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Summary.—This report contains some fairly simple and economic methods for calculating the load distribution on wings of any plan form based on the conceptions of lifting-surface theory. The computer work required is only a small fraction of that of existing methods with comparable accuracy. This is achieved by a very careful choice of the positions of pivotal points, by plotting once for all those parts of the downwash integral which occur frequently and by a consequent application of approximate integration methods similar to those devised by the author for lifting-line problems.

The basis of the method is to calculate the local lift and pitching moment at a number of chordwise sections from a set of linear equations satisfying the downwash conditions at two pivotal points in each section. Interpolation functions of trigonometrical form are used for spanwise integration both in setting up the downwash equations and in getting the resultant forces on the wing from the local forces. The preliminary chordwise integrations for the downwash are predigested in a series of charts (Figs. 1 to 6); it is these which make the method a practical computing proposition.

The theory is outlined in sections 2 to 5; section 6 deals with the solution of the linear equation and section 7 with the resultant forces on the wing. Some examples are worked out in section 8 to compare with other methods; one solution is given in full detail in Tables 8 to 30 as a guide for computers. Appendices I to VI discuss more carefully some salient points of the mathematical theory, and Appendix VII is intended to instruct the computer how to carry out the steps of the calculation.

1. *Introduction.*—The basic problem of aerofoil theory is the analysis and prediction of the aerodynamic forces and their distribution on wings and winglike bodies such as tailplanes, fins, etc. It seems hardly necessary to emphasize the importance of simple methods for solving this problem; so many questions of aircraft design depend on its solution.

From the mathematical point of view the problem appears by no means simple. Even after some rigorous restrictions—we neglect the effects of the viscosity of the fluid and assume all velocities produced by the action of the wing as small compared with the speed of the aircraft—there remains a potential problem in three-space dimensions with very arbitrary boundary conditions. In the first period of aircraft development which is now fading out the worst difficulties could be avoided by using Prandtl's conception of the 'lifting line'. Thus the interference between different parts of a wing was approximately reduced to two such problems in two dimensions: the flow in any plane parallel to the plane of symmetry was considered as essentially the same as that about the infinite cylindrical wing with the same cross-section;

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the interference between different wing sections was approximately represented by a local modification of the direction of flow, the 'induced angle of incidence' to be estimated for straight large aspect ratio wings as half the downwash angle far enough downstream at the same spanwise station. The calculation of this downwash was again a matter of two dimensions only. One definite advantage of this scheme is the possibility of introducing measured aerofoil section characteristics instead of the theoretical values, thus including at least to some extent the effects of the originally neglected friction. But even this simplification led to an integral equation with a nuclear function so complicated as to forbid the application of all methods for integral equations to be found in mathematical textbooks.

Among the many methods devised for the solution of this particular problem of lifting-line aerofoil theory one suggested by the author became the routine method at least in Germany and the U.S.A. thanks to its practical simplicity[1]. The appreciable reduction of computer work was mainly achieved by a consequent application of approximate integration methods, *i.e.*, the downwash integral was replaced by sums containing the circulation values at certain stations multiplied by constant once-for-all-calculable coefficients. This led to a linear system of equations with the circulations at those stations as unknowns which could be solved easily by an iteration process. No sacrifices in accuracy or limitations in the range of wing shapes to which it should be applied were necessary because the number of control stations could be so high as to meet all requirements.

It was not the mathematical desire for absolute correctness but the fundamental failure of the lifting-line conception in dealing with modern trends in wing design which aroused a new wave of interest in the lifting-surface theory. An early attempt at a lifting-surface calculation by Blenk[2] was mainly to show that the results of the lifting-line theory were quite good—at least for wings of rectangular plan form and some aspect ratios. In its physical conception the lifting surface is more simple and direct than the lifting line which by its very nature is restricted to nearly straight wings of a fairly large aspect ratio—just the type of wing which was suitable in the past as long as one had not to bother about compressibility and wing-elasticity problems and the resulting limitations. With the progress of aircraft development into the sonic zone we have hardly any choice but to abandon both the straight wing and the large aspect ratio, the first in order to avoid unnecessary wave drags and the second for structural reasons because of the very thin wing sections to which we are restricted.

This is, of course, not the first attempt to find a convenient way of dealing with lifting-surface problems. One group of investigations concentrated on wing plan forms for which some mathematical advantages existed. Thus, Kinner[3] and Krienes[4] calculated the special cases of wings with circular or elliptical plan forms by developing suitable classes of Mathieu's functions. Similarly, Fuchs[5] applied von Kármán's suggestion[6] to use Fourier integrals for the representation of the general solution to the special case of the rectangular wing. But it seems not very promising to extend any of these methods to wings of arbitrary plan forms. Therefore, many attempts have been made to use simplifying physical models for the approach to the general problem, *e.g.*, a lifting line in the quarter-chord line for calculating the downwash on the three-quarter-chord line (Weissinger[7], Mutterperl[8], Schlichting and Kahlert[9], Thwaites[10], etc.). Indeed, these methods are modifications and extensions of the lifting-line conception rather than real lifting-surface solutions. Although the reduction of computer work by such simplifications should be considerable it always raises the doubt whether the particular physical model is really a suitable substitute for the given wing. The distribution of the lift on more than one lifting line as in Schlichting and Kahlert[9] is, of course, another step towards the lifting-surface conception, but it implies still more arbitrary assumptions requiring justification.

A further important step towards a generally suitable lifting-surface method was Falkner's proposal[11] which stands half-way between a really continuous lifting surface and the vortex models mentioned above. His vortex lattice is not meant as a simplifying model of the actual wing but should be regarded as a rather crude method of approximate integration. The differences between the results of this method, which is so far the only workable lifting-surface approximation, and those of the second-order lifting-line methods do justify a further progress in that direction although the calculus is rather cumbersome. As a first notable contribution of this kind we have to consider Garner's calculations[12, 13] which are, apart from the present paper, the only genuine attempt to operate with a continuous lifting surface, *i.e.*, without physical or mathematical assumptions and models whose application needs to be justified. The results are a valuable basis for comparison although the length of time required for the calculation forbids its use as a routine method for which it was indeed not intended.

The present report aims at filling the still-existing gap: a method based on lifting-surface conceptions to calculate the lift distributions on arbitrary wings which is convenient enough for all practical needs without using auxiliary assumptions and artificial vortex configurations which seem not strictly necessary. For this purpose the approximate integration developed by the author for lifting-line calculations should again be the most powerful tool.

2. *The Integral Equation of the Lifting Surface.*—The problem of finding the wing shape for a given load distribution is relatively easy and can be solved directly by integration. But for the more interesting inverse problem—to find the aerodynamic load distribution for a given aerofoil geometry, no direct solution seems possible except for very special plan forms. This is the case with almost all such coupled problems of the potential theory: if it is easy the one way round then it leads to an integral equation for the inverse question. It is not always the more interesting problem which is more difficult; *e.g.*, the pressure distribution about a wing due to its thickness distribution may be found by integrating over sinks and sources proportional to the slope of the local thickness only, whereas the solution of an integral equation is required in order to determine the thickness distribution for a given pressure pattern.

As an introduction to the mathematical problem a concise derivation of the basic equations of lifting-surface theory does not seem out of place here, although it is not intended to indulge too much in conceptional details for which the reader is referred to Prandtl[14] and Burgers[15].

Let x, y, z be an orthogonal system of axes so that the x-axis coincides with the direction of undisturbed flow relative to the wing in its plane of symmetry (if it has one, otherwise the origin must be arbitrarily chosen in the wing surface); the z-axis points upwards almost perpendicular to the wing area which is supposed to have zero thickness and small camber and twist. The local velocity vector is split up into the undisturbed velocity U and the three small additional components u, v, w in the direction of the respective axes. The assumption that u, v, w are small compared with U is a necessary condition for a linearisation of the whole problem; as we already know from two-dimensional aerofoil theory this is not a serious restriction except near the speed of sound.

To the pressure field around the wing belongs a velocity field which can be found by integrating the linearised Euler's equations of the motion of a fluid relative to a body:

$$\left.\begin{aligned} U\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} &= 0 \\ U\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial y} &= 0 \\ U\frac{\partial w}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial z} &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

These equations are derived from the complete Euler's equations by neglecting the terms which contain products of the u, v, w and their derivatives. In the same way we obtain the continuity equation:

$$\frac{U}{\rho}\frac{\partial \rho}{\partial x} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \,. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Since no heat transfer within the fluid is assumed, and frictional effects are neglected we can consider $\rho = \rho(p)$ or $p = p(\rho)$ the connecting relation being

$$\frac{dp}{d\rho} = a^2 \qquad \ldots \qquad (3)$$

where a is the speed of sound, which again comes out as constant for $u << U$. The differential dp/ρ of the Euler's equations can also be considered as the differential of the enthalpy I of the flow element so that the Euler's equations are transformed into

$$\left.\begin{aligned} U\frac{\partial u}{\partial x} + \frac{\partial I}{\partial x} &= 0 \\ U\frac{\partial v}{\partial x} + \frac{\partial I}{\partial y} &= 0 \\ U\frac{\partial w}{\partial x} + \frac{\partial I}{\partial z} &= 0 \end{aligned}\right\} \qquad \ldots \qquad (4)$$

while the continuity equation gives:

$$\frac{U}{a^2}\frac{\partial I}{\partial x} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 . \qquad \ldots \qquad (5)$$

These four equations can be combined by differentiating the first three with respect to x, y and z and the last again to x so as to give

$$\frac{\partial^2 I}{\partial x^2}\left(1 - \frac{U^2}{a^2}\right) + \frac{\partial^2 I}{\partial y^2} + \frac{\partial^2 I}{\partial z^2} = 0 \qquad \ldots \qquad (6)$$

which for $U/a =$ constant, can be transformed into the usual Laplace equation in three dimensions by reducing *e.g.*, y and z proportional to $\sqrt{(1 - U^2/a^2)}$ (Prandtl-Glauert rule). We can concentrate, therefore, in the following entirely on the incompressible case $U/a << 1$.

Within the limits of the linearised theory I is given by

$$I - I_\infty = \frac{p - p_\infty}{\rho_\infty} \qquad \ldots \qquad (7)$$

I_∞, p_∞, ρ_∞ representing the undisturbed flow far enough away from the wing. Thus for all practical purposes the enthalpy field is equivalent to the pressure field. A sufficiently thin wing, the aerodynamic load of which is different from zero, may, therefore, be described as a discontinuity surface in the enthalpy potential field which satisfies the Laplace equation. A discontinuity surface in a potential field is usually built up by a sheet of doublets with their axes normal to it; the intensity per unit area of these doublets is proportional to the enthalpy difference on either side and, thus, to the local load density. We may call

$$l = \frac{\Delta p}{\frac{1}{2}\rho U^2} \qquad \ldots \qquad (8)$$

the non-dimensional load per unit area; the I-field is, then, determined by the condition that the discontinuity of I through the wing surface is

$$\Delta I = \frac{U^2}{2} l . \qquad \ldots \qquad (9)$$

Since I is a solution of the Laplace equation in three dimensions we may write down the equation of the $I(x,y,z)$-field by integrating over the elementary fields of doublets; for a doublet at $x = x_0$, $y = y_0$, $z = 0$ with its axis in the z-direction, this elementary field is known as

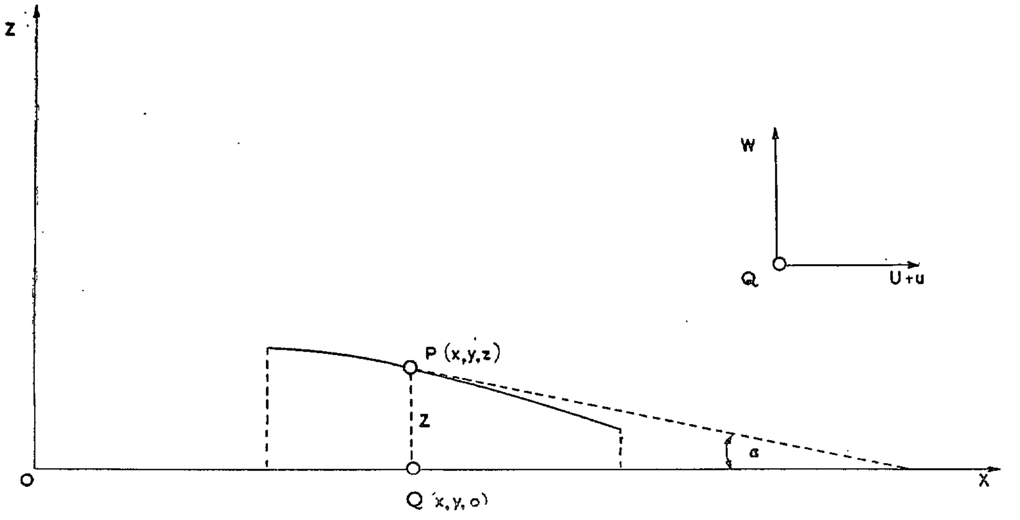
$$\frac{-1}{4\pi} \frac{z}{[(x - x_0)^2 + (y - y_0)^2 + z^2]^{3/2}} .$$

Thus we have

$$I(x,y,z) = \frac{-zU^2}{8\pi} \iint_S \frac{l(x_0,y_0)\, dx_0\, dy_0}{[(x-x_0)^2 + (y-y_0)^2 + z^2]^{3/2}} \quad \ldots \quad (10)$$

x_0, y_0 being the co-ordinates in the wing plane. In using (10) for the enthalpy we are in effect replacing the curved sheet of the wing by its projection on the plane $z = 0$, *see* sketch. To this approximation the flow given by (10) at the point Q must be the same as the actual flow at the point P of the wing. In particular if the local incidence $-\partial z/\partial x$ of the surface at P is α we must have

$$\alpha = -w/U . \quad \ldots \quad (11)$$



Section through $y =$ const.

The downwash velocity w is now obtained by integrating the third of the Euler's equations (4):

$$w(x,y,z) = \frac{-1}{U} \int_{-\infty}^{x} \frac{\partial I}{\partial z}(x',y,z)\, dx' \quad \ldots \quad (12)$$

This implies the obvious assumption that $w = 0$ at $x = -\infty$, *i.e.*, undisturbed flow far enough upstream of the wing. In the wing plane, *i.e.*, for $z = 0$ we find:

$$\frac{\partial I}{\partial z}(x',y,0) = \frac{-1}{4\pi} \frac{U^2}{2} \iint_S \frac{l(x_0,y_0)\, dx_0\, dy_0}{[(x'-x_0)^2 + (y-y_0)^2]^{3/2}} \quad \ldots \quad (13)$$

The essential part in the integral (12) is then

$$\int_{-\infty}^{x} \frac{dx'}{[(x'-x_0)^2 + (y-y_0)^2]^{3/2}} = \frac{1}{(y-y_0)^2} \left\{ 1 + \frac{x-x_0}{\sqrt{[(x-x_0)^2 + (y-y_0)^2]}} \right\} . \quad \ldots \quad (14)$$

Thus we find the downwash integral from equations (12) and (13) as

$$\alpha(x,y) = \frac{-1}{8\pi} \iint_S \frac{l(x_0,y_0)}{(y-y_0)^2} \left\{ 1 + \frac{x-x_0}{\sqrt{[(x-x_0)^2 + (y-y_0)^2]}} \right\} dx_0\, dy_0 . \quad \ldots \quad (15)$$

This integral contains a strong singularity at $y \to y_0$ which makes it intractable in this form. By a more careful approach than it was thought necessary to give in detail here, it can be shown that a 'principal value' for these integrals can be defined; thus, we are going to understand that

$$\int_a^b \frac{f(y_0)\, dy_0}{(y-y_0)^2} = \lim_{\varepsilon \to 0} \left\{ \int_a^{y-\varepsilon} \frac{f(y_0)\, dy_0}{(y-y_0)^2} + \int_{y+\varepsilon}^{b} \frac{f(y_0)\, dy_0}{(y-y_0)^2} - 2\frac{f(y)}{\varepsilon} \right\} \quad \ldots \quad (16)$$

wherever integrals of this type occur in this report. Dr. Mangler was kind enough to contribute a less objectionable derivation of these relations which is added in Appendix I.

Thus, equation (15) gives us the local incidence distribution of the wing for a given load distribution l. If, as usual, not the load distribution but the local incidence distribution is given we may consider equation (15) as the integral equation for determining the load distribution $l(x,y)$. But this is not enough. We must bear in mind that there may be many functions $l(x,y)$ which satisfy equation (15). As in two-dimensional theory we have to add a trailing-edge (Kutta-Joukowsky) condition stating the simple physical fact that at sufficiently high Reynolds numbers and moderate angles of attack we do not observe any flow around the trailing edge, *i.e.*, the downwash behind the wing is a continuation of the angle of incidence at the trailing edge. The easiest way to respect this condition is to admit only such load functions which disappear towards the trailing edge.

3. *The Distribution of Pivotal Points.*—There is little hope of finding direct and complete solutions of our integral equation (15). The best thing we can do is to choose a limited number of independent load distributions out of which we construct linear combinations so as to satisfy the integral at a certain number of so called ' pivotal points '. The more of these points we take into account the more independent load distributions are available, *i.e.*, the more accurate will be the resulting load distribution. On the other hand the computing effort is roughly proportional to the square of the number of pivotal stations ; therefore, we have to find a reasonable compromise.

The pivotal points should not be arbitrarily chosen because the calculation methods depend largely on this choice. It is fairly obvious that a concentration of most or all of these pivotal points in one part of the wing will hardly give a reliable solution in other parts of the wing. An absolutely equi-distant distribution of them might also not produce the best results; the kernel function of our integral equation is hardly of a type to suggest that.

As to the spanwise distribution of pivotal stations it is very unlikely that we can find a better distribution than that which we used in the lifting-line theory, because the dominating part of the kernel function of our integral equation is the term $1/(y - y_0)^2$ which occurs also in the lifting-line theory. There we found the best distribution of pivotal points by equally dividing the semi-circle over the wing span, Ref. 1.

This choice had some important advantages over any other distribution of the same number of stations, namely:

(*a*) half the coefficients of the sum representing the downwash integral in our approximate integration method were zero

(*b*) the system of equations that replaced the integral equation could always be solved by an iteration process instead of by elimination.

If we succeed in securing these advantages also for our lifting-surface downwash integrals this would mean a reduction of the computer work required to a small fraction of what was needed otherwise.

Because the aspect ratio of most of the wings under discussion is, although small, nevertheless greater than one, the number of chordwise stations must be even more restricted than the number of spanwise stations, for what counts most towards the computing effort is the total number of pivotal points, which is the product of the number of spanwise and chordwise stations. This means that it matters very much where the chordwise position of these pivotal stations will be. We know already from two-dimensional aerofoil theory that with regard to forces and moments the rear parts of the wing skeleton count much more than the front parts and this tendency is even more pronounced with wings of small aspect ratio. Since we have a fully developed theory only for the infinite aspect ratio we take these results to decide about the chordwise position of our pivotal stations, the number of which is supposed to be very small.

