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## RESEARCH MEMORANDUM

THE WAVE DRAG OF ARBITRARY CONFIGURATIONS IN  
 LINEARIZED FLOW AS DETERMINED BY AREAS  
 AND FORCES IN OBLIQUE PLANES

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CLASSIFICATION CANCELLED

Authority *NACA Res. Rep.* Date *12/14/53*  
*RM No. 94*  
 By *H. Lomax* 1/16/56 See

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NATIONAL ADVISORY COMMITTEE  
 FOR AERONAUTICS

WASHINGTON

March 24, 1955

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## SUMMARY

The wave drag, based on linearized theory, of any lifting or non-lifting planar or nonplanar object in a steady supersonic flow is shown to be identical at a fixed Mach number to the average wave drag of a series of equivalent bodies of revolution. The streamwise gradient of area, measured normal to the free stream, at a section of each of these equivalent bodies of revolution is given by the sum of two quantities:

1. The streamwise gradient of area, measured in parallel oblique planes tangent to the Mach cones, along the given object
2. A term proportional to the resultant force on the object measured in the same oblique planes.

## INTRODUCTION

The evaluation of the wave drag of airplanes traveling at transonic speeds has, recently, been greatly simplified by the discovery of what has been termed the transonic area rule which can be stated as follows:

A (reasonably smooth) airplane flying at a near sonic speed has the same drag as a body of revolution with the same cross-sectional area in planes normal to the flight direction.

The original statement of this rule (ref. 1) has since had many experimental verifications. From a theoretical viewpoint, a paper written by Hayes (ref. 2) also contains the concept in the sense that it is a result of linearized supersonic flow theory when the Mach number is allowed to approach 1. More recently, several papers have developed theoretical

techniques which make it possible to include the study of thickness effects in slender airplane theory, a linearized flow theory devoted specifically either to airplanes flying at any speed but geometrically slender in the streamwise direction, or to airplanes flying near the speed of sound but smooth enough and having an aspect ratio low enough for the pressure coefficient to be small almost everywhere on their surface. The rule stated above lies at least implicitly in published results based on the latter theory. See, for example, references 3 and 4.

When linearized supersonic flow theory was shown to be consistent with the transonic area rule as the Mach number approached unity, the question naturally arose as to whether or not it could be used to develop an analogous rule applicable to the evaluation of wave drags on airplanes flying at supersonic speeds. Thereafter, a formula, often referred to as the supersonic area rule, was presented (refs. 5 and 6) which did extend the transonic area rule, again expressing the drag in terms of the streamwise distribution of airplane cross-sectional area - but this time in terms of cross sections taken in planes tangent to the characteristic Mach cones. However, this supersonic area rule, when applied to general airplane shapes, gives only an approximation to the correct linearized-theory value of the wave drag. The degree of approximation ranges from very good, for cases such as a nonlifting wing centrally mounted on a slender body (essentially the case to which Jones (ref. 5) limited his result), to very poor, for cases such as the Busemann biplane or a shrouded body similar to that studied by Ferri in reference 7. Further, the quantitative error made in applying the rule to general shapes flying at supersonic speeds was unknown.

The object of this report is to present a formula which gives the correct (linearized theory) value of the wave drag for any lifting or nonlifting object in a steady supersonic flow as the average of a series of wave drags of equivalent bodies of revolution

#### LIST OF IMPORTANT SYMBOLS

$C_p$	pressure coefficient, $\frac{\text{local pressure} - \text{free-stream pressure}}{q}$
$D$	wave drag
$l(x, \theta)$	oblique section lift (see eq. (17))
$M$	free-stream Mach number
$n$	normal to airplane surface

$\left. \begin{array}{l} \bar{n}_1 \\ \bar{n}_2 \\ \bar{n}_3 \end{array} \right\}$	direction cosines with respect to the $x, y, z$ axes
$q$	free-stream dynamic pressure, $\frac{1}{2} \rho_0 U_0^2$
$r_1$	$\sqrt{(y-y_1)^2 + (x-x_1)^2}$
$S(x, \theta)$	oblique section area (see eq. (12))
$U_0$	speed of free stream
$x_0$	$x - \beta r$
$x, y, z$	Cartesian coordinates in wind-axes system
$x, r, \theta$	cylindrical coordinates in wind-axes system
$\beta$	$\sqrt{M^2 - 1}$
$v$	conormal to airplane surface
$\xi, \sigma$	coordinates defined in equation (10)
$\sigma_\theta$	coordinate defined in sketch (d)
$\rho_0$	free-stream density
$\phi$	perturbation velocity potential

