.

As $x \rightarrow 0$, the variation of $R(x)$ is such that

$$
\frac{d R}{d x} \sim a_{1} a_{0} x^{-1 / 2}
$$

so that even though $R(0)$ is finite, $R_{x}(0)$ is not. As a result, $R(x)$ does not appear to converge to $a_{0}^{2} / 2$ until $x$ is very near zero, and then the curve for $R(x)$ approaches $a_{0}^{2} / 2$ along the axis $x=0$. Furthermore, because $a_{1}<0$ (for 4-digit or 4-digit-modified airfoil), $R_{x}(0)<0$ and thus $\mathrm{R}(x)$ must have a minimum $R_{\min }(x)<R(0)$. A general expression for this minimum is probably not worth the labor of the derivation. For the NACA 0020 airfoil, it is approximately $R_{\min }=R(0.000345)=0.04304$ compared with $R(0)=a_{0}^{2} / 2=0.044075$ in nondimensional coordinates.

## References

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11. Theodorsen, Theodore: Theory of Wing Sections of Arbitrary Shape. NACA Rep. 411, 1931.
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Table I. Sample Computer File "64012.rpt" for Symmetric Airfoil

| AIRFOLS ORDINATE INFORMATION REPORT |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| for 64012 airfoil |  |  |  |  |  |
| PROFILE: | 6-SERIES | CAMBER: - none - |  |  |  |
| PROFILE PARAMETERS: |  |  |  |  |  |
| $0.120000=$ THICKNESS / CHORD (TOC) |  |  |  |  |  |
| $0.010000=$ BASIC X INTERVAL (DX) |  |  |  |  |  |
| $6.000000=$ MODEL CHORD (CHI |  |  |  |  |  |
| $\operatorname{RAT}(\mathrm{I})=1.000$ |  | 00.581 | 12720.9 | 0.999700 |  |
| 4 = NUMBER OF ITERATIONS (IT) |  |  |  |  |  |
| $0.999994=\mathrm{T} / \mathrm{C}$ RATIO, INPUT TO COMPUTED (SF) |  |  |  |  |  |
| 0.571965 = CUMULATIVE SCALING OF EPS,PSI |  |  |  |  |  |
| $0.060000=$ MAXIMUM Y/C (YMA |  |  |  |  |  |
| 0.375975 = PEAK LOCATION (X |  |  |  |  |  |
| $0.3748420 .060000=$ SLOPE SIGN CHANGE LOCATION (XTP, |  |  |  |  |  |
| $0.006717=$ X/C FIT OF ELLIPSE (XT (12) |  |  |  |  |  |
| $0.011216=Y / C$ FIT OF ELLIPSE (YT (12)) |  |  |  |  |  |
| $0.780074=$ SLOPE FIT OF ELLIPSE (YTP (12) |  |  |  |  |  |
| $0.010034=$ RADIUS OF ELLIPSE, ORIGIN TO XT (12)/C,YT(12)/ |  |  |  |  |  |
| X/C | Y/C | X | Y | DY/DX | D2Y/DX2 |
| 0.000000 | 0.000000 | 0.000000 | 0.000000 | ************* | ************* |
| 0.000250 | 0.002217 | 0.001500 | 0.013300 | 4.582653 | -10133.497070 |