For reference we begin by repeating some well-known results of the two-dimensional aerofoil theory. The skeleton line being given by $z(x)$ between $0 < x < c$ we obtain the local downwash $w/U = dz/dx$ from the integral

$$\frac{w}{U}(x) = \frac{-1}{4\pi} \int_0^c \frac{l(x')\,dx'}{x - x'} . \qquad (17)$$

It has been useful to introduce the angular co-ordinate

$$\varphi = \cos^{-1}\left(1 - 2\frac{x}{c}\right) . \qquad (18)$$

Assessing the lift distribution as

$$l = a_0 \cot\frac{\varphi}{2} + \sum_1^\infty a_n \sin n\varphi \qquad (19)$$

we obtain for the local downwash by working out equation (17)

$$\alpha\left(\frac{x}{c}\right) = -\frac{w}{U} = \frac{a_0}{4} - \tfrac{1}{4} \sum_1^\infty a_n \cos n\varphi . \qquad (20)$$

Lift and pitching moments about the quarter-chord point are given as follows

$$C_L = \frac{\pi}{2}\left(a_0 + \frac{a_1}{2}\right) \qquad (21)$$

$$C_m = -\frac{\pi}{16}(a_1 - a_2) . \qquad (22)$$

These results are fairly well known from the classical analysis of the thin aerofoil in two-dimensional flow by Birnbaum, Munk and Glauert. If we express a_0, a_1 and a_2 by Fourier analysis from equation (20) we obtain the well-known Munk's integrals for lift and pitching moment. But the problem that interests us is somewhat different: supposing we have only a limited number of points along the chord where α can be given, how are these to be placed so as to obtain lift and perhaps also the pitching moment as accurately as possible?

Let us first consider the case of only one chordwise station. Since it is the curvature of the skeleton line which determines the moment we cannot expect to obtain that from the incidence at one station only. But we can demand that

$$C_L = K \,.\, \alpha\left(\frac{x_1}{c}\right) \qquad (23)$$

should be fulfilled as best possible; comparing equations (23) and (21) we see that for

$$K = 2\pi$$

and

$$\frac{x_1}{c} = 0\cdot75 \qquad (\cos\varphi = -0\cdot5 \text{ or } \varphi = \tfrac{2}{3}\pi) \qquad (24)$$

equation (23) holds if $\alpha(\varphi)$ is given by the first two terms of equation (20).

This is the whole story of the three-quarter–chord point, which means that if we have only one chordwise station for measuring the incidence this should be done at three-quarter chord in order to have the most reliable value for the lift coefficient. We can interpret it also in a somewhat different way: if we measure the downwash at three-quarter chord it does not matter how we assess the lift distribution: whether we take only the cot $\varphi/2$-term or only the sin φ-term or any linear combination of both to represent a certain total lift, in every case the downwash at three-quarter chord is exactly the same. Thus, if we represent the chordwise lift distribution by the first term only the second is implicitly taken into account.

It is a lucky accident and should be appreciated as such that even the concentration of the chordwise lift distribution into a single vortex at quarter-chord induces the same downwash at three-quarter chord. This is the main justification of some second-order lifting-line theories, *e.g.*, those of Weissinger and Mutterperl for swept wings and the low aspect-ratio theories by the author and Helmbold.

If we can choose two chordwise pivotal stations we may admit three terms of the series development (19) and (20) and demand both the lift and the moment to be fully valid within this restriction to the first three terms; *i.e.*, we write

$$C_L = K_1 \,.\, \alpha\left(\frac{x_1}{c}\right) + K_2 \,.\, \alpha\left(\frac{x_2}{c}\right) \qquad \ldots \quad (25)$$

and

$$C_m = K_3 \,.\, \alpha\left(\frac{x_1}{c}\right) + K_4 \,.\, \alpha\left(\frac{x_2}{c}\right). \qquad \ldots \quad (26)$$

The condition that these two equations are compatible with equations (21) and (22) for any a_0, a_1, and a_2 values gives six equations for the unknown $K_1 \ldots K_4$ and (x_1/c), (x_2/c). The solution is:

$$\left.\begin{aligned} K_1 &= +\pi\left(1 + \frac{1}{\sqrt{5}}\right) = +4\cdot5466 \\ K_2 &= +\pi\left(1 - \frac{1}{\sqrt{5}}\right) = +1\cdot7366 \\ K_3 &= -K_4 = -\frac{\pi}{2\sqrt{5}} = -0\cdot7025 \end{aligned}\right\} \qquad \ldots \quad (27)$$

and

$$\left.\begin{aligned} \frac{x_1}{c} &= \frac{5 + \sqrt{5}}{8} = 0\cdot9045 \qquad (\varphi_1 = \tfrac{4}{5}\pi) \\ \frac{x_2}{c} &= \frac{5 - \sqrt{5}}{8} = 0\cdot3455 \qquad (\varphi_2 = \tfrac{2}{5}\pi) \end{aligned}\right\} \qquad \ldots \quad (28)$$

With this choice of the pivotal stations we have the advantage that we can assess the lift distribution with the first two terms $a_0 \cot \varphi/2 + a_1 \sin \varphi$ and the omitted third term cannot affect the lift coefficient nor the moment coefficient, *i.e.*, this particular choice of two chordwise stations is roughly as good as any arbitrary choice of three such stations.

As a rule we can regard the representation of the skeleton line by a parabola of the second or third order as quite satisfactory. The main practical case requiring some more care are wings with flaps or aileron. This case will be dealt with in Appendix II.

If we want to take more than two chordwise pivotal points into account in order to find out some more details about the pressure distribution along the chord it can be shown that for p pivotal points their best distribution is given by

$$\varphi_n = \frac{2\pi n}{2p + 1} \qquad n = 1, 2, 3 \ldots p\,.$$

4. *The Chordwise Integration of the Downwash.**—To do the chordwise integration first and the spanwise part afterwards seems the most obvious way in dealing with equation (15). The main advantage of this sequence is the fact that the spanwise integral is thus brought very close to the downwash integral of the lifting line for which excellent numerical methods are already existing.

* These influence functions have been recalculated by the Mathematics Division, N.P.L., and are now available in the form of tables. For accurate work the use of these tables is recommended, and copies can be obtained on application to the Aerodynamics Division, N.P.L.

It may be worth mentioning that in supersonic aerofoil theory another possible way—to begin with conical fields which are integrated mainly in spanwise direction—yields the better results, owing to the fact that the downwash in any pivotal station is only affected by singularities in its forecone.

The first thing we have to do is to choose some appropriate function for the chordwise load distribution which may be put into the downwash integral equation (15). The most natural choice is the load functions which occur in two-dimensional aerofoil theory, equation (19). For the greatest part of the wing a couple of these functions should describe the chordwise pressure distribution at least as well as any other arbitrarily chosen family of functions. The only practical exception is the very central section of a swept-back wing where the pressure at the leading edge does not run up into the usual suction peak; but it seems to be better to stick to some uniformity in the assumptions of our calculations and make some readjustments later if necessary rather than to over-emphasise what is only a very local effect.

The integrals we have to deal with are of the type

$$\int_{x_{0l}}^{x_{0t}} l(x_0,y_0)\left\{1+\frac{x-x_0}{\sqrt{[(x-x_0)^2+(y-y_0)^2]}}\right\}dx_0 \quad \ldots \quad (29)$$

(x_{0l} = leading edge, x_{0t} = trailing edge of inducing wing section, y_0 = const). For our further convenience we can introduce some auxiliary non-dimensional co-ordinates X, Y

$$X=\frac{x-x_{0l}}{c(y_0)}, \qquad Y=\frac{y-y_0}{c(y_0)}, \text{ and } X_0=\frac{x_0-x_{0l}}{c(y_0)}. \quad \ldots \quad (30)$$

The first chordwise load distribution which we are going to consider is

$$l_0 = a_0 \cot \varphi/2 \quad \ldots \quad (31)$$

with

$$\varphi = \cos^{-1}(1-2X_0) \quad \ldots \quad (18)$$

which gives the lift

$$C_L = a_0 \,.\, \pi/2 \quad \ldots \quad (32)$$

but no pitching moment about the quarter-chord point. We are relating the chordwise part of the downwash integral with the load distribution l_0 equation (31), to the $(C_L c)$-value at y_0; this integral may be called the influence function i:

$$i(x,y,y_0)=\frac{1}{C_L \,.\, c}\int_{x_{0l}}^{x_{0t}} l_0(x_0,y_0)\left\{1+\frac{x-x_0}{\sqrt{[(x-x_0)^2+(y-y_0)^2]}}\right\}dx_0 \quad \ldots \quad (33)$$

which may be transferred into the above-defined non-dimensional co-ordinates as follows:

$$i(X,Y)=\frac{1}{\pi}\int_0^{\pi}\cot\frac{\varphi}{2}\sin\varphi\left\{1+\frac{X-\dfrac{1-\cos\varphi}{2}}{\sqrt{\left[\left(X-\dfrac{1-\cos\varphi}{2}\right)^2+Y^2\right]}}\right\}d\varphi$$

$$=1+\frac{1}{\pi}\int_0^{\pi}\frac{(1+\cos\varphi)(2X-1+\cos\varphi)\,d\varphi}{\sqrt{[(2X-1+\cos\varphi)^2+4Y^2]}}. \quad \ldots \quad (34)$$

This is an elliptic integral the reduction of which to tabulated standard integrals is so complicated and lengthy that direct computation by graphical and numerical methods was indicated except for a few special cases.

A second load distribution may be given by the second term of equation (19) or any combination of the first two terms. A fairly convenient distribution is the one which gives a pitching moment but no lift, namely

$$l_1 = a_1\left[\cot\frac{\varphi}{2} - 2\sin\varphi\right] \qquad \ldots \qquad (35)$$

with

$$C_{L1} = 0 \qquad C_{m1} = a_1 \,.\, \pi/8\,. \qquad \ldots \qquad (36)$$

For this second load distribution we define another influence function j :

$$j(x,y,y_0) = \frac{1}{C_{m1}\,.c\,(y_0)}\int_{x_{0l}}^{x_{0t}} l_1(x_0,y_0)\left\{1 + \frac{x - x_0}{\sqrt{[(x - x_0)^2 + (y - y_0)^2]}}\right\} dx_0 \quad \ldots \quad (37)$$

which may be transformed into

$$j(X,Y) = \frac{4}{\pi}\int_0^{\pi}\frac{(2\cos^2\varphi + \cos\varphi - 1)(2X - 1 + \cos\varphi)\,d\varphi}{\sqrt{[(2X - 1 + \cos\varphi)^2 + 4Y^2]}} \qquad \ldots \qquad (38)$$

which again, as the previous one, had to be calculated numerically and graphically.

For large Y-values we have calculated the i- and j-functions by series expansions for $Y = \infty$. It is already obvious that for large Y the induction of a wing section loaded according to equation (33) may approximately be represented by placing a concentrated load in the quarter-chord point; this gives the asymptotic value for i

$$i_\infty = 1 + \frac{X - \frac{1}{4}}{\sqrt{[(X - \frac{1}{4})^2 + Y^2]}}\,. \qquad \ldots \qquad (39)$$

If we introduce for simplicity

$$A = \frac{X - \frac{1}{4}}{Y} \qquad \ldots \qquad (40)$$

equation (37) may be written in the form

$$i(X,Y) = 1 + \frac{1}{\pi}\int_0^{\pi}\frac{(1 + \cos\varphi)\left(A + \dfrac{2\cos\varphi - 1}{4Y}\right)d\varphi}{\sqrt{\left[1 + \left(A + \dfrac{2\cos\varphi - 1}{4Y}\right)^2\right]}}\,. \qquad \ldots \qquad (41)$$

This suggests a power series development

$$i = i_\infty + \frac{f_1(A)}{Y} + \frac{f_2(A)}{Y^2} + \frac{f_3(A)}{Y^3} + \ldots \qquad \ldots \qquad (42)$$

with

$$\left.\begin{aligned}
f_1(A) &= \left[\frac{\partial i}{\partial(1/Y)}\right]_{Y=\infty} = \frac{1}{4\pi}\int_0^{\pi}\frac{(1 + \cos\varphi)(2\cos\varphi - 1)\,d\varphi}{(1 + A^2)^{3/2}} = 0\\
f_2(A) &= \frac{1}{2!}\left[\frac{\partial^2 i}{\partial(1/Y^2)}\right]_{Y=\infty} = \frac{-3A}{32\pi(1 + A^2)^{5/2}}\int_0^{\pi}(1 + \cos\varphi)(2\cos\varphi - 1)^2\,d\varphi\\
&= \frac{-3A}{32(1 + A^2)^{5/2}}\\
f_3(A) &= \frac{1}{128}\,\frac{1 - 4A^2}{(1 + A^2)^{7/2}}\\
f_4(A) &= \frac{15A}{2048}\,\frac{3 - 4A^2}{(1 + A^2)^{9/2}}\\
f_5(A) &= \frac{-9}{4096}\,\frac{1 - 12A^2 + 8A^4}{(1 + A^2)^{11/2}} \text{ etc.}
\end{aligned}\right\}. \quad (43)$$

A similar series development for large Y-values can be made with $j(X,Y)$ and gives:

$$j\left(A, \frac{1}{Y}\right) = \frac{g_1(A)}{Y} + \frac{g_2(A)}{Y^2} + \frac{g_3(A)}{Y^3} + \ldots \quad \ldots \quad (44)$$

with

$$\left.\begin{aligned} g_1 &= \frac{1}{(1+A^2)^{3/2}} & g_4 &= -\frac{15A}{256}\frac{3-4A^2}{(1+A^2)^{9/2}} \\ g_2 &= \frac{3A}{8(1+A^2)^{5/2}} & g_5 &= \frac{90}{4096}\frac{1-12A^2+8A^4}{(1+A^2)^{11/2}} \\ g_3 &= \frac{-3}{32}\frac{1-4A^2}{(1+A^2)^{7/2}} & &\text{etc.} \end{aligned}\right\} . \quad \ldots \quad (45)$$

The results of these calculations are plotted in a series of diagrams, Figs. 1 to 6, in which i and j are shown as functions of Y with X as parameter. The scales are so chosen as to suit the accuracy requirements of the final calculation where these diagrams are widely used.

It is difficult to obtain a series development for small Y-values. The values of i and j at $Y = 0$ are easy: in equation (38) and (37) the factor in brackets is either 2 or zero:

$$\lim_{y \to y_0}\left[1 + \frac{x - x_0}{\sqrt{[(x-x_0)^2 + (y-y_0)^2]}}\right] = \begin{cases} 2 & x_0 < x \\ 0 & x_0 < x\,. \end{cases} \quad \ldots \quad (46)$$

Thus, with

$$\varphi_1 = \cos^{-1}(1 - 2X)$$

we obtain

$$i(X,0) = \frac{2}{\pi}\int_0^{\varphi_1}(1 + \cos\varphi)\,d\varphi = \frac{2}{\pi}(\varphi_1 + \sin\varphi_1)$$

$$= \frac{2}{\pi}\left\{\cos^{-1}(1-2X) + 2\sqrt{[X(1-X)]}\right\} \quad \ldots \quad (47)$$

and

$$j(X,0) = \frac{8}{\pi}\int_0^{\varphi_1}(2\cos^2\varphi + \cos\varphi - 1)\,d\varphi = \frac{8}{\pi}\left(\frac{\sin 2\varphi_1}{2} + \sin\varphi_1\right)$$

$$= \frac{32}{\pi}X^{1/2}(1-X)^{3/2}. \quad \ldots \quad (48)$$

The derivatives of i and j with respect to X are also a matter of course:

$$\frac{\partial i}{\partial X}(X,0) = 2\frac{l_0}{C_L} = \frac{4}{\pi}\sqrt{\left(\frac{1-X}{X}\right)} \quad \ldots \quad (49)$$

$$\frac{\partial j}{\partial X}(X,0) = 2\frac{l_1}{C_m} = \frac{16}{\pi}(1-4X)\sqrt{\left(\frac{1-X}{X}\right)} \quad \ldots \quad (50)$$

and:

$$\frac{\partial^2 i}{\partial X^2}(X,0) = -\frac{2}{\pi}\frac{1}{X^{3/2}\sqrt{(1-X)}} \quad \ldots \quad (51)$$

$$\frac{\partial^2 j}{\partial X^2}(X,0) = -\frac{8}{\pi}\frac{1+4X-8X^2}{X^{3/2}\sqrt{(1-X)}}. \quad \ldots \quad (52)$$

But the derivatives with respect to Y are more difficult. From their definitions it is obvious that i and j are symmetrical in Y because they contain only Y^2 in the integral; therefore, we have $\partial i/\partial Y = 0$ and $\partial j/\partial Y = 0$ for $Y = 0$. The difficulty begins with the next derivative $\partial^2 i/\partial Y^2$ which does not exist for $Y = 0$. Thus, we cannot develop a Taylor series for $i(Y)$ from $Y = 0$; this is a usual feature of some elliptical integrals with the modulus k near 1, to which our integrals i and j are much akin. But it is still possible to develop some other series containing also logarithmic terms; the series development of $i(X,Y)$ begins thus with

$$i(X,Y) = i(X,0) + Y^2 [K_1 \ln |Y| + K_2] + \ldots \quad (53)$$

and it will be shown in Appendix III that

$$K_1 = \frac{-1}{C_L} \frac{dl_0}{dX} = \frac{1}{\pi X^{3/2} \sqrt{(1-X)}} \quad (54)$$

and accordingly in the analogous series development for j:

$$K_1 = \frac{-1}{C_m} \frac{dl_1}{dX} = \frac{4}{\pi} \frac{1 + 4X - 8X^2}{X^{3/2} \sqrt{(1-X)}} . \quad (55)$$

Although not very apparent in any plotting of i or j over Y, *see* Figs. 2 and 4, this logarithmic term does affect the eventual downwash integral to some extent. The spanwise integration includes a factor proportional $1/Y^2$ in the integrand which thus contains a logarithmic singularity for $y = y_0$ not covered by the 'principal value' of equation (16).

Any further terms of the chordwise load distribution can, of course, be dealt with in quite the same manner, but there seems to be no direct need for their calculation especially with regard to our particular choice of chordwise control stations.