## DEVELOPMENT

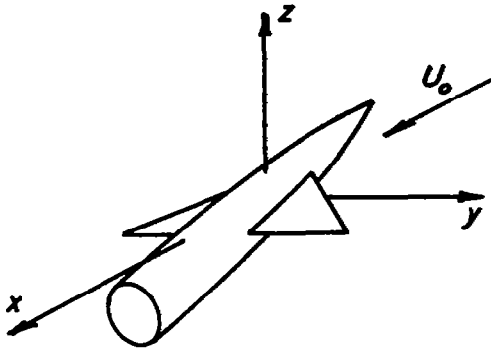
Consider an airplane in a steady supersonic stream. Let the surfaces of the airplane be inclined to the free stream at angles small enough for the disturbed flow field to be adequately approximated by solutions to the wave equation

$$\beta^2 \phi_{xx} - \phi_{yy} - \phi_{zz} = 0 \quad (1)$$

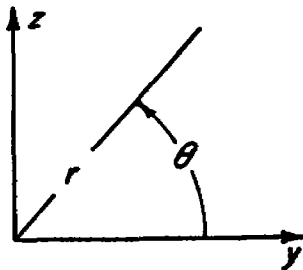
where the free stream is moving in the positive  $x$  direction and  $\beta^2 = M^2 - 1$ . A general solution to equation (1), developed by Volterra, is given by

$$\phi(x, y, z) = -\frac{1}{2\pi} \frac{\partial}{\partial x} \int_T \int \left( \frac{\partial \phi}{\partial v_1} - \phi \frac{\partial}{\partial v_1} \right) \ln \frac{x - x_1 + \sqrt{(x - x_1)^2 - \beta^2 r_1^2}}{\beta r_1} dS_1 \quad (2)$$

where  $r_1^2 = (y - y_1)^2 + (z - z_1)^2$ ,  $dS_1$  is an element of surface area on the airplane,  $v_1$  is the outward conormal (the conormal to the characteristic cone lies along the cone) to that element, and  $\tau$  is that portion of the airplane surface within the Mach forecone from the point  $x, y, z$ .



We wish to use equation (2) to calculate the wave drag of a given object. For this purpose the wave drag is expressed in terms of the perturbation velocities induced by the object on an enclosing cylindrical control surface of infinite radius. This equation (see, e.g., ref. 8) can be written in terms of the cylindrical coordinate system defined in sketch (a) (notice that the control surface is parallel to the free-stream direction, that is, we are using wind axes) as



Sketch (a)

$$D = -\rho_0 \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dx \left( \lim_{r \rightarrow \infty} \phi_r \phi_x \right) \quad (3)$$

The wave drag of an arbitrarily shaped object (arbitrary except, of course, for the condition that equation (1) must adequately represent the flow field) can be calculated by inserting equation (2) into equation (3).

First let us inspect the conormal derivative  $\partial/\partial v_1$  and the differential area  $dS_1$ . By definition (see ref. 9, p. 246)

$$\frac{\partial}{\partial v_1} = \frac{-\beta^2 \bar{n}_1 \frac{\partial}{\partial x_1} + \bar{n}_2 \frac{\partial}{\partial y_1} + \bar{n}_3 \frac{\partial}{\partial z_1}}{\sqrt{\bar{n}_1^2 \beta^4 + \bar{n}_2^2 + \bar{n}_3^2}}$$

where  $\bar{n}_1$ ,  $\bar{n}_2$ , and  $\bar{n}_3$  are the direction cosines of an outward normal to the airplane surface. For surfaces making small angles with the free stream

$$\bar{n}_2^2 + \bar{n}_3^2 \approx 1$$

$$\bar{n}_1 \approx \text{streamwise surface slope}$$

Hence, within the framework of linearized theory

$$\frac{\partial}{\partial v_1} = \frac{\partial}{\partial n_1} \quad (4)$$

where  $n_1$  is either the normal to the airplane surface or the normal to the surface in the  $y_1, z_1$  plane. Similarly, the differential area  $dS_1$  can be expressed as

$$dS_1 = \frac{ds_1 dx_1}{\sqrt{1 - \bar{n}_1^2}}$$

where  $ds_1$  is an element of arc along the airplane surface in an  $x_1 = \text{constant}$  plane; so, again, within the framework of linearized theory, one can write