| 0.000500 | 0.003153 | 0.003000 | 0.018918 | 3.151760 | -3253.991943 |
| 0.000750 | 0.003861 | 0.004500 | 0.023166 | 2.557617 | -1787.510742 |
| 0.001000 | 0.004446 | 0.006000 | 0.026674 | 2.218630 | -1187.118042 |
| 0.001250 | 0.004972 | 0.007500 | 0.029834 | 1.960144 | -813.015991 |
| 0.001500 | 0.005434 | 0.009000 | 0.032604 | 1.791810 | -637.092957 |
| 0.001750 | 0.005867 | 0.010500 | 0.035199 | 1.646746 | -497.648315 |
| 0.002000 | 0.006262 | 0.012000 | 0.037571 | 1.535877 | -408.647339 |
| 0.002250 | 0.006633 | 0.013500 | 0.039798 | 1.443982 | -346.249725 |
| 0.002500 | 0.006986 | 0.015000 | 0.041916 | 1.361467 | -291.917206 |
| 0.002750 | 0.007316 | 0.016500 | 0.043893 | 1.295341 | -255.805206 |
| 0.003000 | 0.007631 | 0.018000 | 0.045788 | 1.236006 | -225.954163 |
| 0.003250 | 0.007935 | 0.019500 | 0.047611 | 1.181388 | -199.190826 |
| 0.003500 | 0.008224 | 0.021000 | 0.049344 | 1.134461 | -178.795593 |
| 0.003750 | 0.008501 | 0.022500 | 0.051005 | 1.093128 | -162.901077 |
| 0.004000 | 0.008770 | 0.024000 | 0.052620 | 1.053807 | -147.842957 |
| 0.004250 | 0.009030 | 0.025500 | 0.054180 | 1.017726 | -134.695694 |
| 0.004500 | 0.009279 | 0.027000 | 0.055675 | 0.986034 | -124.466530 |
| 0.004750 | 0.009521 | 0.028500 | 0.057127 | 0.956632 | -115.500816 |
| 0.005000 | 0.009758 | 0.030000 | 0.058545 | 0.928518 | -106.995934 |
| 0.005250 | 0.009987 | 0.031500 | 0.059923 | 0.902220 | -99.390312 |
| 0.005500 | 0.010209 | 0.033000 | 0.061256 | 0.878265 | -93.122375 |
| 0.005750 | 0.010425 | 0.034500 | 0.062550 | 0.856161 | -87.877068 |
| 0.006000 | 0.010637 | 0.036000 | 0.063819 | 0.834813 | -82.808548 |
| 0.006250 | 0.010843 | 0.037500 | 0.065061 | 0.814474 | -78.015030 |
| 0.006500 | 0.011045 | 0.039000 | 0.066271 | 0.795407 | -73.602821 |
| 0.006750 | 0.011241 | 0.040500 | 0.067446 | 0.777865 | -69.673355 |
| 0.007000 | 0.011433 | 0.042000 | 0.068595 | 0.761235 | -65.977486 |
| 0.007250 | 0.011621 | 0.043500 | 0.069726 | 0.745170 | -62.376503 |
| 0.007500 | 0.011806 | 0.045000 | 0.070836 | 0.729842 | -58.941341 |
| 0.007750 | 0.011987 | 0.046500 | 0.071923 | 0.715419 | -55.742962 |
| 0.008000 | 0.012164 | 0.048000 | 0.072983 | 0.702071 | -52.852318 |
| 0.008250 | 0.012337 | 0.049500 | 0.074022 | 0.689594 | -50.213284 |