5. *The Spanwise Integration of the Downwash.*—5.1. *The Regular Part of the Downwash Integral.*—After having concentrated the chordwise part of our downwash integral (15) into the influence functions i and j we are now on fairly well-known ground with the rest of the integration because something very similar has already been worked out in the lifting-line theory. With the influence functions i and j the downwash integral is reduced to

$$\alpha(x,y) = \frac{-1}{8\pi} \int_{-b/2}^{b/2} \frac{(C_L c)(y') \, . \, i(x,y,y') \, dy'}{(y-y')^2} - \frac{1}{8\pi} \int_{-b/2}^{b/2} \frac{(C_m c)(y') - j(x,y,y') \, dy'}{(y-y')^2} \quad (56)$$

plus further expressions of a similar pattern if we consider more than two independent chordwise load distributions. With the restriction to only one of them we have to deal with only the first integral. For convenience we may introduce the non-dimensional co-ordinates

$$\eta = \frac{y}{b/2} \text{ and } \xi = \frac{x}{b/2} \quad (57)$$

and a non-dimensional lift per unit span (circulation)

$$\gamma = \frac{\Gamma}{bU} = \frac{C_L c}{2b} \quad (58)$$

to which a corresponding expression for the pitching moment per unit span may be added

$$\mu = \frac{C_m c}{2b} . \quad (59)$$

Written in these non-dimensional units the downwash integral (15) is now

$$\alpha(\xi,\eta) = \frac{-1}{2\pi} \int_{-1}^{1} \frac{\gamma(\eta') \, . \, i(\xi,\eta,\eta') \, d\eta'}{(\eta-\eta')^2} - \frac{1}{2\pi} \int_{-1}^{1} \frac{\mu(\eta') \, . \, j(\xi,\eta,\eta') \, d\eta'}{(\eta-\eta')^2} . \quad (60)$$

Since both integrals have very much the same shape we treat them in quite the same manner. The following discussion of the first integral applies to the second with the substitution of μ and j for γ and i respectively.

The problem of finding a convenient method of working out integral (60) may have, and has indeed, many possible solutions. As usual in such problems there exists no absolute criterion to decide which is the best possible method. The only thing we can do is to compare the time required for computing with the accuracy reached. The method presented here is the result of such comparisons; in the author's opinion it gives the most favourable correlation between accuracy and computing effort for a given number of stations at which the integral equation is to hold exactly. An alternative method is described in Appendix V; it needs much more work but might be slightly more accurate.

The main part of our spanwise integration is based on the approximate integration method for the 'induced angle of attack' of the lifting-line aerofoil theory as developed by the author some time ago[1]. This induced angle of attack is given by

$$\alpha_i(\eta) = \frac{-1}{2\pi}\int_{-1}^{1} \frac{\gamma(\eta')\, d\eta'}{(\eta - \eta')^2} . \qquad (61)$$

This integral is usually written as

$$\frac{1}{2\pi}\int_{-1}^{1} \frac{d\gamma}{d\eta'} \frac{d\eta'}{\eta - \eta'} .$$

One can easily prove the identity of both expressions by partial integration keeping in mind the definition of the second-order principal value, equation (16). The downwash integral equation (15) and this induced angle of attack differ only in the factor $i(\eta,\eta')$ which is a fairly continuous quantity varying only within the limits $0 < i < 2$. It is, therefore, only natural to apply the integration methods developed for α_i also to our downwash integral by considering the $(\gamma \,.\, i)$-distribution as roughly equivalent to another γ-distribution. Due to the continuity and finiteness of i this $(\gamma \,.\, i)$-distribution has all the typical features of a γ-distribution, especially its proportionality to the square root of the distance from the wing tips near these tips.

Thus, if we introduce the usual angular co-ordinate

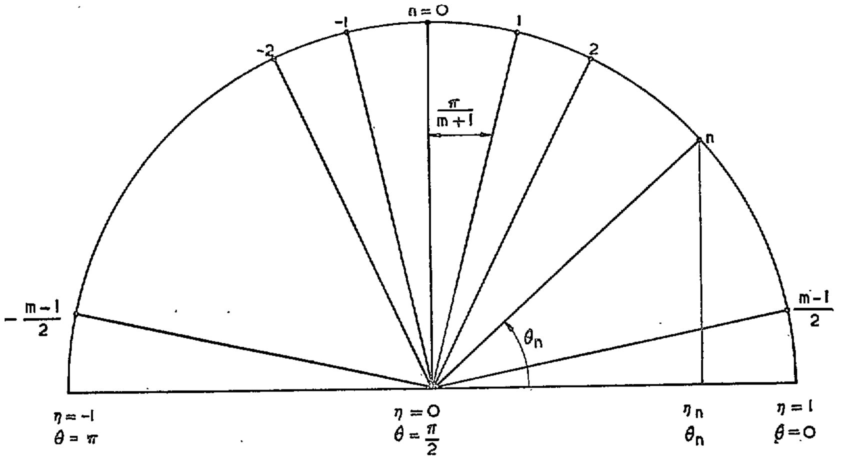
$$\theta = \cos^{-1} \eta \qquad (62)$$

we may well assume that not only γ but also $(\gamma \,.\, i)$ may be represented by a sum

$$(\gamma \,.\, i) = \Sigma\, a_\lambda \sin \lambda\theta \qquad (63)$$

to a fair degree of accuracy without too many terms of this development being needed. The a_λ are, of course, functions of the pivotal station.

Although fairly simple for general discussions such series developments are usually not so convenient when it comes to the actual calculation. If we consider the values of $(\gamma \,.\, i)$ to be given at a limited number of stations we can find values between these stations either by working out a series development with as many terms as stations are given or by using different interpolation functions. The latter is the way which is successfully done in the lifting-line theory[1]. Since the original report might not be available it was thought advisable to repeat the essential theory in Appendix IV; the principle results are quoted in the following discussion.



The stations at which the γ or $(\gamma \,.\, i)$ values are supposed to be known result from an equidistant subdivision of the angular co-ordinate range $0 < \theta < \pi$. We always work with an odd number m of stations, so as to include the span centre $\theta = \pi/2$ as one of them. The stations are numbered positive to the right and negative to the left of the zero central station, as shown in the sketch.

Thus n takes the m values $0, \pm 1, \pm 2, \ldots \pm (m-1)/2$.

The angular interval is $\pi/(m+1)$ and so

$$\theta_n = \frac{\pi}{2} - \frac{n\pi}{m+1}$$

and

$$\eta_n = \cos\theta_n = \sin\frac{n\pi}{m+1} \,. \qquad \ldots \qquad (64)$$

This numbering of the stations is more convenient than the system used in the lifting-line theory, since it works from the essential symmetry of the system, $\eta_n = -\eta_{-n}$.

We can now write

$$(\gamma \, . \, i) = \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} (\gamma \, . \, i)_n \, . \, g_n(\theta) \qquad \ldots \qquad (65)$$

$(\gamma \, . \, i)_n$ being the value of $(\gamma \, . \, i)$ at the n-th interpolation station and $g_n(\theta)$ the interpolation function belonging to this n-th station. Its essential feature is the fact that its value is unity at the n-th station and zero at all the others. A function which goes that way and has the required tendency towards the wing tips is given by

$$g_n(\theta) = -\frac{\sin\theta_n}{(m+1)\cos(m+1)\theta_n} \, . \, \frac{\sin(m+1)\theta}{\cos\theta - \cos\theta_n} \,. \qquad \ldots \qquad (66)$$

This function, being proportional to $\sin(m+1)\theta$, vanishes at all stations except θ_n, and it can easily be proved that

$$\lim_{\theta \to \theta_n} \frac{\sin(m+1)\theta}{\cos\theta - \cos\theta_n} = -\frac{(m+1)\cos(m+1)\theta_n}{\sin\theta_n} \,. \qquad \ldots \qquad (67)$$

Hence $g_n(\theta) = 1$ at $\theta = \theta_n$. *See* Appendix IV.

By developing $g_n(\theta)$ into a Fourier series we can also show that

$$g_n = \frac{2}{m+1} \sum_{\lambda=1}^{m} \sin\lambda\theta_n \sin\lambda\theta \,. \qquad \ldots \qquad (68)$$

In this form these interpolation functions are frequently used in the harmonic analysis of periodic functions; they are usually derived by a least-square method.

Our downwash integral (60) gives now with $\eta = \cos\theta$

$$\alpha(\theta) = \frac{-1}{2\pi} \int_0^\pi \frac{(\gamma \, . \, i)(\theta') \, . \, \sin\theta' \, d\theta'}{(\cos\theta - \cos\theta')^2} \qquad \ldots \qquad (69)$$

which with equation (65) comes to

$$\alpha(\theta) = \frac{-1}{2\pi} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} (\gamma \, . \, i)_n \, . \int_0^\pi \frac{g_n(\theta') \sin\theta' \, d\theta'}{(\cos\theta - \cos\theta')^2} \,. \qquad \ldots \qquad (70)$$

Since the integrals in the sum do not contain the $(\gamma \, . \, i)$ function any longer but in its stead the interpolation functions $g_n(\theta)$ we can work out these integrals numerically for any value of θ which seems interesting. The most practical proposal is to calculate the downwash at the interpolation stations which are thus chosen as pivotal points; if we do so we see that only the integral over the interpolation term belonging to the pivotal station itself gives a positive contribution to α, all the others give either negative contributions or zero. In agreement with the previous paper on lifting-line theory we write for the downwash at the νth spanwise station:

$$\alpha_\nu = b_{\nu\nu}(\gamma \, . \, i)_\nu - \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, b_{\nu n}(\gamma \, . \, i)_n \quad \ldots \quad (71)$$

where

$$b_{\nu\nu} = \frac{m+1}{4 \sin \theta_\nu} = \frac{m+1}{4 \cos \dfrac{\nu\pi}{m+1}} \quad \ldots \quad (72)$$

$$b_{\nu n} = \frac{\sin \theta_n}{(m+1)(\cos \theta_n - \cos \theta_\nu)^2} \, ; \quad |n - \nu| = 1, 3, 5, \ldots .$$

$$= 0 \qquad |n - \nu| = 2, 4, 6 \ldots .$$

as shown in Appendix IV. The stroke at the sum symbol is to remind us that the value for $\nu = n$ is to be left out. Bearing in mind that the influence function i in the product $(\gamma \, . \, i)$ is in fact also a function of ν we may write

$$(\gamma \, . \, i)_n = \gamma_n \, . \, i_{\nu n} \quad \ldots \quad (73)$$

and obtain thus:

$$\alpha_\nu = b_{\nu\nu} \, i_{\nu\nu} \gamma_\nu - \sum_{\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, b_{\nu n} \, i_{\nu n} \, \gamma_n \, . \quad \ldots \quad (74)$$

The only serious objection we can raise against this approximate method of integration concerns the behaviour of the function $i(\eta', \eta)$ near the pivotal point, *i.e.*, for $\eta' \to \eta$. Apart from the logarithmic singularity of $i/(\eta' - \eta)^2$ which is not covered by the interpolation polynomials we must consider how far the representation by interpolation polynomials may be relied on. It is quite obvious that we can not expect the interpolation functions still to work if the function represented shows an irregular behaviour between two interpolation stations, *i.e.*, the interpolation stations must be placed so closely together as to miss no essential feature of the function. In this respect we have to expect more trouble from the influence functions $i(\eta')$ than from the circulation γ; as a rule $i(\eta')$ has a maximum in or near the pivotal point and two inflection points on either side of it; for the usual chordwise positions of pivotal stations these inflection points are to be found at about

$$Y = 0{\cdot}2 \text{ to } 0{\cdot}25$$

measured from the inducing section, *see* Fig. 2. To avoid an appreciable distortion of the interpolated function the distance between two spanwise stations should not be wider than the distance between the points of inflection. This rule means in practice that the distance between two spanwise stations should be less than about $0{\cdot}4$ or $0{\cdot}5$ wing chords. Accordingly the number of spanwise stations should be about three times the aspect ratio or more, since we apply the lifting-surface calculations mainly to wings with a moderate aspect ratio this is not too heavy a condition.

5.2. *Correction for a Logarithmic Singularity**.—So far we have entirely neglected the influence of the logarithmic singularity in the second derivative of $i(\eta, \eta')$ for $\eta' = \eta$. As shown above, equation (53), near the inducing section $i(X, Y)$ can only be developed into a series beginning with

$$i(X,Y) = i(X,0) + K_1(X) Y^2 \ln |Y| + \ldots \ldots \quad \ldots \quad (53)$$

* The method of calculating the correction to $i_{\nu\nu}$ and $j_{\nu\nu}$ given here is a rather crude approximation, and may lead to appreciable error in special cases. To avoid trouble from this source the alternative method of Mangler and Spencer (1952) is recommended (Ref. 18).

After a change to η or η' co-ordinates there remains still this logarithmic term in a series development of $i(\eta,\eta')$ for constant η near the pivotal station

$$i(\eta,\eta') = i(\eta,\eta) + a_1(\eta) \,.\, (\eta' - \eta) + \ldots$$
$$+ K_1\left(\frac{b}{2c(\eta)}\right)^2 (\eta - \eta')^2 \ln|\eta - \eta'| + \ldots . \qquad (75)$$

This first of the logarithmic terms is not affected by the angle between the line of pivotal points and the direction of undisturbed flow because in a two-dimensional development of $i(X,Y)$ from a point on the wing section $Y = 0$ all lower derivatives (e.g., $\partial i/\partial X$, $\partial^2 i/\partial X^2$, $\partial^2 i/\partial X\,\partial Y$) are finite except $\partial^2 i/\partial Y^2$.

The downwash integral is effected by this first logarithmic term because the factor $(\eta - \eta')^2$ just cancels the denominator so that an integral over $\ln|\eta - \eta'|$ is left. Fortunately this integral is finite even without the principal value although the logarithm becomes infinite at $\eta' = \eta$.

Since the logarithmic infinity of a second derivative is not very noticeable in any plotting of $i(\eta')$, (*see* the $i(Y)$-curves in our diagrams Figs. 1 and 2) we may reasonably assume our interpolation functions to represent $(\gamma \,.\, i)$ fairly well with enough interpolation stations except for the immediate neighbourhood of the pivotal station η_ν. Here we add to the interpolation functions for $\gamma \,.\, i$ a correction $\Delta(\gamma \,.\, i)$ containing mainly the logarithmic singularity, *e.g.*, for the interval $\eta_\nu \leqslant \eta' < \eta_{\nu+1}$:

$$\Delta(\gamma \,.\, i) = \gamma_\nu K_1\left(\frac{b}{2c_\nu}\right)^2 (\eta_\nu - \eta')^2 \ln\frac{\eta' - \eta_\nu}{\eta_{\nu+1} - \eta_\nu} \cdot \left[1 - \left(\frac{\eta' - \eta_\nu}{\eta_{\nu+1} - \eta_\nu}\right)^2\right]^2 \qquad (76)$$

and similarly for the other side of the pivotal station. The last factor and the denominator in the logarithm are included in order to enforce an efficient fading out of $\Delta(\gamma \,.\, i)$ at the next spanwise station $\eta_{\nu+1}$ or $\eta_{\nu-1}$; thus, at these stations we have

$$\Delta(\gamma \,.\, i) = 0\,, \qquad \frac{\partial}{\partial\eta'}\Delta(\gamma \,.\, i) = 0\,, \qquad \frac{\partial^2}{\partial\eta'^2}\Delta(\gamma \,.\, i) = 0$$

without affecting the logarithmic term near η_ν very much.

The correction for the downwash is now fairly simple:

$$\Delta\alpha_\nu = \frac{-1}{2\pi}\int_{\eta_{\nu-1}}^{\eta_{\nu+1}} \frac{\Delta(\gamma \,.\, i)\, d\eta'}{(\eta - \eta')^2} \qquad (77)$$

with

$$u = \frac{\eta' - \eta_\nu}{\eta_{\nu+1} - \eta_\nu} \quad \text{or} \quad u = \frac{\eta_\nu - \eta'}{\eta_\nu - \eta_{\nu-1}} \qquad (78)$$

we obtain :

$$\Delta\alpha_\nu = \frac{-\gamma_\nu}{2\pi} K_1\left(\frac{b}{2c_\nu}\right)^2 (\eta_{\nu+1} - \eta_{\nu-1})\int_0^1 (1 - u^2)^2 \ln u\, du\,. \qquad (79)$$

The numerical value of the last integral can be easily worked out as $-\dfrac{184}{225} = -0\cdot818$; thus $\Delta\alpha_\nu$ is :

$$\Delta\alpha_\nu = 0\cdot1302\, \gamma_\nu K_1\left(\frac{b}{2c_\nu}\right)^2 (\eta_{\nu+1} - \eta_{\nu-1})\,. \qquad (80)$$

Adding this to the integration formula equation (74) we have

$$\alpha_\nu = \gamma_\nu\left[b_{\nu\nu} i_{\nu\nu} + 0\cdot1302\, K_1\left(\frac{b}{2c_\nu}\right)^2 (\eta_{\nu+1} - \eta_{\nu-1})\right] - \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, b_{\nu n}\, i_{\nu n}\, \gamma_n\,. \qquad (81)$$

For simplicity we write

$$\alpha_\nu = b_{\nu\nu}\,\gamma_\nu\,\overline{i_{\nu\nu}} - \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \; b_{\nu n}\,i_{\nu n}\,\gamma_n \qquad (82)$$

or, with two terms for the chordwise lift distribution:

$$\alpha_\nu = b_{\nu\nu}\,(\gamma_\nu\,\overline{i_{\nu\nu}} + \mu_\nu\,\overline{j_{\nu\nu}}) - \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \; b_{\nu n}\,(\gamma_\nu\,i_{\nu n} + \mu_n\,j_{\nu n}) \qquad (83)$$

the $\overline{i_{\nu\nu}}$ being

$$\overline{i_{\nu\nu}} = i_{\nu\nu} + 0\cdot1302\,K_1\left(\frac{b}{2c_\nu}\right)^2 \cdot \frac{4\sin\theta_\nu}{m+1}\,(\eta_{\nu+1} - \eta_{\nu-1}) \qquad (84)$$

with a similar equation for $\overline{j_{\nu\nu}}$.

The last part in this expression

$$\frac{\sin\theta_\nu}{m+1}\,(\eta_{\nu+1} - \eta_{\nu-1}) = \frac{2}{m+1}\cos^2\frac{\nu\pi}{m+1}\sin\frac{\pi}{m+1} \qquad (85)$$

depends only on m and the station number ν. $i_{\nu\nu}$ or $j_{\nu\nu}$ and K_1 are functions of the chordwise position of our pivotal points only according to equations (47, 48) and (54, 55). For the most important positions we have the following numerical values:

(a) $0\cdot75c$:

$$\overline{i_{\nu\nu}} = 1\cdot884_7 + 0\cdot510_3\left(\frac{b}{2c_\nu}\right)^2 \frac{\sin\theta_\nu}{m+1}\,(\eta_{\nu+1} - \eta_{\nu-1})$$

(b) $0\cdot9045c$:

$$\overline{i_{\nu\nu}}' = 1\cdot974_2 + 0\cdot623_4\left(\frac{b}{2c_\nu}\right)^2 \frac{\sin\theta_\nu}{m+1}\,(\eta_{\nu+1} - \eta_{\nu-1})$$

$$\overline{j_{\nu\nu}}' = 0\cdot2958 - 4\cdot805\left(\frac{b}{2c_\nu}\right)^2 \frac{\sin\eta_\nu}{m+1}\,(\eta_{\nu+1} - \eta_{\nu-1}) \qquad (86)$$

(c) $0\cdot3455c$:

$$\overline{i_{\nu\nu}}'' = 1\cdot405_5 + 1\cdot008_7\left(\frac{b}{2c_\nu}\right)^2 \frac{\sin\theta_\nu}{m+1}\,(\eta_{\nu+1} - \eta_{\nu-1})$$

$$\overline{j_{\nu\nu}}'' = 3\cdot170_2 + 5\cdot758\left(\frac{b}{2c_\nu}\right)^2 \frac{\sin\theta_\nu}{m+1}\,(\eta_{\nu+1} + \eta_{\nu-1})$$

The correction terms usually account for about 5 or 10 per cent of the total downwash; this is an effect well worth considering but small enough to be still a correction only.