$$dS_1 = ds_1 dx_1 \quad (5)$$

For convenience equation (2) is divided into two parts such that

$$\varphi(x, y, z) = \varphi_1(x, y, z) + \varphi_2(x, y, z) \quad (6)$$

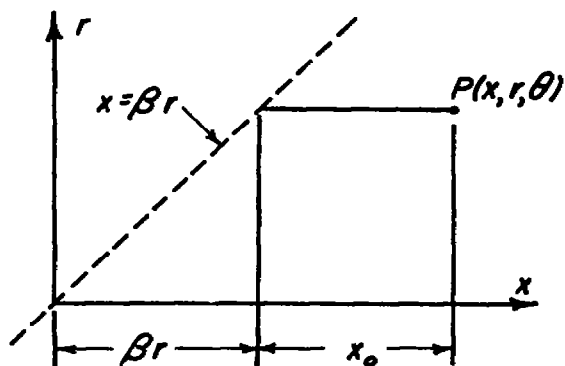
where

$$\varphi_1(x, y, z) = -\frac{1}{2\pi} \frac{\partial}{\partial x} \int_{\tau} \int \varphi_{n_1}(x_1, s_1) \ln \frac{x - x_1 + \sqrt{(x - x_1)^2 - \beta^2 r_1^2}}{\beta r_1} dx_1 ds_1 \quad (7)$$

and

$$\varphi_2(x, y, z) = \frac{1}{2\pi} \frac{\partial}{\partial x} \int_{\tau} \int \varphi(x_1, s_1) \frac{\partial}{\partial n_1} \ln \frac{x - x_1 + \sqrt{(x - x_1)^2 - \beta^2 r_1^2}}{\beta r_1} dx_1 ds_1 \quad (8)$$

Consider, first, equation (7). According to equation (3), we need to find the value of  $\varphi_x$  and  $\varphi_r$  in the limit as  $r$  (which equals  $\sqrt{y^2 + z^2}$ ) goes to infinity. However, since no disturbances can be induced ahead of the foremost Mach cone enveloping the disturbing object, it is convenient to increase  $x$  as  $r$  is increased so that the point  $x, r, \theta$  remains in the vicinity of this Mach cone. Therefore, we first



set (see sketch (b))

$$x = x_0 + \beta r$$

and then let  $r$  approach infinity. In this way, as  $r$  becomes very large, one can show that equation (7) reduces to

Sketch (b)

$$\varphi_1(x, r, \theta) = \frac{-1}{2\pi \sqrt{2\beta r}} \int_{\tau} \int \frac{\varphi_{n_1}(x_1, s_1) dx_1 ds_1}{\sqrt{x_0 - x_1 + \beta y_1 \cos \theta + \beta z_1 \sin \theta}} \quad (9)$$

By means of the transformations

$$\left. \begin{aligned} \xi &= x_1 - \beta y_1 \cos \theta - \beta z_1 \sin \theta \\ \sigma &= s_1 \end{aligned} \right\} \quad (10)$$

equation (9) further simplifies to

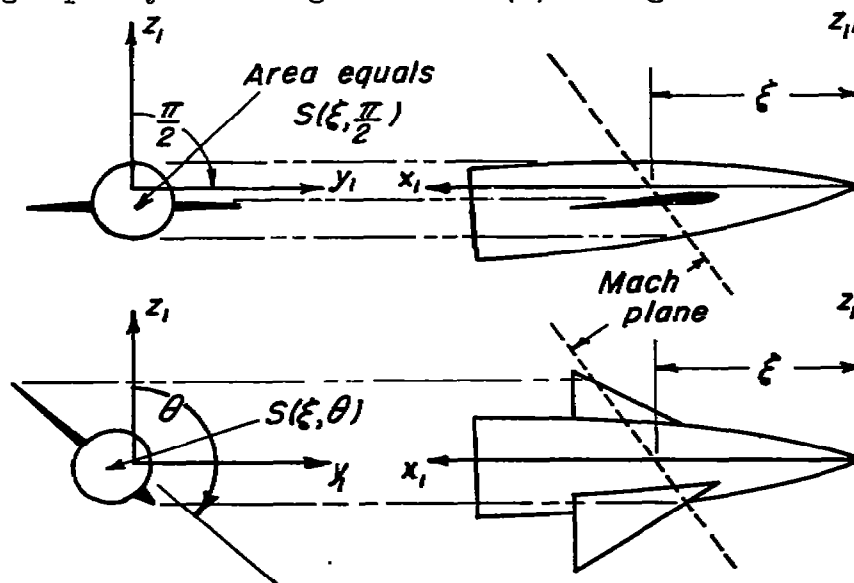
$$\varphi_1(x, r, \theta) = - \frac{1}{2\pi \sqrt{2\beta r}} \int_{-\infty}^{x_0} \frac{d\xi}{\sqrt{x_0 - \xi}} \int_{oc} \varphi_{n_1}[\xi - f(\xi, \sigma), \sigma] d\sigma \quad (11)$$

where  $\int_{oc} d\sigma$  is a line integral around the airplane surface in the oblique cut.