Table I. Continued

| 0.008500 | 0.012508 | 0.051000 | 0.075048 | 0.677483 | -47.681225 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.008750 | 0.012676 | 0.052500 | 0.076058 | 0.665794 | -45.311497 |
| 0.009000 | 0.012842 | 0.054000 | 0.077051 | 0.654589 | -43.162434 |
| 0.009250 | 0.013004 | 0.055500 | 0.078026 | 0.643933 | -41.292366 |
| 0.009500 | 0.013163 | 0.057000 | 0.078981 | 0.633884 | -39.757317 |
| 0.009750 | 0.013320 | 0.058500 | 0.079921 | 0.624186 | -38.392265 |
| 0.010000 | 0.013475 | 0.060000 | 0.080850 | 0.614691 | -37.067284 |
| 0.012500 | 0.014909 | 0.075000 | 0.089454 | 0.536935 | -25.528528 |
| 0.015000 | 0.016179 | 0.090000 | 0.097075 | 0.482244 | -18.988861 |
| 0.017500 | 0.017331 | 0.105000 | 0.103985 | 0.441078 | -14.018546 |
| 0.020000 | 0.018394 | 0.120000 | 0.110366 | 0.410865 | -10.439953 |
| 0.022500 | 0.019392 | 0.135000 | 0.116349 | 0.387887 | -8.075612 |
| 0.025000 | 0.020338 | 0.150000 | 0.122027 | 0.369581 | -6.734070 |
| 0.027500 | 0.021242 | 0.165000 | 0.127449 | 0.353842 | -5.837592 |
| 0.030000 | 0.022109 | 0.180000 | 0.132653 | 0.340221 | -5.192567 |
| 0.032500 | 0.022944 | 0.195000 | 0.137662 | 0.327704 | -4.773360 |
| 0.035000 | 0.023748 | 0.210000 | 0.142490 | 0.316412 | -4.212404 |
| 0.037500 | 0.024527 | 0.225000 | 0.147161 | 0.306573 | -3.777347 |
| 0.040000 | 0.025282 | 0.240000 | 0.151690 | 0.297353 | -3.617945 |
| 0.042500 | 0.026014 | 0.255000 | 0.156082 | 0.288415 | -3.505087 |
| 0.045000 | 0.026724 | 0.270000 | 0.160345 | 0.279875 | -3.305694 |
| 0.047500 | 0.027414 | 0.285000 | 0.164481 | 0.271898 | -3.064729 |
| 0.050000 | 0.028084 | 0.300000 | 0.168504 | 0.264468 | -2.922085 |
| 0.052500 | 0.028736 | 0.315000 | 0.172417 | 0.257319 | -2.802585 |
| 0.055000 | 0.029371 | 0.330000 | 0.176224 | 0.250416 | -2.674942 |
| 0.057500 | 0.029989 | 0.345000 | 0.179933 | 0.243961 | -2.528763 |
| 0.060000 | 0.030591 | 0.360000 | 0.183545 | 0.237823 | -2.358885 |
| 0.062500 | 0.031178 | 0.375000 | 0.187069 | 0.232108 | -2.180080 |
| 0.065000 | 0.031752 | 0.390000 | 0.190512 | 0.226970 | -2.010549 |
| 0.067500 | 0.032313 | 0.405000 | 0.193879 | 0.222084 | -1.917779 |
| 0.070000 | 0.032862 | 0.420000 | 0.197174 | 0.217262 | -1.948437 |
| 0.072500 | 0.033400 | 0.435000 | 0.200398 | 0.212446 | -1.813631 |
| 0.075000 | 0.033925 | 0.450000 | 0.203550 | 0.208084 | -1.664815 |
| 0.077500 | 0.034440 | 0.465000 | 0.206640 | 0.204270 | -1.503816 |
| 0.080000 | 0.034946 | 0.480000 | 0.209678 | 0.200514 | -1.480895 |
| 0.082500 | 0.035443 | 0.495000 | 0.212660 | 0.196759 | -1.525902 |
| 0.085000 | 0.035930 | 0.510000 | 0.215580 | 0.193007 | -1.444046 |
| 0.087500 | 0.036408 | 0.525000 | 0.218446 | 0.189404 | -1.350303 |
| 0.090000 | 0.036878 | 0.540000 | 0.221268 | 0.186288 | -1.246098 |
| 0.092500 | 0.037340 | 0.555000 | 0.224037 | 0.183229 | -1.189765 |
| 0.095000 | 0.037794 | 0.570000 | 0.226761 | 0.180203 | -1.180359 |
| 0.097500 | 0.038241 | 0.585000 | 0.229444 | 0.177304 | -1.181516 |
| 0.100000 | 0.038681 | 0.600000 | 0.232084 | 0.174424 | -1.176315 |
| 0.110000 | 0.040368 | 0.660000 | 0.242209 | 0.163658 | -0.964989 |
| 0.120000 | 0.041955 | 0.720000 | 0.251727 | 0.153665 | -0.953014 |
| 0.130000 | 0.043446 | 0.780000 | 0.260673 | 0.144683 | -0.846358 |
| 0.140000 | 0.044851 | 0.840000 | 0.269105 | 0.136288 | -0.839168 |
| 0.150000 | 0.046174 | 0.900000 | 0.277045 | 0.128548 | -0.769506 |
| 0.160000 | 0.047421 | 0.960000 | 0.284524 | 0.120930 | -0.740389 |
| 0.170000 | 0.048594 | 1.020000 | 0.291562 | 0.113927 | -0.645912 |
| 0.180000 | 0.049701 | 1.080000 | 0.298208 | 0.107368 | -0.689126 |
| 0.190000 | 0.050742 | 1.140000 | 0.304450 | 0.100969 | -0.582494 |
| 0.200000 | 0.051721 | 1.200000 | 0.310329 | 0.094978 | -0.631849 |