The neglect of this correction usually affects more the absolute value of all results than their spanwise distribution. But the additional work required for this correction is such a small fraction of the total calculation that it hardly ever pays to leave it out.

5.3. *The Central Section of Swept Wings.*—We have not yet mentioned any peculiar features of the central section of swept wings although in some other methods of calculating the lift distribution this point is dealt with rather carefully. Indeed the treatment of this region of the wing depends on the basic physical meaning of the model used to represent the wing. If for example, the lifting system is built up of kinked vortices we shall certainly have to reckon with singularities in the downwash along the section in which these kinks occur; thus some compensating functions are necessary to eliminate what is, in fact, the result of an unsuitable physical model.

What is really unusual or difficult to assess in the local distribution in the middle part of a swept wing ? If the local incidence is continuous across the middle section equi-potential lines or lines of constant pressure cannot have a kink on this section as one can easily see from the potential equations. In fact, all available pressure measurements on swept wings show clearly this tendency. Curves of constant pressure are curved and have no kink; if the wing is symmetrical they cross the middle section at right-angles. The only irregularities occur near the leading and trailing-edge corners but these are of a very local nature.

With our method of approach to the lifting-surface problem we are luckier than perhaps we ought to expect. Because we use our interpolation polynomials, equation (65), throughout for the representation of the $(\gamma \, . \, i)$ and $(\mu \, . \, j)$-functions no discontinuity across the middle line can occur. Thus, for the greater part of the median section we can meet the physical reality better than with an artificial vortex system. Because we satisfy the integral equation of the lifting surface only at a limited number of pivotal stations and because by our interpolation polynomials only the geometrical wing characteristics at these sections enter into our calculations, we calculate in fact the downwash of an ' interpolated wing ' which coincides with the real one only in these pivotal sections. All the characteristics of this interpolated wing are accordingly continuous functions of the spanwise co-ordinate, *i.e.*, are rounded off in the middle of the swept wing.

The only question worth considering is whether this rounding off through our interpolation methods is done in the right way. If we represent a thoroughly continuous function, *i.e.*, a function which is continuous even in its derivatives, by series development or interpolation polynomials the approximation can be very good indeed from a certain number of matching stations upwards. With discontinuous functions or functions with a discontinuity in the first derivative we observe some typical defects in the representation by interpolation polynomials near the discontinuity. If the station of a kink is one of the points at which the given and the interpolated function coincide the agreement between the two will be poor in the two neighbouring intervals until the next matching stations however high the number of stations and accordingly the number of polynomials used might be. If the value of the given function has a maximum at the kink the interpolated function will exceed it in these two neighbouring sections. The situation is illustrated in Fig. 7, where the function to be interpolated is $1 - |\eta|$, having a kink at K. The broken curve is the interpolation function when it is arranged to pass through K, and it is clear that we shall get a better approximation to the function between the kink K and the next matching station P if we make the interpolation function pass through some point K′ below K, thus rounding off the kink. We want to know how to choose K′.

To decide this question we are free to introduce an additional condition about the interpolation functions. Because the interpolated function mostly differs from the given one with the kink in one direction only it appears reasonable to demand that the area under the interpolated curve is equal to that under the given one. With our interpolation polynomials this is the case if KK′ is roughly 1/6 of the vertical distance between K and P. This is proved in Appendix VI.

So this seems to be the best we can do about the middle of swept wings: we round them slightly off according to this rule. Instead of calculating with the actual geometrical data of the middle section we substitute a central section the chordwise co-ordinates of which are to be found by:

$$\xi_0 = \tfrac{5}{6}\,\xi_{0\,\mathrm{geom}} + \tfrac{1}{6}\,\xi_1 \qquad \ldots \quad (87)$$

where suffix $_0$ refers to the centre-section and suffix $_1$ to the next interpolation station. This applies to leading and trailing edge as well as to the chordwise pivotal stations.

The thus calculated loads of the middle section must, of course, be transferred again to the actual geometrical section; the natural condition is that the centre of pressure must not be shifted and that the total lift per unit span remains unaffected.

The same 1/6-rule can be applied to other than central kinks in the wing contour if these happen to fall upon any of our spanwise control stations.

6. *The Calculation of the Lift Distribution.*—Instead of considering the equations (82) or (83) as formulae for finding the downwash values α_ν we may just as well take them as a system of equations to determine the γ_ν's, μ_ν's with the downwash α_ν being equal to the given local incidence. Thus, the original integral equation (15) is approximately replaced by a system of linear equations the solution of which is obviously much simpler.

Regarding the different terms of equations (82) or (83) we see that from the wing geometry the $\overline{i_{\nu\nu}}$, $\overline{j_{\nu\nu}}$, $i_{\nu n}$, $j_{\nu n}$ and α_ν are fixed once the system of pivotal points is chosen, thus leaving as unknowns only the γ_n and with two chordwise pivotal stations also the μ_n. The choice of the pivotal points is again mainly a question of the wing geometry. The number of spanwise stations m should be about three times the aspect ration or more as stated before. For low aspect ratios and for wing plan forms differing widely from the conventional straight wing two chordwise stations are always advisable. With respect to the computing effort one will usually reduce the number of pivotal points to the needed minimum; for this we can give at the moment only these rather rough rules for want of enough experience. Fairly detailed comparative calculations for a series of typical wings with varying numbers of stations will bring some more reliable rules. It is clear without a mathematical proof that an increase in the number of pivotal points gives a more accurate result. If one is in doubt whether the number of stations chosen is sufficient a repetition of the calculation with a different number of points should show where we are. If the difference in the results is considerable the number of pivotal points was not enough.

The first preparatory step in the actual calculation consists in the tabulation of the geometrical wing data we need for the calculation of the $i_{\nu\nu}$ and $j_{\nu\nu}$ according to equation (86) and the determination of the $i_{\nu n}$ and $j_{\nu n}$ from charts, Figs. 1 to 6. As entries into these diagrams we compute firstly for each $i_{\nu n}$ or $j_{\nu n}$ required

$$Y_{\nu n} = \frac{|y_\nu - y_n|}{c_n} = \frac{|\eta_\nu - \eta_n|}{2c_n/b} \quad \ldots \quad (88)$$

$$X_{\nu n} = \frac{x_\nu \text{ (pivotal point)} - x_n \text{ (leading edge)}}{c_n} .$$

With two chordwise stations we have two X-values for one Y-value.

Since the course of solving the system of equations differs according to whether one or two chordwise pivotal points are used we will discuss the two solutions separately.

6.1. *One Chordwise Pivotal Point at* $0{\cdot}75c$.—We can rearrange the system of equations (82) by dividing each equation by $b_{\nu\nu}\overline{i_{\nu\nu}}$ and obtain thus :

$$\gamma_\nu = \frac{\alpha_\nu}{b_{\nu\nu}\,\overline{i_{\nu\nu}}} + \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \frac{b_{\nu n}}{b_{\nu\nu}} \cdot \frac{i_{\nu n}}{\overline{i_{\nu\nu}}} \gamma_n . \quad \ldots \quad (89)$$

It is convenient to introduce other coefficients by writing:

$$\gamma_\nu = \frac{a_{\nu\nu}\,\alpha_\nu}{\overline{i_{\nu\nu}}} + \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' a_{\nu n} \frac{i_{\nu n}}{\overline{i_{\nu\nu}}} \gamma_n \quad \ldots \quad (90)$$

with

$$a_{\nu\nu} = \frac{1}{b_{\nu\nu}} = \frac{4}{m+1} \sin\theta_\nu = \frac{4}{m+1} \cos\frac{\nu\pi}{m+1} \quad \ldots \quad (91)$$

and

$$\begin{aligned} a_{\nu n} &= \frac{4 \sin\theta_\nu \sin\theta_n}{(m+1)^2 (\eta_\nu - \eta_n)^2} & |n-\nu| = 1, 3, 5 \ldots . \\ &= \frac{1}{(m+1)^2}\left[\frac{1}{\sin^2\frac{\pi(n-\nu)}{2(m+1)}} - \frac{1}{\sin^2\frac{\pi(n+\nu)}{2(m+1)}}\right] & \ldots \quad (92) \\ a_{\nu n} &= 0 & |n-\nu| = 2, 4, 6 \ldots . \end{aligned}$$

It is easily to be seen that

$$a_{\nu n} = a_{n\nu} = a_{-\nu,\,-n} = a_{-n,\,-\nu}\,. \qquad (93)$$

These coefficients and the factors $[\sin\theta_\nu/(m+1)][\eta_{\nu+1}-\eta_{\nu-1}]$ from equations (85) and (86) are collected in Tables 1 to 7 for $m = 3, 5, 7, 11, 15, 23, 31$ to suit all practical requirements.

The solution of our system of equations is very similar to that already established for the lifting-line problem[1]. If the wing is symmetrical, *i.e.*, if

$$c_\nu = c_{-\nu} \qquad (94)$$

and

$$\dot{i}_{\nu n} = \dot{i}_{-\nu,\,-n} \qquad (95)$$

as this is usual, we may split up the lift distribution and the α-distribution into a symmetrical and an anti-symmetrical component thus in either case reducing the number of unknowns to about one-half. In the symmetrical case we obtain from

$$\left.\begin{aligned}\alpha_\nu &= \alpha_{-\nu}\\ \gamma_\nu &= \gamma_{-\nu}\end{aligned}\right\} \qquad (96)$$

the system of equations

$$\gamma_\nu = A_\nu\alpha_\nu + \sum_0^{\frac{m-1}{2}}{}' A_{\nu n}\gamma_{\nu n}\,, \qquad \nu = 0, 1, 2 \ldots \frac{m-1}{2} \qquad (97)$$

with

$$A_\nu = \frac{a_{\nu\nu}}{\dot{i}_{\nu\nu}} \qquad (98)$$

and

$$\left.\begin{aligned}A_{\nu n} &= \frac{a_{\nu n}\,\dot{i}_{\nu n} + a_{\nu,\,-n}\,\dot{i}_{\nu,\,-n}}{\dot{i}_{\nu\nu}}\,, \qquad n \neq 0\\ A_{\nu 0} &= \frac{a_{\nu 0}\,\dot{i}_{\nu 0}}{\dot{i}_{\nu\nu}}\end{aligned}\right\} \qquad (99)$$

For the antisymmetrical cases we find similarly from

$$\left.\begin{aligned}\alpha_\nu &= -\alpha_{-\nu}\\ \gamma_\nu &= -\gamma_{-\nu}\end{aligned}\right\} \qquad (100)$$

the system of equations:

$$\gamma_\nu = \mathbb{A}_\nu\alpha_\nu + \sum_1^{\frac{m-1}{2}}{}' \mathbb{A}_{\nu n}\gamma_n\,, \qquad \nu = 1, 2, 3 \ldots \frac{m-1}{2} \qquad (101)$$

with

$$\mathbb{A}_{\nu n} = \frac{a_{\nu n}\,\dot{i}_{\nu n} - a_{\nu,\,-n}\,\dot{i}_{\nu,\,-n}}{\dot{i}_{\nu\nu}}\,. \qquad (102)$$

Antisymmetrical load cases are usually due to unsymmetrical aircraft movements (rolling, yawing, etc.) or aileron action.

If the resulting system of equations (97) or (101) contains only a few unknowns (up to five) we solve it best by elimination. Here the fact that half of the $a_{\nu n}$ are zero is an essential help. The equations for the γ_ν with an even suffix contain only γ_n-values with odd suffixes on the right-hand side and *vice versa*. Thus by inserting the γ_ν expressions for odd ν's into the equations for γ_ν with even ν's we have only one or two equations for the γ's with even suffixes left, the solution of which is a matter of routine and needs no further explanation.

With a higher number of unknowns a solution by iteration is the most convenient course. We begin, for example, with a rough guess of the γ-values with an even suffix; these first estimated values may be called $\gamma_0^{[0]}$, $\gamma_2^{[0]}$, $\gamma_4^{[0]}$ .... Putting these values into the equations for the γ_ν with odd suffixes ν we obtain a first approximation of these $\gamma_1, \gamma_3, \gamma_5$ .... which we call

$$\gamma_\nu^{[1]} = A_\nu \alpha_\nu + \sum_{0}^{\frac{m-1}{2}}{}' A_{\nu n} \gamma_n^{[0]}, \qquad \begin{matrix} n = 0, 2, 4, 6 \ldots \\ \nu = 1, 3, 5 \quad \ldots \end{matrix} \qquad \ldots \qquad (103)$$

In the same way we obtain the first approximation of the $\gamma_0, \gamma_2, \gamma_4$ .... with:

$$\gamma_\nu^{[1]} = A_\nu \alpha_\nu + \sum_{1}^{\frac{m-1}{2}}{}' A_{\nu n} \gamma_n^{[1]}, \qquad \begin{matrix} n = 1, 3, 5 \ldots \\ \nu = 0, 2, 4 \ldots \end{matrix} \qquad \ldots \qquad (104)$$

This process must be repeated until the difference between successive approximations becomes negligible:

$$\left.\begin{aligned}
\gamma_\nu^{[2]} &= A_\nu \alpha_\nu + \sum_{0}^{\frac{m-1}{2}}{}' A_{\nu n} \gamma_n^{[1]}, & & n = 0, 2, 4 \ldots \quad \nu = 1, 3, 5 \ldots \\
\gamma_\nu^{[2]} &= A_\nu \alpha_\nu + \sum_{1}^{\frac{m-1}{2}}{}' A_{\nu n} \gamma_n^{[2]}, & & n = 1, 3, 5 \ldots \quad \nu = 2, 4, 6 \ldots \\
\gamma_\nu^{[3]} &= A_\nu \alpha_\nu + \sum_{0}^{\frac{m-1}{2}}{}' A_{\nu n} \gamma_n^{[2]}, & & n = 0, 2, 4 \ldots \quad \nu = 1, 3, 5 \ldots
\end{aligned}\right\} \qquad \ldots \qquad (105)$$

etc.

This process is always converging to the right solution. Slide-rule accuracy should suit most practical requirements; even more helpful would be an automatic calculating machine which allows the store of coefficients $A_{\nu n}$.

We can reduce the computational effort a great deal by using only the differences of the first successive steps, equations (103) and (104), for further calculations: after computing

$$\Delta^{[1]} \gamma_n = \gamma_n^{[1]} - \gamma_n^{[0]}, \qquad n = 0, 2, 4, 6 \ldots \qquad \ldots \qquad (106)$$

at the end of the first to-and-fro calculation we continue with

$$\left.\begin{aligned}
\Delta^{[2]} \gamma_\nu &= \sum_{0}^{\frac{m-1}{2}}{}' A_{\nu n} \Delta^{[1]} \gamma_n, & & n = 0, 2, 4 \ldots \quad \nu = 1, 3, 5 \ldots \\
\Delta^{[2]} \gamma_\nu &= \sum_{0}^{\frac{m-1}{2}}{}' A_{\nu n} \Delta^{[2]} \gamma_n, & & n = 1, 3, 5 \ldots \quad \nu = 0, 2, 4 \ldots \\
\Delta^{[3]} \gamma_\nu &= \sum_{0}^{\frac{m-2}{2}}{}' A_{\nu n} \Delta^{[2]} \gamma_n, & & n = 0, 2, 4 \ldots \quad \nu = 1, 3, 5 \ldots
\end{aligned}\right\} \qquad \ldots \qquad (107)$$

etc. The γ_n are eventually

$$\gamma_n = \gamma_n^{[1]} + \Delta^{[2]} \gamma_n + \Delta^{[3]} \gamma_n \; \Delta^{[4]} \gamma_n + \ldots \qquad \ldots \qquad (108)$$

The main advantage of this modification is the fact that the $\Delta\gamma_n$ are small and need to be computed only to a few valid decimals; the products $A_{\nu n} \Delta\gamma_n$ can be read off at a glance on the slide rule even when the upper pair of scales (scales of squares) are used in order to avoid moving the slide for all products with the same $\Delta\gamma_n$. A disadvantage of this modification is the necessity of a check for reliable calculations; in the first scheme any errors expire automatically, whereas with the differences scheme we have to check the final results by inserting them into the original systems of equations.