The velocity potential  $\varphi_1$  given by equation (11) is exactly the same as that induced on a large cylinder by a line of sources distributed along the  $x_1$  axis from  $-\infty$  to  $x_0$ , the variation of their strengths

being given by  $\int_{oc} \varphi_{n_1} d\sigma$ . (This was first pointed out by Hayes in ref. 2.) The physical significance of the term  $\int_{oc} \varphi_{n_1} d\sigma$  is more

readily grasped by referring to sketch (c). Imagine a series of Mach



Sketch (c)

planes parallel to the  $y_1$  axis each given by the equation  $x_1 - \beta z_1 = \text{constant}$ . Place the airplane in its normal flight attitude. Each Mach plane slices through the airplane, defining, thereby, a certain area composed of the region on the Mach plane within the airplane surface. Project these areas on planes normal to the free stream (i.e.,  $y_1, z_1$  planes) and designate the resulting area distribution by  $S(\xi, \frac{\pi}{2})$ . The integral  $\int_{oc} \varphi_{n_1} d\sigma$  is then proportional to the streamwise rate of change of these normally projected, obliquely cut areas; that is, for the airplane so placed,

$$\int_{oc} \varphi_{n_1} d\sigma = U_0 \frac{\partial}{\partial \xi} S\left(\xi, \frac{\pi}{2}\right)$$

Now, keeping the Mach planes fixed,<sup>1</sup> revolve the airplane about the  $x_1$  axis (not about its own body axis unless the latter happens to coincide with the  $x_1$  axis in our wind-axes system) and repeat the above process for all orientations in a complete  $360^\circ$  rotation. For any given angle

$$\int_{oc} \varphi_{n_1} d\sigma = U_0 \frac{\partial}{\partial \xi} S(\xi, \theta) = U_0 S'(\xi, \theta) \quad (12)$$

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<sup>1</sup>Or holding the airplane fixed, rotate the Mach planes - always, of course, keeping them tangent to the characteristic Mach cone.



Since one can show  $(\varphi_r)_{r \rightarrow \infty} = -\beta(\varphi_x)_{r \rightarrow \infty}$ , combining equations (11) and (12) yields

$$(\varphi_{1x})_{r \rightarrow \infty} = -\frac{1}{\beta} (\varphi_{1r})_{r \rightarrow \infty} = -\frac{U_0}{2\pi\sqrt{2\beta r}} \int_{-\infty}^{x_0} \frac{\delta^{(1)}(\xi, \theta) d\xi}{\sqrt{x_0 - \xi}} \quad (13)$$

and, by means of equation (3), this gives the complete contribution to the wave drag of the first term in equation (2).

Consider next equation (8). Taking the derivative with respect to  $n_1$  as indicated, we have

$$\varphi_2(x, y, z) = \frac{1}{2\pi} \frac{\partial}{\partial x} \iint_{\tau} \frac{\varphi(x_1, s_1) \left( \frac{dy_1}{dn_1} \cos \theta + \frac{dz_1}{dn_1} \sin \theta \right) dx_1 ds_1}{[(y-y_1)^2 + (z-z_1)^2] \sqrt{(x-x_1)^2 - \beta^2(y-y_1)^2 - \beta^2(z-z_1)^2}}$$

Proceeding as before, setting  $x = x_0 + \beta r$ ,  $y = r \cos \theta$ ,  $z = r \sin \theta$ , and letting  $r$  go to infinity, we find

$$\varphi_2(x, r, \theta) = \frac{\beta}{2\pi\sqrt{2\beta r}} \frac{\partial}{\partial x_0} \iint_{\tau} \frac{\varphi(x_1, s_1) \left( \frac{dy_1}{dn_1} \cos \theta + \frac{dz_1}{dn_1} \sin \theta \right) dx_1 ds_1}{\sqrt{x_0 - x_1 + \beta y_1 \cos \theta + \beta z_1 \sin \theta}} \quad (14)$$

Introduce the transformation given by equation (10) and  $\varphi_2$  can be expressed as

$$\varphi_2(x, r, \theta) = \frac{\beta}{2\pi\sqrt{2\beta r}} \frac{\partial}{\partial x_0} \int_{-\infty}^{x_0} \frac{d\xi}{\sqrt{x_0 - \xi}} \int_{0c} \varphi[\xi - f(\xi, \sigma), \sigma] \left( \frac{dy_1}{dn_1} \cos \theta + \frac{dz_1}{dn_1} \sin \theta \right) d\sigma$$

Notice (see sketch (d)) that the term  $\left( \frac{dy_1}{dn_1} \cos \theta + \frac{dz_1}{dn_1} \sin \theta \right) ds_1$  is simply the component of  $ds_1$  normal to the plane  $\theta = \text{constant}$ .