Table I. Continued

| 0.210000 | 0.052639 | 1.260000 | 0.315837 |
| :---: | :---: | :---: | :---: |
| 0.220000 | 0.053499 | 1.320000 | 0.320993 |
| 0.230000 | 0.054302 | 1.380000 | 0.325812 |
| 0.240000 | 0.055048 | 1.440000 | 0.330291 |
| 0.250000 | 0.055739 | 1.500000 | 0.334436 |
| 0.260000 | 0.056377 | 1.560000 | 0.338264 |
| 0.270000 | 0.056960 | 1.620000 | 0.341760 |
| 0.280000 | 0.057493 | 1.680000 | 0.344957 |
| 0.290000 | 0.057972 | 1.740000 | 0.347833 |
| 0.300000 | 0.058400 | 1.800000 | 0.350401 |
| 0.310000 | 0.058781 | 1.860000 | 0.352689 |
| 0.320000 | 0.059112 | 1.920000 | 0.354675 |
| 0.330000 | 0.059395 | 1.980000 | 0.356368 |
| 0.340000 | 0.059628 | 2.040000 | 0.357769 |
| 0.350000 | 0.059809 | 2.100000 | 0.358853 |
| 0.360000 | 0.059931 | 2.160000 | 0.359588 |
| 0.370000 | 0.059993 | 2.220000 | 0.359957 |
| 0.380000 | 0.059991 | 2.279999 | 0.359948 |
| 0.390000 | 0.059916 | 2.339999 | 0.359499 |
| 0.400000 | 0.059772 | 2.399999 | 0.358630 |
| 0.410000 | 0.059548 | 2.459999 | 0.357290 |
| 0.420000 | 0.059260 | 2.519999 | 0.355559 |
| 0.430000 | 0.058897 | 2.579999 | 0.353381 |
| 0.440000 | 0.058475 | 2.639999 | 0.350849 |
| 0.450000 | 0.057991 | 2.699999 | 0.347946 |
| 0.460000 | 0.057450 | 2.759999 | 0.344698 |
| 0.470000 | 0.056858 | 2.819999 | 0.341145 |
| 0.480000 | 0.056218 | 2.879999 | 0.337309 |
| 0.490000 | 0.055534 | 2.939999 | 0.333204 |
| 0.500000 | 0.054806 | 3.000000 | 0.328834 |
| 0.510000 | 0.054032 | 3.060000 | 0.324192 |
| 0.520000 | 0.053222 | 3.120000 | 0.319330 |
| 0.530000 | 0.052372 | 3.180000 | 0.314233 |
| 0.540000 | 0.051484 | 3.240000 | 0.308906 |
| 0.550000 | 0.050561 | 3.300000 | 0.303368 |
| 0.560000 | 0.049607 | 3.360000 | 0.297639 |
| 0.570000 | 0.048621 | 3.420000 | 0.291726 |
| 0.580000 | 0.047601 | 3.480000 | 0.285605 |
| 0.590000 | 0.046553 | 3.539999 | 0.279317 |
| 0.600000 | 0.045476 | 3.599999 | 0.272859 |
| 0.610000 | 0.044374 | 3.659999 | 0.266245 |
| 0.620000 | 0.043249 | 3.719999 | 0.259491 |
| 0.630000 | 0.042098 | 3.779999 | 0.252586 |
| 0.640000 | 0.040923 | 3.839999 | 0.245540 |
| 0.650000 | 0.039733 | 3.899999 | 0.238400 |
| 0.660000 | 0.038520 | 3.959999 | 0.231121 |
| 0.670000 | 0.037287 | 4.019999 | 0.223724 |
| 0.680000 | 0.036040 | 4.079999 | 0.216239 |
| 0.690000 | 0.034776 | 4.139999 | 0.208654 |
| 0.700000 | 0.033499 | 4.199999 | 0.200992 |
| 0.710000 | 0.032206 | 4.259999 | 0.193236 |
| 0.720000 | 0.030905 | 4.319999 | 0.185431 |
| 0.730000 | 0.029592 | 4.379999 | 0.177554 |