If the convergence of the iteration process seems too slow we may cut it short by extrapolating the rest of the $\Delta\gamma_n$ along the geometrical series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

x being the ratio of two consecutive $\Delta\gamma_n$; if $\Delta^{[\lambda]}\gamma_n$ be the last difference computed we write approximately

$$\gamma_n = \gamma_n^{[1]} + \Delta^{[2]}\gamma_n + \Delta^{[3]}\gamma_n + \ldots + \Delta^{[\lambda]}\gamma_n + \frac{(\Delta^{[\lambda]}\gamma_n)^2}{\Delta^{[\lambda-1]}\gamma_n - \Delta^{[\lambda]}\gamma_n} . \quad \ldots \quad (109)$$

6.2. *Two Chordwise Pivotal Stations at* 0·9045*c and* 0·3455*c*.—With two chordwise pivotal points we have for every spanwise station two equations (83) ; one for the 0·9045*c* point in the following marked by a single stroke′, and one for the 0·3455*c* point marked by a double stroke ″ at all the quantities which are different. Each part of equations (83)

$$\alpha_\nu{}' = b_{\nu\nu}\left[\overline{i_{\nu\nu}}{}'\,\gamma_\nu + \overline{j_{\nu\nu}}{}'\,\mu_\nu\right] - \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, b_{\nu n}(\gamma_n i_{\nu n}{}' + \mu_n j_{\nu n}{}')$$

$$\alpha_\nu{}'' = b_{\nu\nu}\left[\overline{i_{\nu\nu}}{}''\,\gamma_\nu + \overline{j_{\nu\nu}}{}''\,\mu_\nu\right] - \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, b_{\nu n}(\gamma_n i_{\nu n}{}'' + \mu_n j_{\nu n}{}'')$$

is better firstly transformed into two equations for γ_ν and μ_ν only. To do so we calculate for every ν:

$$\left.\begin{aligned}
l_\nu{}' &= \frac{\overline{j_{\nu\nu}}{}''}{\overline{i_{\nu\nu}}{}' \,.\, \overline{j_{\nu\nu}}{}'' - \overline{i_{\nu\nu}}{}'' \,.\, \overline{j_{\nu\nu}}{}'} \\
l_\nu{}'' &= \frac{\overline{j_{\nu\nu}}{}'}{\overline{i_{\nu\nu}}{}' \,.\, \overline{j_{\nu\nu}}{}'' - \overline{i_{\nu\nu}}{}'' \,.\, \overline{j_{\nu\nu}}{}'} \\
m_\nu{}' &= \frac{\overline{i_{\nu\nu}}{}''}{\overline{i_{\nu\nu}}{}' \,.\, \overline{j_{\nu\nu}}{}'' - \overline{i_{\nu\nu}}{}'' \,.\, \overline{j_{\nu\nu}}{}'} \\
m_\nu{}'' &= \frac{\overline{i_{\nu\nu}}{}'}{\overline{i_{\nu\nu}}{}' \,.\, \overline{j_{\nu\nu}}{}'' - \overline{i_{\nu\nu}}{}'' \,.\, \overline{j_{\nu\nu}}{}'}
\end{aligned}\right\} \quad \ldots \quad (110)$$

With these figures and the coefficients $a_{\nu\nu}$ and $a_{\nu n}$, equations (91) and (92) each pair of equations (83) is transformed into :

$$\left.\begin{aligned}
\gamma_\nu &= a_{\nu\nu}(l_\nu{}'a_\nu{}' - l_\nu{}''\alpha_\nu{}'') + \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, a_{\nu n}(l_\nu{}'i_{\nu n}{}' - l\,''i_{\nu n}{}'')\gamma_n \\
&\quad + \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, a_{\nu n}(l_\nu{}'j_{\nu n}{}' - l_\nu{}''j_{\nu n}{}'')\mu_n \\
\mu_\nu &= a_{\nu\nu}(m_\nu{}'\alpha_\nu{}'' - m_\nu{}'\alpha_\nu{}') + \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, a_{\nu n}(m_\nu{}''i_{\nu n}{}'' - m_\nu{}'i_{nn}{}')\gamma_\nu \\
&\quad + \sum_{-\frac{m-1}{2}}^{\frac{m-1}{1}}{}' \, a_{\nu n}(m_\nu{}''j_n{}'' - m_\nu{}'j_{\nu n}{}')\mu_n
\end{aligned}\right\} \quad \ldots \quad (111)$$

The solution of this system of equations is carried through in the same way as for one chordwise pivotal station. We split it up into a symmetrical and an antisymmetrical part if the wing plan form is symmetrical.

For symmetrical load distributions:

$$\alpha_\nu = \alpha_{-\nu} \qquad \gamma_\nu = \gamma_{-\nu} \qquad \mu_\nu = \mu_{-\nu}$$

we write abbreviating:

$$\left.\begin{aligned} \gamma_\nu &= a_{\nu\nu}(l_\nu'\alpha_\nu' - l_\nu''\alpha_\nu'') + \sum_{0}^{\frac{m-1}{2}}{}' B_{\nu n}\gamma_n + \sum_{0}^{\frac{m-1}{2}}{}' C_{\nu n}\mu_n \\ \mu_\nu &= a_{\nu\nu}(m_\nu''\alpha_\nu'' - m_\nu'\alpha_\nu') + \sum_{0}^{\frac{m-1}{2}}{}' D_{\nu n}\gamma_n + \sum_{0}^{\frac{m-1}{2}}{}' E_{\nu n}\mu_n \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (112)$$

with

$$\left.\begin{aligned} B_{\nu n} &= a_{\nu n}(l_\nu' i_{\nu n}' - l_\nu'' i_{\nu n}'') + a_{\nu,-n}(l_\nu' i_{\nu,-n}' - l_\nu'' i_{\nu,-n}'') \\ C_{\nu n} &= a_{\nu n}(l_\nu' j_{\nu n}' - l_\nu'' j_{\nu n}'') + a_{\nu,-n}(l_\nu' j_{\nu,-n}' - l_\nu'' j_{\nu,-n}'') \\ D_{\nu n} &= a_{\nu n}(m_\nu'' i_{\nu n}'' - m_\nu' i_{\nu n}') + a_{\nu,-n}(m_\nu'' i_{\nu,-n}'' - m_\nu' i_{\nu,-n}') \\ E_{\nu n} &= a_{\nu n}(m_\nu'' j_{\nu n}'' - m_\nu' j_{\nu n}') + a_{\nu,-n}(m_\nu'' j_{\nu,-n}'' - m_\nu' j_{\nu,-n}') \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (113)$$

for $n = 0$ we put $a_{\nu,-n} = 0$ and retain $a_{\nu,n} = a_{\nu,0}$.

For antisymmetrical load cases

$$\alpha_\nu = -\alpha_{-\nu} \qquad \gamma_\nu = -\gamma_{-\nu} \qquad \mu_\nu = -\mu_{-\nu}$$

we write correspondingly

$$\left.\begin{aligned} \gamma_\nu &= a_{\nu\nu}(l_\nu'\alpha_\nu' - l_\nu''\alpha_\nu'') + \sum_{0}^{\frac{m-1}{2}}{}' \boldsymbol{B}_{\nu n}\gamma_n + \sum_{0}^{\frac{m-1}{2}}{}' \boldsymbol{C}_{\nu n}\mu_n \\ \mu_\nu &= a_{\nu\nu}(m_\nu''\alpha_\nu'' - m_\nu'\alpha_\nu') + \sum_{1}^{\frac{m-1}{2}}{}' \boldsymbol{D}_{\nu n}\gamma_n + \sum_{1}^{\frac{m-1}{2}}{}' \boldsymbol{E}_{\nu n}\mu_n \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (114)$$

The bold italics $\boldsymbol{B}_{\nu n}$, $\boldsymbol{C}_{\nu n}$, $\boldsymbol{D}_{\nu n}$ and $\boldsymbol{E}_{\nu n}$ being

$$\left.\begin{aligned} \boldsymbol{B}_{\nu n} &= a_{\nu n}(l_\nu' i_{\nu n}' - l_\nu'' i_{\nu n}'') - a_{\nu,-n}(l_\nu' i_{\nu,-n}' - l_\nu'' i_{\nu,-n}'') \\ \boldsymbol{C}_{\nu n} &= a_{\nu n}(l_\nu' j_{\nu n}' - l_\nu'' j_{\nu n}'') - a_{\nu,-n}(l' j_{\nu,-n}' - l_\nu'' j_{\nu,-n}'') \\ \boldsymbol{D}_{\nu n} &= a_{\nu n}(m_\nu'' i_{\nu n}'' - m_\nu' i_{\nu n}') - a_{\nu,-n}(m_\nu'' i_{\nu,-n}'' - m_\nu' i_{\nu,-n}') \\ \boldsymbol{E}_{\nu n} &= a_{\nu n}(m_\nu'' j_{\nu n}'' - m_\nu' j_{\nu n}') - a_{\nu,-n}(m_\nu'' j_{\nu,-n}'' - m_\nu' j_{\nu,-n}') \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (115)$$

The iteration process for the systems of equations (112) or (114) is the same as for (97). We substitute only the $B_{\nu n}$, etc., for the $A_{\nu n}$ in equation (103) . . . (107).

The main part of the actual computation of a lift distribution is not the solution of the system of equations but the collection of the $i_{\nu n}$ and $j_{\nu n}$ and the computation of the coefficients $B_{\nu n}$, $C_{\nu n}$ . . . . Thus, if it takes about ten hours to calculate the first load case for a given wing any other case (other α-distribution) takes not more than about two hours if the wing plan form remains the same. This is essential with regard to calculations of the elastic wing which usually needs a step-by-step approach: with the load of the undeformed wing one calculates the deflection due to this load according to the stiffness distribution; the next step is the lift distribution due to this deflection, etc.

6.3. *Wings of Infinite Aspect Ratio.*—A sometimes useful abstraction is the wing of infinite aspect ratio—not just the trivial case of the wing in two-dimensional flow but a wing with the tips so far away that they do not count any longer. This is quite a helpful conception for studying separately and exclusively some local effects, *e.g.*, the central part of a swept wing, cut-outs in the wing contour, wing-fuselage interferences, etc. In the analysis of such effects the wing span bears no relation to any of the interesting quantities, so it is only natural that we should dispense with it as best possible.

If we consider the central part of a wing we may approach the infinite aspect ratio state by increasing both the span and the number of spanwise stations at the same time. Then the control stations in that central part of the wing become more and more equidistant. In the limiting case their distance d is given by

$$d = \lim_{b \to \infty} \frac{\pi}{2} \frac{b}{m+1} \qquad (116)$$

and we may use this distance as the reference length instead of the span. Thus we define the circulation

$$\gamma' = \frac{\Gamma}{dU} = \frac{C_L c}{2d} = \lim_{m \to \infty} \frac{2}{\pi} (m+1)\gamma \qquad (117)$$

and in analogy the moment per unit span:

$$\mu' = \frac{C_m c}{2d} = \lim_{m \to \infty} \frac{2}{\pi} (m+1)\mu \,. \qquad (118)$$

The spanwise stations y_n counting from some assumed wing centre are now:

$$y_n = nd \,. \qquad (119)$$

Of course, the influence functions i and j are not affected by changing the spanwise reference length. Only the correction terms for the $i_{\nu\nu}$ and $j_{\nu\nu}$ in equation (84) contain the span as reference length which must be replaced by d; we thus obtain instead of equation (84):

$$\overline{i_{\nu\nu}} = i_{\nu\nu} - 0\cdot1302K_1 \,.\, \frac{8}{\pi}\left(\frac{d}{c_\nu}\right)^2 . \qquad (120)$$

The equations (86) are, therefore, to be modified; for $(b/2c_\nu)^2$ we write $(d/c_\nu)^2$ and for $[\sin\theta_\nu/(m+1)]\,[\eta_{\nu-1} - \eta_{\nu+1}]$ we have always $2/\pi$, e.g., for the $0\cdot75c$ station:

$$\overline{i_{\nu\nu}} = 1\cdot884_7 + 0\cdot510_3 \frac{2}{\pi}\left(\frac{d}{c_\nu}\right)^2 ,$$

etc.

The coefficients $a_{\nu n}$ from equation (92) are degenerating into

$$a_{\nu n} = \frac{4}{\pi^2(\nu - n)^2} , \qquad |\nu - n| = 1, 3, 5 \ldots \qquad (121)$$

and we find instead of $a_{\nu\nu}$:

$$a_{\nu\nu}' = \frac{8}{\pi} = 2\cdot546_5 \ldots \qquad (122)$$

With one chordwise pivotal point at $0\cdot75c$ we have now the infinite system of equations

$$\gamma_\nu' = \frac{8\alpha_\nu}{\pi \overline{i_{\nu\nu}}} + \sum_{-\infty}^{\infty}{}' a_{\nu n} \frac{i_{\nu n}}{\overline{i_{\nu\nu}}} \gamma_n' \qquad (123)$$

instead of equation (90); in a similar manner we may modify the equations for two chordwise pivotal points. An application of this method is given in section 8.

The solution of such infinite systems of equations needs always an iteration process and one condition must be fulfilled: for large numbers ν or n we must reach asymptotic γ- or μ-values, or the γ- and μ-functions must be periodical, in which case the wavelength should be an integer multiple of d.

6.4. *The Influence of Compressibility.*—As stated at the beginning the influence of the compressibility of the air can be approximately dealt with by a simple co-ordinate reduction (Prandtl-Glauert-rule) within the linearised theory. Equation (6) may be written

$$\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y'^2} + \frac{\partial^2 I}{\partial z'^2} = 0 \qquad \ldots \quad (124)$$

with

$$y' = y\sqrt{\left(1 - \frac{U^2}{a^2}\right)} = y\sqrt{(1 - M^2)}$$

$$z' = z\sqrt{(1 - M^2)} \,.$$

Replacing the y, y_0 and z in equation (10) by $y\sqrt{(1 - M^2)}$, $y_0\sqrt{(1 - M^2)}$ and $z\sqrt{(1 - M^2)}$ we obtain

$$I(x,y,z) = \frac{-z(1 - M^2)}{4\pi} \iint_S \frac{l(x_0,y_0)\, dx_0\, dy_0}{[(x - x_0)^2 + (1 - M^2)(y - y_0)^2 + (1 - M^2)z^2]^{3/2}} \,. \qquad \ldots \quad (125)$$

Since equation (12) as an integration of equation (4) is still valid we have instead of equation (15) eventually:

$$\alpha(x,y) = \frac{-1}{8\pi} \iint_S \frac{l(x_0,y_0)}{(y - y_0)^2} \left\{1 + \frac{x - x_0}{\sqrt{[(x - x_0)^2 + (1 - M^2)(y - y_0)^2]}}\right\} dx_0\, dy_0 \,. \qquad \ldots \quad (126)$$

The compressibility factor $(1 - M^2)$ appears only in the square root but not, for example, as a factor to the denominator $(y - y_0)^2$. This means that only our chordwise integrals i and j are affected. Instead of the expression (29) we find integrals like

$$\int l(x_0,y_0) \left\{1 + \frac{x - x_0}{\sqrt{[(x - x_0)^2 + (1 - M^2)(y - y_0)^2]}}\right\} dx_0$$

so that the non-dimensional Y-co-ordinate is chosen as

$$Y = \sqrt{(1 - M^2)}\, \frac{y - y_0}{c(y_0)} \qquad \ldots \quad (127)$$

in order to leave all the following relations as they are in incompressible flow

Thus the main modification necessary, in the course of our calculation, to take account of compressibility within the linearised theory, is to use the Y-co-ordinate of equation (127) in taking $i_{\nu n}$ and $j_{\nu n}$ from our diagrams. The generalised form of equation (88) is

$$Y_{\nu n} = \sqrt{(1 - M^2)}\, \frac{y_\nu - y_n}{c_n} = \sqrt{(1 - M^2)}\, \frac{\eta_\nu - \eta_n}{2c_n b} \,. \qquad \ldots \quad (128)$$

In addition the correction term of equation (80) has to be multiplied by $(1 - M^2)$, so that we have instead of equation (84):

$$\overline{i_{\nu\nu}} = i_{\nu\nu} - 0{\cdot}1302(1 - M^2)K_1 \left(\frac{b}{2c_\nu}\right)^2 \frac{4 \sin \theta_\nu}{m + 1} (\eta_{\nu+1}\, \eta_{\nu-1}) \,,$$

with a similar equation in $\overline{j_{\nu\nu}}$ and $j_{\nu\nu}$.

7. *Resulting Forces and Moments.*—Almost a matter of course are the forces and moments resulting from our load distributions γ and μ.

For the general case of a cambered and twisted wing there are four quantities to be determined. These are,

$d\bar{C}_L/d\alpha$	rate of change of overall lift coefficient with incidence
α_0	no-lift angle
$dC_M/d\bar{C}_L$	aerodynamic centre
$C_{m\,0}$	C_M at no lift.

This requires two calculations (*a*) with zero incidence of the root chord, when the incidence distribution is defined by the camber and twist, and (*b*) with the same wing plan form at uniform incidence. Suppose $\bar{C}_{L1}$ is overall lift coefficient for (*a*), and C_{M1} the corresponding pitching-moment coefficient then,

$$\bar{C}_{L1} = -\frac{d\bar{C}_L}{d\alpha} \cdot \alpha_0$$

and

$$C_{M1} - C_{M0} = \frac{dC_M}{d\bar{C}_L} \cdot \bar{C}_{L1} .$$

From which α_0 and C_{M0} can be determined when C_{M1}, $dC_M/d\bar{C}_L$, C_{L1} and $d\bar{C}_L/d\alpha$ are known. We shall now consider the calculation of $d\bar{C}_L/d\alpha$ and $dC_M/d\bar{C}_L$ in detail. The calculation of C_{L1} and C_{M1} is on similar lines.

The lift per unit span

$$\frac{dL}{dy} = \tfrac{1}{2}\rho U^2 C_L c = \rho U^2 b\gamma \qquad \ldots \qquad (129)$$

gives the total lift of the wing

$$L = \rho U^2 b \int_{-b/2}^{b/2} \gamma \, dy = \tfrac{1}{2}\rho U^2 b^2 \int_{-1}^{1} \gamma \, d\eta \qquad \ldots \qquad (130)$$

with the usual definitions

$$\bar{C}_L = \frac{L}{\frac{1}{2}\rho U^2 S} \quad \text{and} \quad A = \frac{S}{b^2}$$

this leads to

$$\bar{C}_L = A \int_{-1}^{1} \gamma \, d\eta \,. \qquad \ldots \qquad (131)$$

If γ is represented by our interpolation formula we can apply a simple approximate integration :

$$\bar{C}_L = \frac{\pi A}{m+1} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_n \sin \theta_n = \frac{\pi A}{m+1} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_n \cos \frac{\pi n}{m+1} \,. \qquad \ldots \qquad (132)$$

For the γ-distribution belonging to $\alpha = 1$ everywhere in the wing surface we obtain the $dC_M/d\alpha$ of the wing by this formula.

If γ is symmetrical in η we have instead:

$$\bar{C}_L = \frac{2\pi A}{m+1} \left[\frac{\gamma_0}{2} + \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_n \cos \frac{\pi n}{m+1} \right] . \qquad \ldots \qquad (133)$$

Just as easy is the rolling moment L. From

$$C_l = \frac{L}{\frac{1}{2}\rho U^2 S b} = \frac{A}{2} \int_{-1}^{1} \gamma\eta \, d\eta \qquad \ldots \qquad (134)$$

we find with $\sin 2\theta = 2 \sin \theta \ \cos \theta = 2\eta \sin \theta$

$$C = \frac{\pi A}{4(m+1)} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_n \sin 2\theta_n \qquad \ldots \qquad (135)$$

or for anti-symmetrical γ-values:

$$C_l = \frac{\pi A}{2(m+1)} \sum_{1}^{\frac{m-1}{2}} \gamma_n \sin 2\theta_n \,. \qquad \ldots \qquad (136)$$

The pitching moment of the wing is affected by γ and μ. The chordwise position of the centre of pressure of any section is given by

$$X_{ac} = 0{\cdot}25 - \mu/\gamma \quad \ldots \qquad (137)$$

For the whole wing we find the moment about the y-axis by:

$$M = \tfrac{1}{2}\rho U^2 \int_{-b/2}^{b/2} [C_m c^2 - C_L c x_{c/4}]\, dy \, . \qquad (138)$$

$$= \tfrac{1}{2}\rho U^2 \frac{b^3}{2} \int_{-1}^{1} \left[\mu \frac{2c}{b} - \gamma \, . \, \xi_{c/4}\right] d\eta \, .$$

With the non-dimensional coefficient

$$C_M = \frac{M}{\frac{1}{2}\rho U^2 S \bar{c}} = \frac{Mb}{\frac{1}{2}\rho U^2 S^2} \, .$$

we obtain by again applying the above mentioned approximate integration formula:

$$C_M = \frac{\pi A^2}{2(m+1)} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \left(\mu_n \frac{2c_n}{b} - \gamma_n \xi_{n\,c/4}\right) . \cos \frac{n\pi}{m+1} \, . \qquad (139)$$

For symmetrical load distributions this is simpler:

$$C_M = \frac{\pi A^2}{m+1} \left[\sum_{1}^{\frac{m-1}{2}} \left(\mu_n \frac{2c_n}{b} - \gamma_n \xi_{n\,c/4} \, . \, \frac{\cos n\pi}{m+1} + \frac{1}{2}\left(\mu_0 \frac{2c_0}{b} - \gamma_0 \xi_{0\,c/4}\right)\right]. \qquad (140)$$

It is often useful to define the centre of pressure, *i.e.*, the centre of gravity of the load distribution; for the load belonging to the constant incidence case this is the 'aerodynamic centre'. Its ξ-co-ordinate is:

$$\xi_{\text{c.p.}} = \frac{\sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \left(\gamma_n \xi_{n\,c/4} - \mu_n \frac{2c_n}{b}\right) \cos \frac{n\pi}{m+1}}{\sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_n \cos \frac{n\pi}{m+1}} \, . \qquad (141)$$

If the load distribution is not symmetrical its η-co-ordinate is:

$$\eta_{\text{c.p.}} = \frac{\sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_n \sin \frac{2n\pi}{m+1}}{2 \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_n \cos \frac{n\pi}{m+1}} \, . \qquad (142)$$

More complicated are forces and moments resulting partly or entirely from components in the wing area and not normal to it as induced drag, yawing moment and side force. We will deal here only with the induced drag because those unsymmetrical moments and forces are required mainly in connection with special unsymmetrical motions of the aircraft the investigation of which is beyond the scope of this report.