Designate this direction by the coordinate  $\sigma_\theta$ , as in sketch (d), and equation (14) becomes

$$\varphi_2(x, r, \theta)$$

$$= \frac{\beta}{2\pi \sqrt{2\beta r}} \frac{\partial}{\partial x_0} \int_{-\infty}^x \frac{d\xi}{\sqrt{x_0 - \xi}} \int_{oc} \varphi d\sigma_\theta$$

Integrating by parts (notice that  $\frac{\partial}{\partial \xi} \varphi[\xi - f(\xi, \sigma), \sigma] = u[\xi - f(\xi, \sigma), \sigma]$  where  $u(x, \sigma) = \frac{\partial}{\partial x} \varphi(x, \sigma)$ , since in

linearized theory  $\partial f / \partial \xi$  can be neglected relative to unity) and using the relation for pressure coefficient

$$C_p = \frac{p - p_0}{\frac{1}{2} \rho_0 U_0^2} = - \frac{2\varphi_x}{U_0} \quad (15)$$

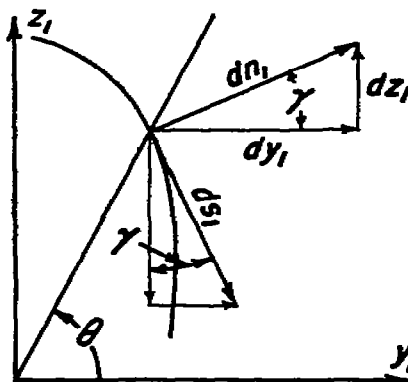
we find

$$\varphi_2(x, r, \theta)$$

$$= \frac{-\beta U_0}{4\pi \sqrt{2\beta r}} \int_{-\infty}^{x_0} \frac{d\xi}{\sqrt{x_0 - \xi}} \int_{oc} C_p d\sigma_\theta$$

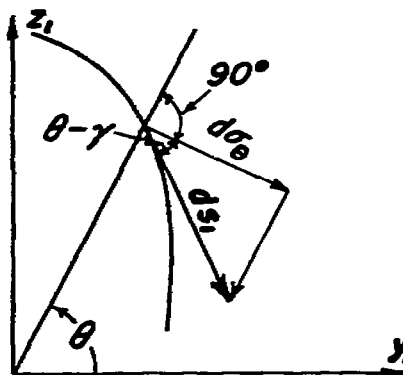
(16)

The velocity potential  $\varphi_2$  is again exactly the same as that induced on a large cylinder by a line of sources distributed along the  $x_1$  axis from  $-\infty$  to  $x_0$ , the variation of their strength this time being given by  $\frac{\beta U_0}{2} \int_{oc} C_p d\sigma_\theta$ . The physical significance of  $\int_{oc} C_p d\sigma_\theta$  can be demonstrated



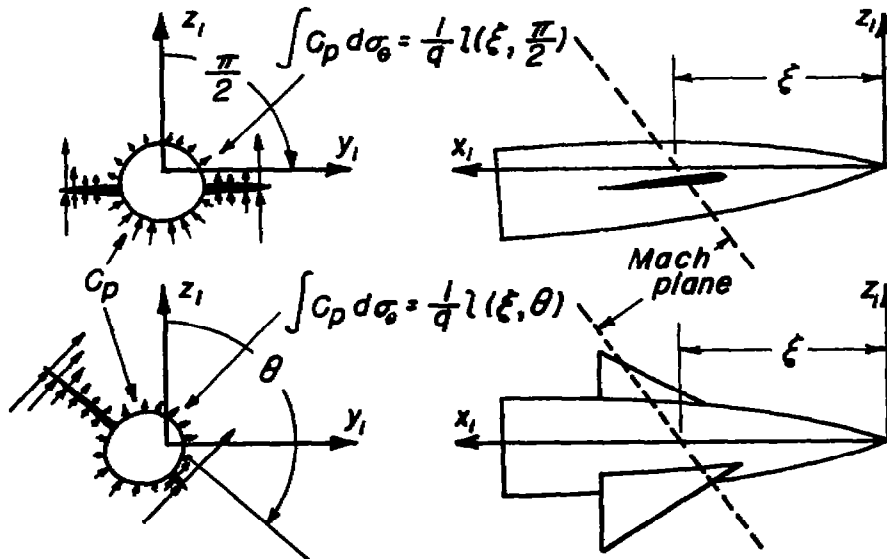
$$\frac{dy_1}{dn_1} = \cos \gamma ; \quad \frac{dz_1}{dn_1} = \sin \gamma$$

$$\cos \gamma \cos \theta + \sin \gamma \sin \theta = \cos(\gamma - \theta)$$



Sketch (d)

with the aid of sketch (e). As before, imagine a series of planes



Sketch (e)