| 0.088733 | -0.591514 |
| :---: | :---: |
| 0.083079 | -0.560491 |
| 0.077596 | -0.569729 |
| 0.071720 | -0.567787 |
| 0.066492 | -0.525667 |
| 0.060994 | -0.569880 |
| 0.055913 | -0.508531 |
| 0.050420 | -0.557001 |
| 0.045442 | -0.506901 |
| 0.040442 | -0.474289 |
| 0.035488 | -0.500163 |
| 0.030786 | -0.465019 |
| 0.025781 | -0.488108 |
| 0.020854 | -0.534137 |
| 0.015181 | -0.583586 |
| 0.009271 | -0.618653 |
| 0.003000 | -0.617353 |
| -0.003694 | -0.728268 |
| -0.010928 | -0.686004 |
| -0.018412 | -0.779822 |
| -0.025435 | -0.657702 |
| -0.032712 | -0.775743 |
| -0.039309 | -0.598958 |
| -0.045457 | -0.617973 |
| -0.051196 | -0.567208 |
| -0.056922 | -0.501878 |
| -0.061447 | -0.466768 |
| -0.066273 | -0.467965 |
| -0.070572 | -0.444957 |
| -0.075264 | -0.459214 |
| -0.079229 | -0.360220 |
| -0.082976 | -0.400605 |
| -0.086970 | -0.369812 |
| -0.090484 | -0.352421 |
| -0.094048 | -0.310705 |
| -0.096797 | -0.298187 |
| -0.100354 | -0.364253 |
| -0.103489 | -0.269771 |
| -0.106118 | -0.299605 |
| -0.109077 | -0.263943 |
| -0.111324 | -0.229142 |
| -0.113851 | -0.262963 |
| -0.116378 | -0.237618 |
| -0.118140 | -0.161841 |
| -0.120230 | -0.234471 |
| -0.122256 | -0.197908 |
| -0.124178 | -0.142382 |
| -0.125457 | -0.160029 |
| -0.127102 | -0.142993 |
| -0.128568 | -0.157516 |
| -0.129713 | -0.076584 |
| -0.130663 | -0.132782 |
| -0.131876 | -0.048956 |

Table I. Concluded

| 0.740000 | 0.028272 | 4.439999 | 0.169634 | -0.132190 | -0.071199 |
| :--- | :--- | :--- | :--- | :--- | ---: |
| 0.750000 | 0.026946 | 4.499999 | 0.161675 | -0.133017 | -0.014410 |
| 0.760000 | 0.025617 | 4.559999 | 0.153701 | -0.132997 | -0.020850 |
| 0.770000 | 0.024285 | 4.619998 | 0.145708 | -0.133368 | 0.003955 |
| 0.780000 | 0.022952 | 4.679998 | 0.137712 | -0.133111 | 0.023193 |
| 0.790000 | 0.021622 | 4.739998 | 0.129730 | -0.133023 | 0.039957 |
| 0.800000 | 0.020294 | 4.799998 | 0.121766 | -0.132388 | 0.072903 |
| 0.810000 | 0.018974 | 4.859998 | 0.113846 | -0.131488 | 0.116263 |
| 0.820000 | 0.017665 | 4.919998 | 0.105989 | -0.130493 | 0.107131 |
| 0.830000 | 0.016366 | 4.979998 | 0.098198 | -0.129051 | 0.177069 |
| 0.840000 | 0.015084 | 5.039998 | 0.090504 | -0.127493 | 0.174549 |
| 0.850000 | 0.013819 | 5.099998 | 0.082913 | -0.125490 | 0.201894 |
| 0.860000 | 0.012574 | 5.159998 | 0.075443 | -0.123375 | 0.252569 |
| 0.870000 | 0.011354 | 5.219998 | 0.068122 | -0.120714 | 0.275818 |
| 0.880000 | 0.010160 | 5.279998 | 0.060962 | -0.118055 | 0.318184 |
| 0.890000 | 0.008996 | 5.339998 | 0.053979 | -0.114893 | 0.301135 |
| 0.900000 | 0.007864 | 5.399998 | 0.047186 | -0.107152 | 0.392689 |
| 0.910000 | 0.006772 | 5.459998 | 0.040631 | -0.102876 | 0.412245 |
| 0.920000 | 0.005721 | 5.519998 | 0.034324 | -0.097767 | 0.584972 |
| 0.930000 | 0.004720 | 5.579998 | 0.028319 | -0.092050 | 0.588944 |
| 0.940000 | 0.003771 | 5.639997 | 0.022626 | -0.085061 | 0.725263 |
| 0.950000 | 0.002884 | 5.699997 | 0.017303 | -0.076628 | 0.904850 |
| 0.960000 | 0.002073 | 5.759997 | 0.012437 | -0.066925 | 1.038957 |
| 0.970000 | 0.001351 | 5.819997 | 0.008106 | -0.055016 | 1.359224 |
| 0.980000 | 0.000739 | 5.879997 | 0.004435 | -0.039331 | 1.985244 |
| 0.990000 | 0.000264 | 5.939997 | 0.001581 | -0.001854 | 21.079264 |
| 1.000000 | 0.000000 | 5.999997 | 0.000000 |  |  |