As to the induced drag we can rely on Munk's law that the induced drag is independent of the chordwise position of the lift components; this appears understandable if we consider the work done by the induced drag as the equivalent of the energy in the velocity field behind the wing and normal to the direction of flight. Thus, with the monoplane wing which we are only considering here it is the spanwise distribution of the lift, *i.e.*, the $\gamma(\eta)$ distribution which counts, and we may safely go back in this special question to lifting-line conceptions. With the 'induced angle of attack' α_i, equation (61), the induced drag is calculated from the integral

$$C_{D\,i} = A \int_{-1}^{1} \gamma \, . \, \alpha_1 \, d\eta$$

$$= \frac{\pi A}{m+1} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_\nu \alpha_{i\,\nu} \cos \frac{\nu\pi}{m+1} \, . \qquad (143)$$

At the ν-th station α_i is given by

$$\alpha_i = b_{\nu\nu}\gamma_\nu - \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} b_{\nu n}\gamma_n \qquad (144)$$

(equation (71) for $i = 1$); with the $a_{\nu n}$ introduced in equation (92) we derive then from $a_{\nu n} = 4b_{\nu n}/(m+1) \, . \cos \nu\pi/(m+1)$:

$$C_{D\,i} = \frac{\pi A}{4} \left[\sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_\nu^2 - \sum_{\frac{m-1}{2}}^{\frac{m-1}{2}} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} a_{\nu n}\gamma_\nu\gamma_n \right] \qquad (145)$$

since every product $\gamma_\nu\gamma_n$ appears twice and $a_{\nu n} = a_{n\nu}$ we may also write

$$C_{D\,i} = \frac{\pi A}{4} \left[\sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \gamma_\nu^2 - 2 \sum_{\substack{-\frac{m-1}{2} \\ \nu = \pm 1, 3, 5 \ldots}}^{\frac{m-1}{2}} \; \sum_{\substack{-\frac{m-1}{2} \\ n = 0, \pm 2, 4, 6 \ldots}}^{\frac{m-1}{2}} a_{\nu n}\gamma_\nu\gamma_n \right] . \qquad (146)$$

With the tabulated $a_{\nu n}$-values this summation is fairly simple. A further simplification for symmetrical loads gives:

$$C_{D\,i} = \pi A \left[\frac{\gamma_0^2}{4} + \frac{1}{2} \sum_{1}^{\frac{m-1}{2}} \gamma_\nu^2 - \sum_{\nu = 1, 3, 5}^{\frac{m-1}{2}} \; \sum_{\substack{-\frac{m-1}{2} \\ n = 0, \pm 2, 4, 6}}^{\frac{m-1}{2}} a_{\nu n}\gamma_\nu\gamma_n \right] . \qquad (147)$$

The thus calculated induced drag can be compared with the difference of the component of the lift in the direction of flow and the suction force at the leading edge which can be approximately assessed as follows; the suction force involves only the term in cot $\varphi/2$ in the expression for the chordwise load, or in other words the lift corresponding to the flat plate. We thus have,

$$\frac{1}{\frac{1}{2}\rho U^2 c} \frac{dS}{dy} = \frac{C_{L\,0}^2}{2\pi} \qquad (= C_{L\,0}\,\alpha \text{ for flat plate})$$

where $C_{L\,0}$ is the lift due to the cot $\varphi/2$ term. Now from equations (31) and (35), and using equations (32) and (36),

$$C_{L\,0} = \frac{\pi}{2}(a_0 + a_1)$$

$$= C_L + 4C_M$$

$$= \frac{2b}{c}(\gamma + 4\mu) \, .$$

Inserting this in the equation for dS/dy we have,

$$C_S = \frac{1}{\frac{1}{2}\rho U^2 c} \frac{dS}{dy} = \frac{1}{2\pi} \left(\frac{2b}{c}\right)^2 (\gamma + 4\mu)^2 \quad \ldots \quad (148)$$

so that the total suction force is

$$\frac{S}{\frac{1}{2}\rho U^2} = \frac{b^2}{2\pi} \int_{-1}^{1} \frac{(\gamma + 4\mu)^2\, d\eta}{2c/b} \quad \ldots \quad (149)$$

$$\bar{C}_S = \frac{A}{2(m+1)} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} (\gamma_n + 4\mu_n)^2 \left(\frac{b}{2c_n}\right) \cos \frac{n\pi}{m+1} . \quad \ldots \quad (150)$$

For a flat plate it should be

$$C_{Di} = \alpha C_L - \overline{C_S} . \quad \ldots \quad (151)$$

The validity of this equation can be used for checking whether the reduction of the chordwise load distribution to only two independent terms seems justified. If the discrepancy is appreciable further terms are needed.

The spanwise distribution of the induced drag cannot be derived from equation (143); it may be entirely different from the $\gamma \, . \, \alpha_i$-distribution. In this respect the resultant from lift per unit span and suction force projected into the $z = 0$ plane is more reliable.

8. *Examples*.—As an illustration of the methods derived here a few examples have been calculated. They are so chosen as to give a good comparison with other theoretical or experimental results. Owing to the complexity of the problem there are only very few theoretical solutions of a reliable degree of accuracy available and these are not very typical.

A first example of this kind is a wing of circular plan form which was worked out by Kinner[3] by means of special potential functions entirely different from our load functions and suitable only for this plan form. Kinner gives as the result of his calculations $dC_L/d\alpha = 1{\cdot}82$ and the aerodynamic centre at $0{\cdot}515$ radius ahead of the wing centre. The calculation for 2×5 pivotal points gave

$$\frac{dC_L}{d\alpha} = 1{\cdot}799 \quad \text{and} \quad \frac{-x_{ac}}{r} = 0{\cdot}528 .$$

The local γ- and μ-values for $\alpha = 1$ were:

n	0	1	2
η_n	0	$0{\cdot}5$	$0{\cdot}866$
γ_n	$0{\cdot}907_1$	$0{\cdot}774_8$	$0{\cdot}440_8$
μ_n	$0{\cdot}048_1$	$0{\cdot}044_7$	$0{\cdot}045_6$

i.e., towards the wing tips we find the section aerodynamic centre farther forward than in the middle of the wing and the local lift coefficient is in the wing centre slightly higher than the mean value. This is in good agreement with Kinner's results. But the differences in the total lift and the moment were more than anticipated and since no mistake could be found in our calculations we had to check Kinner's computations. Kinner's solution is not exact but is an approximation by a series of functions suitable only for this special case of which he considered 3 or 4 as enough. The result of the repeated calculation with some more valid decimals was as follows:

Number of functions taken into account	1	2	3	4	5
$dC_L/d\alpha$	$2{\cdot}286$	$1{\cdot}883$	$1{\cdot}832$	$1{\cdot}811$	$1{\cdot}804$
$-x_{a.c.}/r$	$0{\cdot}424$	$0{\cdot}499$	$0{\cdot}511$	$0{\cdot}516$	$0{\cdot}519$

The convergence towards values fairly close to our results is obvious.

A second case which appeared worth considering was a 'delta' wing which was previously calculated by Garner[12,13] and Falkner[11]. The wing has a 45-deg swept-back leading edge and a straight trailing edge: the aspect ratio is 3, the taper ratio 7 : 1. The lift distribution is calculated for $\alpha = 1$ with 1×7 and 2×15 pivotal points. The agreement between Garner's results and the author's for 2×15 points in the lift distribution and the location of the sectional aerodynamic centres is remarkedly good especially in the central part of the wing, Fig. 8. Some agreement could be expected because the basic conception of both methods is not very different. Garner's distribution of pivotal points is somewhat different; *e.g.*, he has four such points in the central section. The fact that here the agreement is particularly good between both methods may be considered as some justification by results of our location of pivotal stations. Also remarkable is the fairly good agreement between the 1×7 points solution and the much more laborious 2×15 points solution for the spanwise γ-distribution. Of course, the calculation with 1×7 points does not give the local pitching moments and, therefore, not the sectional aerodynamic centres which must be *faute de mieux* assumed at $c/4$, but even so the resulting aerodynamic centre of the whole wing is not too badly estimated. What improvement is actually achieved by our methods may be seen from the comparison with Falkner's results, for Falkner's methods could be considered as the best approximate method so far available. The computer work required for wings like this delta is less than one hour for 1×7 pivotal points and about two days for 2×15 points.

A few numerical values are not uninteresting:

Method of calculation	$\dfrac{dC_L}{d\alpha}$	$-\xi_{\text{a.c.}}$ (ahead of trailing edge)
Garner's method c	3·038	0·545
Falkner's 6-point solution	3·21	0·556
Falkner's 8-point solution	3·19	0·547
This method with 2×15 points ..	3·057	0·542
This method with 1×7 points ..	3·040	(0·555)
Lifting-line, 7 points	3·68	(0·570)
R.A.E. measurements	3·048	0·538

A further case which was calculated with our method for two chord-wise pivotal stations is an infinite 45-deg swept wing of constant chord, Fig. 9. The loss in the lift distribution in the wing centre is considerable and extends to spanwise stations far away from the middle. The local aerodynamic chord line approaches more quickly the $c/4$ line. No other theoretical values for this case are available except Schlichting's middle function[9] which appears to be a rather crude approximation. Some new more detailed calculations of this 'middle function' which are not yet published come much nearer to our load distribution. The only experiments which are comparable to some extent (wind-tunnel corrections are difficult to assess) show fairly good agreement (J. Weber, Ref. 17).

A last example is added in order to show the practical course of calculation. The whole calculation is set out in Tables 8 to 30, which are used in Appendix VI to develop a detailed set of instructions for computers' use. In Tables 8 to 12 we find the details of the computation for the symmetrical load distribution of an arbitrarily chosen wing calculated with 1×15 pivotal points. The arrangement shows the three stages of the calculation: in Form 1 all the geometrical data of the wing are collected together with those coefficients which are only a function of one station. Form 2 contains the computation of the coefficients which depend on two stations; the $X_{\nu n}$, $Y_{\nu n}$ are the co-ordinates of the ν-th pivotal point measured in chords at the n-th section with the leading edge of that section as origin. The $i_{\nu n}$ are taken out of the Figs. 1 to 3. In Form 3 the solution of the system of equations is demonstrated.

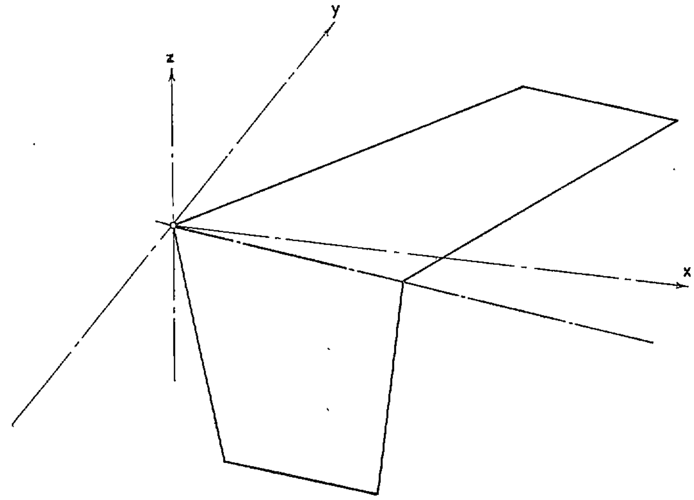
Tables 13 to 22 contain the calculation for the same wing with 2×15 pivotal stations. In Tables 23 to 30, the calculations have been extended to give the asymetric load distribution produced by ailerons on the same wing (*see* diagram of Table 23). The results of the whole calculation are shown in Fig. 10.

These sheets may be used as a guide for those who merely want to calculate special cases without bothering about the underlying theory. In these examples the calculation is carried through with about one decimal more than usually required mainly because a calculating machine was used. As a rule the whole computation can be done with a slide rule although calculating machines may reduce the work still further. For calculations with a slide rule only, the time required for the first load distribution for a given wing plan form was found to be roughly one minute times the square of the numbers of pivotal points considered, provided one knows how the calculation is to be done.

9. *Conclusion.*—A general and convenient method of calculating the load distribution of arbitrary wings based on lifting-surface conceptions is developed. The foundation is the downwash integral for a wing with given lift distribution as it is obtained by integrating the Euler's equations of the motion of a flow without friction. This downwash integral is dealt with in two stages: firstly the chordwise integration which leads to complicated elliptical integrals is done numerically for the whole range of interesting positions of the inducing section relative to any pivotal points or vice versa, assuming the most important chordwise load distributions only. The spanwise integration of the downwash is achieved by a method of approximate integration similar to that developed by the author for lifting-line problems. Special care is taken in the choice of the position of pivotal stations, *i.e.*, points in the wing surface at which the boundary conditions of the integral equation are exactly fulfilled. The result is a linear system of equations with the lifts and, if required, also the moments per unit span at certain spanwise stations as unknowns. The coefficients of this system of equations can be easily calculated from the geometrical data of the wing using tabulated factors which depend only on the arrangement of pivotal points and the diagrams for the chordwise components of the downwash integral. The solution of this system of equations can always be done by iteration, so that even fairly large numbers of pivotal stations can be employed. The computing effort is fairly moderate and compares favourably with other methods which achieve or aim at a similar degree of accuracy.

I wish to acknowledge here the valuable help I had from my assistant M. Winter in many parts of the numerical calculation and from S. B. Gates and K. W. Mangler in a critical review of the general arrangement.

10. NOTATION



x,y,z		Rectangular co-ordinates system attached to the wing
x		In the direction of undisturbed flow in the plane of symmetry of the wing
y		In the starboard direction
z		Upwards in the plane of symmetry
U		Velocity of undisturbed flow relative to the wing
u,v,w		Additional velocities produced by the wing in the direction of the x,y,z-axes
α	$=$	$-\,\partial z_0/\partial x$ (local) wing incidence, $z_0(x,y)$ describing the wing skeleton
p		Pressure (of the flow about the wing)
ρ		Density (of the flow about the wing)
a		Speed of sound (of the flow about the wing)
I		Enthalpy of the unit volume (of the flow about the wing)
l	$=$	$\dfrac{p_{\text{pressure side}} - p_{\text{suction side}}}{\frac{1}{2}\rho U^2}$ Non-dimensional load per unit area of the wing
b		Wing span
c		Wing chord (usually a function of y)
Λ		Angle of sweep
ξ	$=$	$x/\frac{1}{2}b$ (Non-dimensional co-ordinates related to the semispan)
η	$=$	$y/\frac{1}{2}b$ Non-dimensional co-ordinates related to the semispan
ζ	$=$	$z/\frac{1}{2}b$ (Non-dimensional co-ordinates related to the semispan)
θ	$=$	$\cos^{-1}\eta$ angular spanwise co-ordinate ($y = \frac{1}{2}b\cos\theta$)
X	$=$	$(x - x_{0\,\text{L.E.}})/c(y_0)$ Non-dimensional wing co-ordinates related to the inducing wing section (suffix $_0$)
Y	$=$	$(y - y_0)/c(y_0)$ $x_{0\,\text{L.E.}}$ co-ordinate of the leading edge of the inducing wing section
φ	$=$	$\cos^{-1}(1 - 2X)$ angular chordwise co-ordinate
i,j		Influence functions (chordwise downwash integrals)

NOTATION—*continued*

m Number of wing sections taken into account

ν,n Suffixes numerating the spanwise stations, ν giving the pivotal station, n the inducing station

$$-(m-1)/2 \leqslant \nu, \eta \leqslant (m-1)/2$$

$$\theta_\nu = \frac{\pi}{2} - \frac{\nu\pi}{m+1}$$

$$\eta_\nu = \cos\theta_\nu = \sin\frac{\nu\pi}{m+1}$$

$a_{\nu n}, b_{\nu n}$, etc. Coefficients for approximate integration

$\gamma = C_L c/2b$ Non-dimensional lift per unit span

$\mu = C_m c/2b$ Non-dimensional moment per unit span

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APPENDIX I

Contributed by W. Mangler

On the Second-Order Principal Value for the Downwash Integral, Equations (15) *and* (16)

For small $(y - y_0)$ it is better to integrate the downwash first for $z \neq 0$ and to go to the limit $z \to 0$ later on. Thus we obtain

$$\frac{w}{U}(x,y,z) = \frac{-1}{U^2}\int_{-\infty}^{x} \frac{\partial I}{\partial z}(x',y,z)\, dx'$$

$$= \frac{-1}{8\pi}\iint_S l(x_0,y_0) \cdot \left\{\int_{-\infty}^{x} \frac{\partial}{\partial z} \frac{z}{[(x'-x_0)^2 + (y-y_0)^2 + z^2]^{3/2}}\, dx'\right\} dx_0\, dy_0$$

$$= \frac{-1}{8\pi}\iint_S l(x_0,y_0) \cdot \frac{\partial}{\partial z}\left\{\frac{z}{(y-y_0)^2 + z^2}\left(1 + \frac{x - x_0}{\sqrt{[(x-x_0)^2 + (y-y_0)^2 + z^2]}}\right)\right\} dx_0\, dy_0 .$$