( $x_1 - \beta z_1 = \text{constant}$ ) parallel to the  $y_1$  axis. Again place the airplane in its normal flight attitude. Then if we define  $l(\xi, \pi/2)$  as the lift (the component of net resultant force in the  $z_1$  direction, positive upward) on a given section formed by the intersection of a Mach plane with the airplane surface, one can show, for the airplane so placed,

$$\int_{oc} C_p d\sigma_\theta = \frac{1}{q} l\left(\xi, \frac{\pi}{2}\right)$$

where  $q$  is the free-stream dynamic pressure. If we keep the coordinate system and Mach planes fixed and revolve the airplane about the  $x_1$  axis, for each  $\theta$  the term  $q \int_{oc} C_p d\sigma_\theta$  represents at a given  $x$  the net lift<sup>2</sup> on the obliquely cut section. In general,

$$\int_{oc} C_p d\sigma_\theta = \frac{1}{q} l(\xi, \theta) \quad (17)$$

Now combining equations (16) and (17), one can show

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<sup>2</sup>If the airplane is fixed and the Mach planes are rotated,  $l(\xi, \theta)$  represents the resultant obliquely cut section force normal to the free stream and parallel to the plane  $\theta = \text{constant}$ , and  $d\sigma_\theta$  is an element of length normal to that plane.

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$$(\varphi_{2x})_{r \rightarrow \infty} = -\frac{1}{\beta} (\varphi_{2r})_{r \rightarrow \infty} = \frac{U_0}{2\pi \sqrt{2\beta r}} \int_{-\infty}^{x_0} \frac{\frac{\beta}{2q} l'(\xi, \theta) d\xi}{\sqrt{x_0 - \xi}} \quad (18)$$

where  $l'(\xi, \theta) = \frac{\partial}{\partial \xi} l(\xi, \theta)$ , that is, the streamwise gradient of the lift

on the obliquely cut section. Finally, if equations (13) and (18) are placed into equation (3) and the  $x$  integration is carried out, there results

$$\frac{D}{q} = -\frac{1}{4\pi^2} \int_0^{2\pi} d\theta \int_{-L_1(\theta)}^{L(\theta)} dx_1 \int_{-L_1(\theta)}^{L(\theta)} dx_2 \left[ S''(x_1, \theta) - \frac{\beta}{2q} l'(x_1, \theta) \right] \left[ S''(x_2, \theta) - \frac{\beta}{2q} l'(x_2, \theta) \right] \ln |x_1 - x_2| \quad (19)$$

where for constant  $\theta$  the intersecting Mach planes extend from  $x = -L_1(\theta)$  to  $x = L(\theta)$ .

Equation (19) gives the wave drag of any lifting or nonlifting air-plane in a steady supersonic stream, the only approximations being those basic to linearized theory.

#### SOME SIMPLE APPLICATIONS

##### A Plane Wing

In order to demonstrate the general applicability of equation (19), let us use it to solve some simple problems. Some of the simplest kinds of aerodynamic problems are those concerning single planar systems, that is, systems composed of thin wings, the surfaces of which are everywhere close to a given plane. For such cases the area and lift terms in equation (19) have certain symmetry properties. Specifically

$$S''(x, \theta) = S''(x, -\theta)$$

$$l'(x, \theta) = -l'(x, -\theta)$$

or  $l'(x, \theta)$  is an odd and  $S''(x, \theta)$  is an even function of  $\theta$ . Hence,

$$\int_0^{2\pi} S''(x, \theta) l'(x, \theta) d\theta = 0$$

and equation (19) reduces to

$$\begin{aligned} \frac{D}{q} = & -\frac{1}{4\pi^2} \int_0^{2\pi} d\theta \int_{-L_1(\theta)}^{L(\theta)} dx_1 \int_{-L_1(\theta)}^{L(\theta)} dx_2 S''(x_1, \theta) S''(x_2, \theta) \ln|x_1 - x_2| - \\ & \frac{1}{4\pi^2} \int_0^{2\pi} d\theta \int_{-L_1(\theta)}^{L(\theta)} dx_1 \int_{-L_1(\theta)}^{L(\theta)} dx_2 \left(\frac{\beta}{2q}\right)^2 l'(x_1, \theta) l'(x_2, \theta) \ln|x_1 - x_2| \quad (20) \end{aligned}$$