Table II. Sample Computer File "64412.rpt" for Cambered Airfoil

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AIRFOLS ORDINATE INFORMATION REPORT
for 64412 airfoil for 64412 airfoil
PROFILE: 6 -SERIES PROFILE PARAMETERS: $0.120000=$ THICKNESS $/$ CHORD (TOC) $0.010000=$ BASIC $X$ INTERVAL
$6.000000=$ MODEL CHORD (CHD) $\operatorname{RAT}(I)=1.0000000$. 0.999700
 0.984284 $0.060000=$ MAXIMUM Y/C (YMAX) $0.3748420 .060000=$ SLOPE SIGN CHANGE LOCATION (XT (12))
(YT (12)) $0.010034=$ RADIUS OF ELLIPSE, ORIGIN TO XT (12)/C,YT(12)/C (RNP)
OCAMBER PARAMETERS: F ELLIPSE
(XT (12))
(YT (12))

UB

 $1=$ NUMBER OF
CLI (I)
$=$
$=$ A(I) $=$ UNCAMBERED
$\cup \circ$






































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[^2]| REPORT DOCUMENTATION PAGE |  |  |  | Form Approved OMB No. 0704-0188 |
| :---: | :---: | :---: | :---: | :---: |
| Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of thiscollection of information, including suggestions for reducing this burden to Washington Headquarters sevvices, Directorate eor Information operation sand Reports, 12115 JeffersonDavis Highway, Suite 1204, Aring |  |  |  |  |
| 1. AGENCY USE ONLY (Leave blank) | 2. REPORT DATE December 1996 | 3. REPORT TYPE AND DATES COVERED Technical Memorandum |  |  |
| 4. TITLE AND SUBTITLE Computer Program To Obtain Ordinates for NACA Airfoils |  |  | 5. FUNDING NUMBERS WU 505-59-10-31 |  |
| 6. AUTHOR(S) Charles L. Ladson, Cuyler W. Brooks, Jr., Acquilla S. Hill, and Darrell W. Sproles |  |  |  |  |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <br> NASA Langley Research Center <br> Hampton, VA 23681-0001 |  |  | 8. PERFORMING ORGANIZATION REPORT NUMBERL-17509 |  |
| 9. SPONSORING/MONITORING AGEN <br> National Aeronautics and Space <br> Washington, DC 20546-0001 | CY NAME(S) AND ADDRESS(ES) ace Administration |  | 10. SPON AGEN <br> NAS | ORING/MONITORING Y REPORT NUMBER TM-4741 |
| 11. SUPPLEMENTARY NOTES <br> Ladson, Brooks, and Hill: Langley Research Center, Hampton, VA; Sproles: Computer Sciences Corp., Hampton, VA. <br> Computer program is available electronically from the Langley Software Server |  |  |  |  |
| 12a. DISTRIBUTION/AVAILABILITY STATEMENT <br> Unclassified-Unlimited <br> Subject Category 02 <br> Availability: NASA CASI (301) 621-0390 |  |  | 12b. DIST | IBUTION CODE |
| 13. ABSTRACT (Maximum 200 words) <br> Computer programs to produce the ordinates for airfoils of any thickness, thickness distribution, or camber in the NACA airfoil series were developed in the early 1970's and are published as NASA TM X-3069 and TM X-3284. For analytic airfoils, the ordinates are exact. For the 6 -series and all but the leading edge of the 6A-series airfoils, agreement between the ordinates obtained from the program and previously published ordinates is generally within $5 \times 10^{-5}$ chord. Since the publication of these programs, the use of personal computers and individual workstations has proliferated. This report describes a computer program that combines the capabilities of the previously published versions. This program is written in ANSI FORTRAN 77 and can be compiled to run on DOS, UNIX, and VMS based personal computers and workstations as well as mainframes. An effort was made to make all inputs to the program as simple as possible to use and to lead the user through the process by means of a menu. |  |  |  |  |
| 14. SUBJECT TERMS <br> NACA airfoils; 4-digit; 6-series; Platform-independent; FORTRAN |  |  |  | 15. NUMBER OF PAGES 23 |
|  |  |  |  | 16. PRICE CODE A03 |
| 17. SECURITY CLASSIFICATION OF REPORT Unclassified | 18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified | 19. SECURITY CLASS OF ABSTRACT Unclassified | FICATION | 20. LIMITATION OF ABSTRACT |
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