Now we may use the identity

$$\frac{\partial}{\partial z}\frac{z}{(y-y_0)^2 + z^2} = -\frac{\partial}{\partial y_0}\frac{y_0 - y}{(y_0 - y)^2 + z^2} = \frac{(y_0 - y)^2 - z^2}{[(y_0 - y)^2 + z^2]^2}$$

to write :

$$\frac{w}{U}(x,y,z) = \frac{1}{8\pi}\iint_S l(x_0,y_0)\left\{1 + \frac{x - x_0}{\sqrt{[(x-x_0)^2 + (y-y_0)^2 + z^2]}}\right\}\frac{\partial}{\partial y_0}\left(\frac{y_0 - y}{(y_0 - y)^2 + z^2}\right) dx_0\, dy_0$$

$$+ \frac{1}{8\pi}\iint_S l(x_0,y_0)\frac{z^2(x - x_0)\, dx_0\, dy_0}{[(y-y_0)^2 + z^2][(x-x_0)^2 + (y-y_0)^2 + z^2]^{3/2}} .$$

The first of these integrals can be integrated by parts with respect to y_0:

$$\frac{w}{U}(x,y,z) = \left|\frac{1}{8\pi}\frac{y_0 - y}{(y_0 - y)^2 + z^2} \cdot \int_{x_l(y_0)}^{x_t(y_0)} l(x_0,y_0)\left\{1 + \frac{x - x_0}{\sqrt{[(x-x_0)^2 + (y-y_0)^2 + z^2]}}\right\} dx_0\right|_{y_0=-b/2}^{y_0=b/2}$$

$$- \frac{1}{8\pi}\int_{-b/2}^{b/2}\frac{\partial}{\partial y_0}\left[\int_{x_l(y_0)}^{x_t(y_0)} l(x_0,y_0)\left\{1 + \frac{x - x_0}{\sqrt{[(x-x_0)^2 + (y-y_0)^2 + z^2]}}\right\} dx_0\right]\frac{(y_0 - y)\, dy_0}{(y_0 - y^2) + z^2}$$

$$+ \frac{z^2}{8\pi}\iint_S l(x_2,y_0)\frac{(x - x_0)\, dx_0\, dy_0}{[(y-y_0)^2 + z^2][(x-x_0)^2 + (y-y_0)^2 + z^2]^{3/2}} .$$

The first of these three terms is zero because the lift $l(x_0,y_0)$ disappears towards the wing tips $(y_0 \to \pm\, b/2)$. If we now consider the limit $z \to 0$ the third integral too is vanishing and the second is given by the Cauchy's principal value:

$$\frac{w}{U}(x,y,z) = \frac{-1}{8\pi}\lim_{\varepsilon \to 0}\left\{\int_{-b/2}^{y-\varepsilon}\frac{\partial}{\partial y_0}\left[\int_{x_l(y_0)}^{x_t(y_0)} l(x_0,y_0)\left(1 + \frac{x - x_0}{\sqrt{[(x-x_0)^2 + (y-y_0)^2]}}\right) dx_0\right]\frac{dy_0}{y_0 - y}\right.$$

$$\left. + \int_{y+\varepsilon}^{b/2}\frac{\partial}{\partial y_0}\left[\int_{x_l(y_0)}^{x_t(y_0)} l(x_0,y_0)\left(1 + \frac{x - x_0}{\sqrt{[(x-x_0)^2 + (y-y_0)^2]}}\right) dx_0\right]\frac{dy_0}{y_0 - y}\right\} .$$

With

$$f(y_0) = \int_{x_l(y_0)}^{x_t(y_0)} l(x_0,y_0) \left(1 + \frac{x - x_0}{\sqrt{[(x - x_0)^2 + (y - y_0)^2]}}\right) dx_0$$

we obtain by another partial integration:

$$\lim_{\varepsilon \to 0} \left\{ \int_{-b/2}^{y-\varepsilon} \frac{\partial f}{\partial y_0} \frac{dy_0}{y - y_0} + \int_{y+\varepsilon}^{b/2} \frac{\partial f}{\partial y_0} \frac{dy_0}{y - y_0} \right\}$$

$$= \lim_{\varepsilon \to 0} \left\{ \frac{f(y - \varepsilon)}{\varepsilon} - \frac{f\left(-\frac{b}{2}\right)}{y + \frac{b}{2}} + \frac{f\left(\frac{b}{2}\right)}{y - \frac{b}{2}} - \frac{f(y + \varepsilon)}{-\varepsilon} - \int_{-b/2}^{y-\varepsilon} \frac{f(y_0)\, dy_0}{(y - y_0)^2} - \int_{y+\varepsilon}^{b/2} \frac{f(y_0)\, dy_0}{(y - y_0)^2} \right\}.$$

Because the lift disappears towards the wing tips we have $f(-b/2) = f(b/2) = 0$. If $f(y_0)$ is a continuous function we have also for a small ε

$$\frac{f(y - \varepsilon)}{\varepsilon} - \frac{f(y + \varepsilon)}{-\varepsilon} = \frac{f(y)}{\varepsilon} - f' + \frac{\varepsilon f''}{2} - \ldots + \frac{f(y)}{\varepsilon} + f' + \frac{\varepsilon f''}{2} + \ldots$$

$$= \frac{2f(y)}{\varepsilon} + \varepsilon f'' \ldots .$$

The downwash integral may, therefore, be written as equation (15) if we define

$$\int_{-b/2}^{b/2} \frac{f(y_0)\, dy_0}{(y - y_0)^2} = \lim_{\varepsilon \to 0} \left(\int_{-b/2}^{y-\varepsilon} \frac{f(y_0)\, dy_0}{(y - y_0)^2} + \int_{y+\varepsilon}^{b/2} \frac{f(y_0)\, dy_0}{(y - y_0)^2} - 2\frac{f(y)}{\varepsilon} \right\}. \quad \ldots \quad (16)$$

APPENDIX II

Wings with Flaps

To satisfy the integral equation in a limited number of pivotal stations only seems justifiable if the boundary conditions, *i.e.*, the $\alpha(x,y)$-distribution, are fairly continuous. An obvious case of practical importance where this is not true is the wing with flaps to be used as ailerons, high-lift devices, etc. Within the area of these flaps $\alpha(x,y)$ has a different value from the remaining wing area and for practical reasons there is a sudden change from one level to the other, i.e., *the flap contours are discontinuity lines.* It is purely accidental whether any of our pivotal points are just inside or outside the flap area. To increase the number of pivotal stations until the flap contours are fixed within sufficiently close limits is hopeless if we consider the computing effort required. Splitting off the discontinuities is at least not simple because the whole wing contour must be taken into account. Thus the only possible way seems to be an adjustment of the α-values at the chosen pivotal points, *i.e.*, we replace the discontinuous chordwise $\alpha(x,y)$-distribution by a continuous one which gives roughly the right forces and moments in two-dimensional flow.

With only one chordwise pivotal point at each spanwise station we can only follow the procedure of the lifting-line theory: over the span occupied by the flap we choose the value of α which in two-dimensional flow produces the same lift as the flap deflection δ. It is usual to define a quantity

$$\frac{d\alpha}{d\delta} = \frac{\partial C_L}{\partial \delta} \Big/ \frac{\partial C_L}{\partial \alpha}$$

as the factor by which the flap angle is to be multiplied to obtain the equivalent angle of incidence. There exists ample theoretical and experimental material about this quantity for many types of flap.

With two chordwise pivotal points in the standard positions we can do rather better. We may choose the α values (α' at the rear pivotal point and α'' at the front one) so as to obtain in two-dimensional flow the lift and pitching moment which are produced by the flap. α' and α'' are obtained from equations (25), (26) if for C_L and C_m we substitute $C_{L\,\delta}$ and $C_{m\,\delta}$, the values produced by a flap angle δ:—

$$C_{L\,\delta} = K_1\alpha' + K_2\alpha''$$
$$C_{M\,\delta} = K_3\alpha' + K_4\alpha'' .$$

Using the values of the K's given in equation (27) we obtain

$$\alpha' = \frac{C_{L\,\delta}}{2\pi} - \frac{C_{M\,\delta}}{\pi}(\sqrt{5} - 1)$$

$$\alpha'' = \frac{C_{L\,\delta}}{2\pi} + \frac{C_{M\,\delta}}{\pi}(\sqrt{5} + 1)$$

but $C_{L\,\delta}/2\pi$ is α, the incidence required to produce the same lift as δ.

Hence

$$\alpha' = \alpha - 0{\cdot}393C_{m\,\delta}$$
$$\alpha'' = \alpha + 1{\cdot}030C_{m\,\delta}$$

or in differential form

$$\frac{d\alpha'}{d\delta} = \frac{d\alpha}{d\delta} - 0{\cdot}393\,\frac{dC_{m\,\delta}}{d\delta}$$

$$\frac{d\alpha''}{d\delta} = \frac{d\alpha}{d\delta} + 1{\cdot}030\,\frac{dC_{m\,\delta}}{d\delta} .$$

$\partial C_m/\partial\delta$ is also frequently measured. Since it is usually negative the flap deflection appears mainly in the rear pivotal point (α') as we should expect. A few typical values are calculated with the theoretical values of $d\alpha/d\delta$ and $dC_m/\partial\delta$ for hinged flaps:

$\frac{c_{\text{flap}}}{c}$	0·1	0·2	0·3	0·4	0·5
$\frac{\partial\alpha'}{\partial\delta}$	0·608	0·802	0·913	0·979	1·015
$\frac{\partial\alpha''}{\partial\delta}$	−0·160	−0·109	0	0·143	0·303

We cannot expect to find also the whole pressure distribution of the wing with flaps by this simplified method but the resulting forces and moments should be fairly accurate as long as we have no flow separation, etc., since the application of the simpler lifting-line theory gave already fairly reliable values.

If more than two chordwise pivotal points are used at each spanwise station we can satisfy some more conditions about the equivalent incidences. Falkner[16] who had to deal with the analogous problem for his vortex-lattice method takes also the hinge moment into account besides the lift and pitching moment.

It follows from this chordwise representation of incidence that if the wing is at zero incidence with a flap angle, pivotal points inboard of the flap will have zero incidence, while those in the part of the span occupied by the flap will have incidences α', α''. This leaves us with the problem of adjusting the pivotal values near the inner edge of the flap to deal as far as possible with the spanwise discontinuity there. A simple rule to deal with this is illustrated by the diagram of Table 23. We allocate to each spanwise station a chordwise strip bounded by the mean lines between this point and its two neighbours, see strip 3 of the diagram. The incidence at the pivotal station is taken as the mean value over this strip, calculated with respect to the angular co-ordinate θ. Thus there will always be one chordwise station to be so adjusted, unless the inner edge of the flap happens to bisect the distance between two successive chordwise stations.

APPENDIX III

The Logarithmic Singularity of the Influence Functions i and j

The influence functions i and j are defined by equation (37) and (38) which may be written in non-dimensional co-ordinates, X, Y, equation (30) as follows:

$$i(X,Y) = \frac{1}{C_L}\int_0^1 l_0(X_0)\left\{1 + \frac{X - X_0}{\sqrt{[(X - X_0)^2 + Y^2]}}\right\} dX_0$$

and

$$j(X,Y) = \frac{1}{C_m}\int_0^1 l_1(X_0)\left\{1 + \frac{X - X_0}{\sqrt{[(X - X_0)^2 + Y^2]}}\right\} dX_0 .$$

For small distances Y we may consider now the difference between this value $i(X,Y)$, (or $j(X,Y)$), and the limiting value for $Y = 0$; it may be called

$$\Delta i(X,Y) = i(X,Y) - i(X,0) = \frac{-1}{C_L}\int_0^X l(X_0)\left\{1 - \frac{X - X_0}{\sqrt{[(X - X_0)^2 + Y^2]}}\right\} dX_0$$

$$+ \frac{1}{C_L}\int_X^1 l(X_0)\left\{1 - \frac{X_0 - X}{\sqrt{[(X - X_0)^2 + Y^2]}}\right\} dX_0 .$$

It is easily seen that for small Y the factors in brackets in these two integrals are of an appreciable size only near $X_0 = X$. It seems, therefore, reasonable to develop $l_0(X_0)$ into a Taylor series from the point $X_0 = X$:

$$l_0(X_0) = l_0(X) + (X_0 - X)\frac{dl_0}{dX_0}(X) + \frac{(X_0 - X)^2}{2\,!}\frac{d^2l_0}{dX_0^2}(X) + \frac{(X_0 - X)^3}{3\,!}\frac{d^3l_0}{dX_0^3}(X) + \cdots .$$

We can try, then, to integrate Δi term by term; for convenience we introduce

$$X_1 = X_0 - X \text{ for } X_0 > X$$
$$X_1 = X - X_0 \text{ for } X_0 < X .$$

Δi is now given by:

$$\Delta i(X,Y) = \frac{l_0(X)}{C_L}\int_X^{1-X}\left[1 - \frac{X_1}{\sqrt{(X_1^2 + Y^2)}}\right] dX_1$$

$$+ \frac{1}{C_L}\frac{dl_0}{dX_0}(X)\left\{\int_0^{1-X} + \int_0^X X_1\left[1 - \frac{X_1}{\sqrt{(X_1^2 + Y^2)}}\right] dX_1\right\}$$

$$+ \frac{1}{2C_L}\frac{d^2l_0}{d_0X^2}(X)\int_X^{1-X} X_1^2\left[1 - \frac{X_1}{\sqrt{(X_1^2 + Y^2)}}\right] dX_1$$

$$+ \frac{1}{3\,!\,C_L}\frac{d^3l_0}{dX_0^3}(X)\left\{\int_0^{1-X} + \int_0^X X_1^3\left[1 - \frac{X_1}{\sqrt{(X_1^2 + Y^2)}}\right] dX_1\right\}$$

$$+ \cdots .$$

All these integrals are easily evaluated. By expanding the resulting expressions in powers of Y^2 we obtain a regular series development of $\Delta i(Y)$ but in addition to that we see some terms with the factor $\ln Y$ which can not be developed into a series of powers of Y^2; thus we obtain

$$\Delta i(X,Y) = a_1Y^2 + a_2Y^4 + \cdots$$

$$+ \ln Y\left\{\frac{-Y^2}{C_L}\frac{dl_0}{dX_0} + \frac{Y^4}{8C_L}\frac{d^3l_0}{dX_0^3} - \frac{Y^6}{1152C_L}\frac{d^5l_0}{dX_0^5} + - \cdots\right\}.$$

The coefficients of the regular part of this series development are not worked out because they are no longer needed; of the irregular part only the first term can appreciably affect the downwash integral because the factor Y^2 is cancelled by the denominator in the spanwise integration. The coefficient for this term is

$$\frac{-1}{C_L}\frac{dl_0}{dX_0}(X)$$

for the development of i; correspondingly we find in the development of the second influence function j:

$$\frac{-1}{C_m}\frac{dl_1}{dX_0}(X)\,.$$

APPENDIX IV

Development of the Approximate Integration Formulae for the Downwash

With the interpolation stations chosen at equidistant subdivisions of the $0 \ldots \pi$ range of the angular co-ordinate $\theta = \cos^{-1} \eta$

$$\eta_n = \cos \theta_n = \cos\left(\frac{\pi}{2} - \frac{n\pi}{m+1}\right) = \sin \frac{n\pi}{m+1} = \sin\left(\frac{\pi}{2} - \theta_n\right)$$

we wrote the interpolation formula for γ or $(\gamma \, . \, i)$ as

$$(\gamma \, . \, i)(\theta) = \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} (\gamma \, . \, i)_n g_n(\theta) \, .$$

The $g_n(\theta)$ should be determined by the condition that

$$g_n(\theta) = 1 \, , \qquad \theta = \theta_n$$

$$g_x(\theta) = 0 \, , \qquad \theta = \theta_p \, , \qquad n \neq p = 0, \pm 1, \pm 2, \ldots \pm \frac{m-1}{2} \, .$$

To develop a function which satisfies these conditions we may consider the function $\sin (m + 1)\theta$ which disappears at all m stations; to make $g_n = 1$ at η_n we divide $\sin (m + 1)\theta$ by its tangent at η_n:

$$g_n(\theta) = \frac{\sin (m+1)\theta}{(\cos \theta - \cos \theta_n) \, . \, \dfrac{d \sin (m+1)\theta}{d \cos \theta} (\theta_n)} \, .$$

Now

$$\frac{d \sin (m+1)\theta}{d \cos \theta} (\theta_n) = \frac{(m+1) \cos (m+1)\theta_n}{- \sin \theta_n} \, .$$

This gives immediately:

$$g_n(\theta) = - \frac{\sin \theta_n}{(m+1) \cos (m+1)\theta_n} \cdot \frac{\sin (m+1)\theta}{\cos \theta - \cos \theta_n}$$

$$\cos (m+1)\theta_n = (-1)^{\{(m+1)/2\}-n} \, .$$

This is not the only way to represent this interpolation function $g_n(\theta)$. Since $\sin (m + 1)\theta$ is equal to $\sqrt{(1 - \eta^2)} \times$ polynominal in η up the to m-th power we may also develop $g_n(\theta)$ into a trigonometrical series:

$$g_n(\theta) = \sum_1^\infty a_\lambda \sin \lambda\theta \, .$$

Integrating both sides after a multiplication with $(2/\pi) \sin \lambda\theta$ we obtain the well-known Fourier formula:

$$\frac{2}{\pi} \int_0^\pi g_n(\theta) \sin \lambda\theta \, d\theta = a_\lambda$$

which gives:

$$a_\lambda = - \frac{2}{\pi(m+1)} \frac{\sin \theta_n}{\cos (m+1)\theta_n} \int_0^\pi \frac{\sin (m+1)\theta \sin \lambda\theta \, d\theta}{\cos \theta - \cos \theta_n} \, .$$

For $\lambda < m + 1$ this leads to

$$a_\lambda = \frac{1}{\pi(m+1)} \frac{\sin\theta_n}{\cos(m+1)\theta_n} \int_0^\pi \frac{\cos(m+1+\lambda)\theta - \cos(m+1-\lambda)\theta \, d\theta}{\cos\theta - \cos\theta_n} .$$

Now using Glauert's formula

$$\frac{1}{\pi}\int_0^\pi \frac{\cos p\theta \, d\theta}{\cos\theta - \cos\theta_n} = \frac{\sin p\theta_n}{\sin\theta_n} \qquad p \text{ integer} > 0$$

(compare H. Glauert: *The Elements of Aerofoil and Airscrew Theory*, pp. 92–93) we obtain

$$a_\lambda = \frac{1}{m+1} \frac{\sin\theta_n}{\cos(m+1)\theta_n} \left[\frac{\sin(m+1+\lambda)\theta_n}{\sin\theta_n} - \frac{\sin(m+1-\lambda)\theta_n}{\sin\theta_n}\right]$$

$$= \frac{2}{m+1} \sin\lambda\theta_n .$$

For $\lambda \geqslant m + 1$ we can show similarly that

$$a_\lambda = \frac{1}{\pi(m+1)} \frac{\sin\theta_n}{\cos(m+1)\theta_n} \cdot \int_0^\pi \frac{\cos(\lambda+m+1)\theta - \cos(\lambda-m-1)\theta \, d\theta}{\cos\theta - \cos\theta_n}$$

$$= \frac{1}{m+1} \frac{\sin\theta_n}{\cos(m+1)\theta_n} \left[\frac{\sin(\lambda+m+1)\theta_n - \sin(\lambda-m-1)\theta_n}{\sin\theta_n}\right]$$

$$= \frac{2}{m+1} \frac{\cos\lambda\theta_n \sin(m+1)\theta_n}{\cos(m+1)\theta_n} \equiv 0$$

because $\sin(m+1)\theta_n = 0$.