which simply exemplifies the well-known fact that in the study of a thin planar wing, lifting and thickness effects can be analyzed separately. Further, since one can prove (for any  $f(x)$  that can be expanded in a Fourier series)

$$-\int_{-1}^1 \int_{-1}^1 f(x_1) f(x_2) \ln|x_1 - x_2| dx_1 dx_2 \geq 0$$

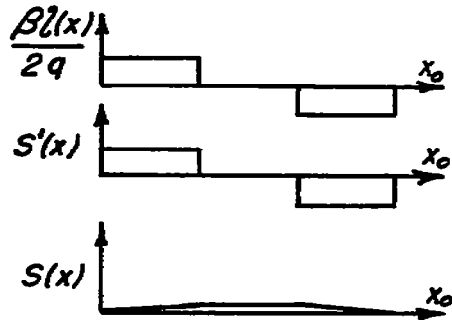
equation (20) shows that the only way the wave drag of a single wing can be zero is for both  $S''(x, \theta)$  and  $l'(x, \theta)$  to be identically zero, which is the trivial case of no wing at all.

#### A Biplane

Consider next a biplane, in particular, the Busemann type biplane shown in sketch (f) traveling at the highest Mach number at which the wave drag is known (a priori) to be zero, that is, at a Mach number

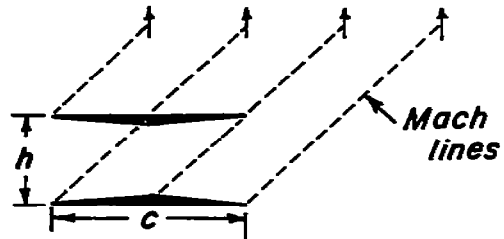
equal to  $\sqrt{\left(\frac{c}{2h}\right)^2 + 1}$ . Since the flow is two-dimensional, we need only study the cut represented by  $\theta = \frac{\pi}{2}$  and consider strips of unit width

in the spanwise direction. Sketch (f) shows the variation of  $S(x)$ ,  $S'(x)$ , and  $\frac{\beta l}{2q}(x)$  with  $x$ . One can show that the two terms  $S'(x)$  and  $\frac{\beta}{2f} l(x)$  are exactly equal for any given value of  $x$  and thus, by equation (19), the total wave drag is zero - the correct result according to linearized theory.



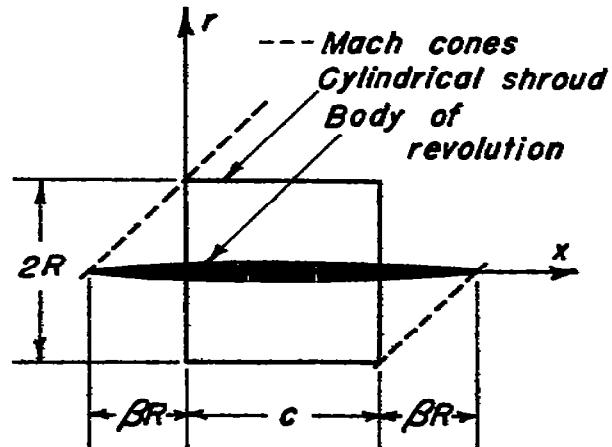
Shrouded Bodies of Revolution

As a final example, let us develop a method for calculating the shape of a body of revolution which when enshrouded by a thin cylindrical tube (having negligible thickness) will result in zero wave drag for the combination. Let the length of the tube be  $c$  and its radius  $R$  (see sketch (g)). Assume a certain loading on the cylinder (i.e., a value of  $\Delta C_p(x) = C_{Pinner} - C_{Pouter}$ ). The equation of the projection in the  $xy$  plane of the curve formed by the intersection of the oblique plane,  $x = \xi + \beta z$ , and the cylinder,  $y^2 + z^2 = R^2$ , is given by



Sketch (f)

$$y^2 + \left(\frac{x - \xi}{\beta}\right)^2 = R^2$$



Sketch (g)

Hence, the oblique section lift can be written

$$\frac{l(\xi)}{q} = 2 \int_0^{\sqrt{R^2 - \left(\frac{\xi}{\beta}\right)^2}} \Delta C_p(\xi + \beta \sqrt{R^2 - y^2}) dy; \quad -\beta R \leq \xi \leq 0 \quad (21a)$$

$$\frac{l(\xi)}{q} = 2 \int_0^R \Delta C_p(\xi + \beta \sqrt{R^2 - y^2}) dy - 2 \int_{\sqrt{R^2 - (\frac{\xi}{\beta})^2}}^R \Delta C_p(\xi - \beta \sqrt{R^2 - y^2}) dy;$$

$$0 < \xi < \beta R \quad (21b)$$

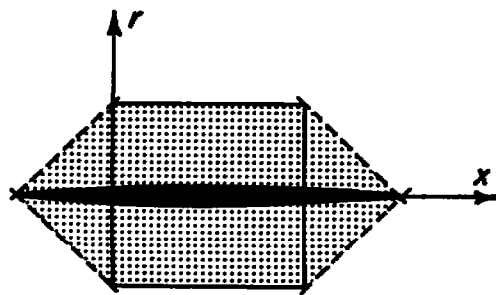
$$\frac{l(\xi)}{q} = 2 \int_0^R [\Delta C_p(\xi + \beta \sqrt{R^2 - y^2}) - \Delta C_p(\xi - \beta \sqrt{R^2 - y^2})] dy;$$

$$\beta R < \xi < c - \beta R \quad (21c)$$

and so forth. Now if the interior body of revolution is slender enough to be analyzed by slender-body theory (i.e., source strength is proportional to  $S'(x)$ ), the oblique section loading on the body itself can be neglected and the body shape for zero total wave drag is, therefore, simply

$$S'(\xi) = \frac{\beta}{2q} l(\xi) \quad (22)$$

where the values of  $l(\xi)$  are given by equations (21).



Sketch (h)

If the body determined by the above process is real and closes, the enshrouding tube will be a perfect cylinder and the velocity of the air everywhere exterior to the shaded region in sketch (h) will be equal in magnitude and direction to the free-stream velocity.<sup>3</sup>

The solution given by equations (21) and (22) has been applied to a tube for which

$$\Delta C_p = \frac{2\lambda}{\beta^2} \left(\frac{x}{c}\right) \left(1 - \frac{x}{c}\right)$$

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<sup>3</sup>This follows directly from a theorem due to E. W. Graham (unpublished) which states that the flow fields of all finite systems having a minimum wave drag under the same specified restraint are identical outside their enclosing fore-and-after Mach cones.

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where  $\lambda$  is a constant. There results

$$S'(x) = \frac{\lambda R^2}{c} \left\{ \left( 1 - 2 \frac{x}{c} \right) \left[ \frac{x}{\beta R} \sqrt{1 - \left( \frac{x}{\beta R} \right)^2} + \cos^{-1} \left( \frac{-x}{\beta R} \right) \right] - \frac{4}{3} \left( \frac{\beta R}{c} \right) \left[ 1 - \left( \frac{x}{\beta R} \right)^2 \right]^{3/2} \right\}; \quad -\beta R \leq x \leq \beta R$$

$$S'(x) = \frac{\lambda R^2}{c} \pi \left( 1 - \frac{2x}{c} \right); \quad \beta R \leq x \leq c - \beta R$$

and these values are identical<sup>4</sup> to those derived in reference 10 by means of a different method.

#### CONCLUDING REMARKS

The wave drag of any object in a supersonic flow field governed by the wave equation is identical at a fixed Mach number to the average wave drag of a series of "equivalent" bodies of revolution. The streamwise gradient of area, measured normal to the free stream, at a section of each of these equivalent bodies of revolution is given by the sum of two quantities:

1. The streamwise gradient of area, measured in parallel oblique planes tangent to the Mach cones, along the given object
2. A term proportional to the resultant force on the object measured in the same oblique planes.

Obviously, a formal general application of the above result requires a complete knowledge of the shape of the disturbing object and its pressure distribution - a knowledge which always, of course, fixes the drag of the object. However, in many special applications one or the other of the two terms mentioned above is small or can be estimated accurately enough without a detailed knowledge of the entire airplane or its surface pressures. For example, if one wishes to find the wave drag of a wing-body combination that is symmetrical about a horizontal plane (e.g., a thin nonlifting wing mounted centrally on a body of revolution), it is not necessary to know the pressures anywhere on the wing, since (if the wing is thin enough) their contributions to the resultant forces on oblique sections are negligible. Hence, for such an example the force term in the drag equation depends only on the pressure over the body and,

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<sup>4</sup>In the equation for radial force in Graham's report the exponent 2 was omitted from the  $\beta$  term.



in fact, only on the asymmetry of these pressures in the oblique (Mach) planes. If the body is slender, the latter effect is small relative to the oblique-area gradient, and, for such configurations, the form of the supersonic area rule posed by Jones is seen to be a good approximation.

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Moffett Field, Calif., Jan. 18, 1955

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