Thus, we have

$$g_n(\theta) = \frac{2}{m+1} \sum_{\lambda=1}^{m} \sin\lambda\theta_n \, \sin\lambda\theta$$

If we have any function f which can be so represented

$$f(\theta) = \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n g_n(\theta)$$

it follows immediately that

$$\int_{-1}^{1} f(\eta)\, d\eta = \int_0^\pi f(\theta) \sin\theta \, d\theta = \frac{\pi}{m+1} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n \sin\theta_n = \frac{\pi}{m+1} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n \cos\frac{n\pi}{m+1}$$

and

$$\int_{-1}^{1} f(\eta) \cdot \eta \, d\eta = \frac{1}{2}\int_0^\pi f(\theta) \sin 2\theta \, d\theta = \frac{\pi}{2(m+1)} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n \sin 2\theta_n$$

$$= \frac{\pi}{m+1} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n \sin\frac{n\pi}{m+1} \cos\frac{n\pi}{m+1} .$$

It is easily shown that these approximate integration formulae allow for something more than just the interpolation polynominals. If we put

$$f(\theta) = \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n g_n(\theta) + \sin(m+1)\theta \,.\, \sum_0^p a_\nu \cos q\theta$$

which means additional terms for the function $f(\theta)$ in the intervals between the θ_n we see that

$$\int_0^\pi f(\theta) \sin\theta \, d\theta = \frac{\pi}{m+1} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n \sin\theta_n$$

still holds if $p < m$, because

$$\int_0^\pi \sin(m+1)\theta \, \sin\theta \, \cos q\theta \, d\theta = \frac{1}{2}\int_0^\pi \sin(m+1)\theta \, [\sin(q+1)\theta - \sin(q-1)\theta] \, d\theta$$

$$= \begin{cases} \pi/4 \quad , & q = m \\ -\pi/4 \, , & q = m+2 \\ 0 \quad , & q \neq m, \quad q \neq m+2 \, . \end{cases}$$

Now, putting the interpolation formula for (γi)

$$\gamma i = \Sigma\, (\gamma i)_n g_n$$

into our downwash integral

$$\alpha(\eta) = \frac{-1}{2\pi}\int_{-1}^{1} \frac{\gamma i \, d\eta'}{(\eta - \eta')^2} = \frac{-1}{2\pi}\Sigma(\gamma i)_n \int_0^\pi \frac{g_n(\theta') \sin\theta' \, d\theta'}{(\cos\theta - \cos\theta')^2}$$

we see that the factors by which the $(\gamma i)_n$ are to be multiplied are functions of n and $\eta = \cos\theta$ only. In particular if α is calculated for the same set of spanwise stations

$$\eta = \eta_\nu = \cos\theta_\nu = \sin\frac{\pi\nu}{m+1}$$

we may write

$$\alpha_\nu = \alpha(\eta_\nu) = b_{\nu\nu}(\gamma i)_\nu - \Sigma' b_{\nu n}(\gamma i)_\nu$$

with

$$b_{\nu\nu} = \frac{-1}{2\pi}\int_0^\pi \frac{g_\nu(\theta') \sin\theta' \, d\theta'}{(\cos\theta_\nu - \cos\theta')^2}$$

$$b_{\nu n} = \frac{1}{2\pi}\int_0^\pi \frac{g_n(\theta') \sin\theta' \, d\theta'}{(\cos\theta_\nu - \cos\theta')^2}\, .$$

Using the above series for $g_n(\theta)$ we have to deal with integrals of the form

$$\frac{-1}{2\pi}\int_0^\pi \frac{\sin\lambda\theta' \, \sin\theta' \, d\theta'}{(\cos\theta_\nu - \cos\theta')^2}$$

which may be integrated in parts to give

$$\frac{\sin\lambda\theta'}{2\pi(\cos\theta_\nu - \cos\theta')}\Bigg|_0^\pi - \frac{\lambda}{2\pi}\int_0^\pi \frac{\cos\lambda\theta' \, d\theta'}{\cos\theta_\nu - \cos\theta'} = \frac{\lambda}{2}\frac{\sin\lambda\theta_\nu}{\sin\theta_\nu}\, .$$

It follows that

$$b_{\nu\nu} = \frac{1}{(m+1)\sin\theta_\nu}\sum_{\lambda=1}^{m}\lambda\sin^2\lambda\theta_\nu = \frac{1}{2(m+1)\sin\theta_\nu}\sum_{1}^{m}\lambda(1-\cos 2\lambda\theta_\nu)$$

$$b_{\nu n} = \frac{-1}{(m+1)\sin\theta_\nu}\sum_{\lambda=1}^{m}\lambda\sin\lambda\theta_\nu\,\sin\lambda\theta_n$$

$$= -\frac{1}{2(m+1)\sin\theta_\nu}\sum_{1}^{m}\lambda[\cos\lambda(\theta_\nu+\theta_n) - \cos\lambda(\theta_\nu-\theta_n)]\,.$$

To evaluate these sums we may consider

$$2(1-\cos x)\sum_{1}^{m}\lambda\cos\lambda x = \sum_{1}^{m}\lambda[2\cos\lambda x - \cos(\lambda-1)x - \cos(\lambda+1)x]$$

$$= \sum_{1}^{m}\lambda[-\cos(\lambda-1)x + 2\cos\lambda x - \cos(\lambda+1)x]\,.$$

Combining always the terms with the same angle we see that most of them cancel each other so that only a few of the first and the last remain:

$$2(1-\cos x)\sum_{1}^{m}\lambda\cos\lambda x = -1 + (m+1)\cos mx - m\cos(m+1)x\,.$$

This gives:

$$b_{\nu\nu} = \frac{1}{2(m+1)\sin\theta_\nu}\left[\frac{m(m+1)}{2} + \frac{1-(m+1)\cos 2m\theta_\nu + m\cos 2(m+1)\theta_\nu}{2(1-\cos 2\theta_\nu)}\right].$$

Since $\cos 2(m+1)\theta_\nu = 1$

$$\cos 2m\theta_\nu = \cos 2(m+1)\theta_\nu\cos 2\theta_\nu - \sin 2(m+1)\theta_\nu\,\sin 2\theta_\nu = \cos 2\theta_\nu$$

we obtain

$$b_{\nu\nu} = \frac{m+1}{4\sin\theta_\nu}.$$

In the same way we find for the $b_{\nu n}$:

$$b_{\nu n} = \frac{1}{2(m+1)\sin\theta_\nu}\left\{\frac{-1+(m+1)\cos m(\theta_\nu+\theta_n) - m\cos(m+1)(\theta_\nu+\theta_n)}{2[1-\cos(\theta_\nu+\theta_n)]}\right.$$
$$\left.-\frac{-1+(m+1)\cos m(\theta_\nu-\theta_n) - m\cos(m+1)(\theta_\nu-\theta_n)}{2[1-\cos(\theta_\nu-\theta_n)]}\right\}.$$

Since

$$\cos(m+1)(\theta_\nu\pm\theta_n) = (-1)^{\nu-n}$$

$$\cos m(\theta_\nu\pm\theta_n) = (-1)^{\nu-n}\cos(\theta_\nu\pm\theta_n)$$

and

$$[1-\cos(\theta_n-\theta_\nu)][1-\cos(\theta_n+\theta_\nu)] = (\cos\theta_\nu - \cos\theta_n)^2$$

this reduces to two different results for $|\nu - n|$ odd or even, namely:

when $\nu - n = \pm 1, \pm 3, \pm 5 \ldots.$

$$b_{\nu n} = \frac{1}{2(m+1)\sin\theta_\nu}\left\{\frac{(m-1)-(m+1)\cos(\theta_\nu+\theta_n)}{2[1-\cos(\theta_\nu+\theta_n)]} - \frac{(m-1)-(m+1)\cos(\theta_\nu-\theta_n)}{2[1-\cos(\theta_\nu-\theta_n)]}\right\}$$

$$= \frac{1}{2(m+1)\sin\theta_\nu}\cdot\frac{-[1+\cos(\theta_\nu+\theta_n)][1-\cos(\theta_\nu-\theta_n)] + [1+\cos(\theta_\nu-\theta_n)][1-\cos(\theta_\nu+\theta_n)]}{2(\cos\theta_\nu-\cos\theta_n)^2}$$

$$= \frac{1}{2(m+1)\sin\theta_\nu}\cdot\frac{[\cos(\theta_\nu-\theta_n) - \cos(\theta_\nu+\theta_n)]}{(\cos\theta_\nu-\cos\theta_n)^2} = \frac{\sin\theta_n}{(m+1)(\cos\theta_\nu-\cos\theta_n)^2}$$

and when $\nu - n = \pm 2, \pm 4, \pm 6 \ldots.$

$$b_{\nu n} = 0\,.$$

A somewhat different approach which leads to the same results goes as follows: we split the influence function $i(\eta,\eta')$ up into a constant value $i(\eta,\eta)$ valid for the pivotal station itself and a residue

$$i(\eta,\eta') = i(\eta,\eta) + i^*(\eta,\eta') = i(\eta,\eta) + [i(\eta,\eta') - i(\eta,\eta)]\,.$$

The downwash integral consists then of two parts

$$\alpha(\eta) = -\frac{i(\eta,\eta)}{2\pi}\int_{-1}^{1}\frac{\gamma(\eta')\,d\eta'}{(\eta-\eta')^2} - \frac{1}{2\pi}\int_{-1}^{1}\frac{\gamma i^*(\eta,\eta')\,d\eta'}{(\eta-\eta')^2}$$

of which the first fits exactly into the approximate integration formula developed above; apart from the factor $i(\eta,\eta)$ it is identical with the ' induced angle of attack ' of the lifting-line theory. As to the second part we see at once that

$$f(\eta,\eta') = \frac{\gamma\,.\,i^*}{\eta'-\eta}$$

is a continuous function even at $\eta' = \eta$. Towards the wing tips ($\eta' = 1$) it behaves like γ; we may, therefore, try to represent it also by interpolation functions:

$$f = \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n g_n(\theta')\,.$$

The second part of the downwash integral is then for the station η'

$$\varDelta\alpha_\nu = \frac{1}{2\pi}\sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n \int_0^\pi \frac{g_n(\theta')\sin\theta'\,d\theta'}{\cos\theta_\nu - \cos\theta'}\,.$$

From Glauert's formula we derive

$$\frac{1}{\pi}\int_0^\pi\frac{\sin\lambda\theta'\,\sin\theta'\,d\theta'}{\cos\theta_\nu-\cos\theta'} = \frac{\sin(\lambda+1)\theta_\nu - \sin(\lambda-1)\theta_\nu}{2\sin\theta_\nu} = \cos\lambda\theta_\nu$$

which gives

$$\varDelta\alpha_\nu = \frac{1}{m+1}\sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} f_n \sum_{\lambda=1}^{m}\sin\lambda\theta_n\cos\lambda\theta_\nu$$

$$= \frac{1}{m+1}\Sigma f_n \sum_{\lambda=1}^{m}\frac{[\sin\lambda(\theta\ +\theta_\nu) + \sin\lambda(\theta\ -\theta_\nu)]}{2}\,.$$

Now we have

$$2(1-\cos x)\sum_1^m \sin\lambda x = \sum_1^m[-\sin(\lambda-1)x + 2\sin\lambda x - \sin(\lambda+1)x]\,.$$

Of this lengthly sum there remain only a few terms after rearranging so that the lines of equal angles are put together:

$$2(1-\cos x)\sum_1^m\sin\lambda x = \sin x + \sin mx - \sin(m+1)x\,.$$

This gives:

$$\sum_1^m\sin\lambda\theta_n\,\cos\lambda\theta_\nu = \frac{\sin(\theta_n+\theta_\nu) + \sin m(\theta_n+\theta_\nu) - \sin(m+1)(\theta_n+\theta_\nu)}{4[1-\cos(\theta_n+\theta_\nu)]}$$

$$+\frac{\sin(\theta_n-\theta_\nu) + \sin m(\theta_n-\theta_\nu) - \sin(m+1)(\theta_n-\theta_\nu)}{4[1-\cos(\theta_n-\theta_\nu)]}\,.$$

Since

$$\sin (m+1)(\theta_n \pm \theta_\nu) = 0$$

$$\sin m(\theta_n \pm \theta_\nu) = (-1)^{n-\nu+1} \sin (\theta_n \pm \theta_\nu)$$

we obtain

$$\sum_1^m \sin \lambda\theta_n \cos \lambda\theta_\nu = \frac{1-(-1)^{n-\nu}}{4} \left\{ \frac{\sin (\theta_n + \theta_\nu)}{1-\cos (\theta_n + \theta_\nu)} + \frac{\sin (\theta_n - \theta_\nu)}{1-\cos (\theta_n - \theta_\nu)} \right\}$$

$$= \frac{1-(-1)^{n-\nu}}{4} \frac{2 \sin \theta_n \cos \theta_\nu - \sin 2\theta_n}{(\cos \theta_n - \cos \theta_\nu)^2}$$

$$= \frac{1-(-1)^{n-\nu}}{2} \frac{\sin \theta_n}{\cos \theta_\nu - \cos \theta_n} .$$

Thus we find:

$$\Delta\alpha_\nu = \frac{1}{m+1} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} \frac{f_n \sin \theta_n}{\cos \theta_\nu - \cos \theta_n} \qquad n = \nu \pm 1, 3, 5 \ldots \text{only.}$$

If we now remember that

$$f_n = \gamma_n \frac{i_{\nu n} - i_{\nu\nu}}{\cos \theta_n - \cos \theta_\nu}$$

we obtain finally:

$$\alpha_\nu = i_{\nu\nu} \left[b_{\nu\nu}\gamma_\nu - \Sigma' \, b_{\nu n}\gamma_n \right] - \frac{1}{m+1} \Sigma' \frac{\gamma_n(i_{\nu n} - i_{\nu\nu}) \sin \theta_n}{(\cos \theta_n - \cos \theta_\nu)^2}$$

$$= b_{\nu\nu} i_{\nu\nu} \gamma_\nu - \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, b_{\nu n} i_{\nu n} \gamma_n$$

as before, although the original assumptions are different.

APPENDIX V

An Alternative Method for the Spanwise Part of the Downwash Integral

As stated before the approximate method used for the spanwise part of the downwash integral is not the only possible one but appeared more convenient than any other tried by the author. The only objection that might be brought against it on theoretical grounds concerns the over-employment of the interpolation polynominals g_n. A linear combination of m independent interpolation polynominals has, of course, m degrees of freedom. If we represent γ by an interpolation formula

$$\gamma = \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} g_n \gamma_n$$

we have used them up so to say; if we demand that also for any point ν the product γi_ν be represented in the same way

$$\gamma i_\nu = \sum_{-\frac{m-1}{2}}^{\frac{m=1}{2}} g_n \gamma_n i_{\nu n}$$

we thus fix i_ν as

$$i_\nu = \frac{\Sigma g_n \gamma_n i_{\nu n}}{\Sigma g_n \gamma_n}$$

which gives $i_\nu(\eta_n) = i_{\nu n}$ as it should be but relates i_ν in the intervals between these stations to the γ_n. This is rather against the principles of interpolation but since we are not concerned in any case about the i-values in these intervals it is difficult to see why the representation used in the report should be less accurate than any other.

Every attempt to represent the influence functions independently from the γ-values by interpolation functions leads to something quite similar to the Weissinger[7] procedure: the coefficients of the system of equations must be found by another approximate integration. This may not be deterrent with only a few pivotal stations although even then the work involved is more than with our scheme but it is definitely prohibitive at a greater number of pivotal points since the computing effort increases roughly with the third power of that number instead of the second.

After many trials at least one method has been found which keeps some more degrees of freedom open for the representation of the influence function; it may, therefore, be that the downwash integral is slightly more accurate with the same number of pivotal points. This method goes as follows:

We split the influence function $i(\eta_\nu, \eta)$ up into three parts so that the first two give the tangent of $i(\eta)$ at $\eta = \eta_\nu$;

$$i(\eta_\nu, \eta') = i_{\nu\nu} + (\eta' - \eta_\nu) \frac{di_\nu}{d\eta}(\eta_\nu) + \Delta i_\nu(\eta') \, .$$

Accordingly the downwash integral consists of three components:

$$\alpha_\nu(\eta) = \frac{-i_{\nu\nu}}{2\pi} \int_{-1}^{1} \frac{\gamma(\eta')\, d\eta'}{(\eta_\nu - \eta')^2} + \frac{\dfrac{di_\nu}{d\eta}^{\eta_\nu}}{2\pi} \int_{-1}^{1} \frac{\gamma\, d\eta'}{\eta - \eta'} - \frac{1}{2\pi} \int_{-1}^{1} \frac{\gamma(\eta')\, \Delta i_\nu(\eta')\, d\eta'}{(\eta - \eta')^2} \, .$$

The first two integrals fit perfectly into the integration methods developed before so that we have

$$\alpha_\nu = b_{\nu\nu} i_{\nu\nu} \gamma_\nu - \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \, b_{\nu n} \gamma_n \left[i_{\nu\nu} + \frac{di_\nu}{d\eta} (\eta_n - \eta_\nu) \right] - \frac{1}{2\pi} \int_{-1}^{1} \frac{\gamma\, \Delta i_\nu}{(\eta_\nu - \eta')^2}\, d\eta' \, .$$

As to the third integral if we firstly ignore the logarithmic singularity of $\Delta i_\nu/(\eta_\nu - \eta')^2$ we may consider its integrand

$$\frac{\gamma(\eta')\,\Delta i_\nu(\eta')}{(\eta_\nu - \eta')^2}$$

as continuous throughout the integration range. For this case we have already an approximate integration formula; by using our interpolation functions g_n we may write

$$\frac{\gamma\,\Delta i_\nu}{(\eta_\nu - \eta')^2} = \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}} g_n(\theta')\,\frac{\gamma_n\,\Delta i_{\nu n}}{(\eta_\nu - \eta_n)^2}\,.$$

The integral over this function is then

$$-\frac{1}{2\pi}\int_{-1}^{1}\frac{\gamma\,\Delta i_\nu}{(\eta_\nu - \eta')^2}\,d\eta' = \frac{-1}{2(m+1)}\sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}\frac{\gamma_n\,\Delta i_{\nu n}\sin\theta_n}{(\cos\theta_\nu - \cos\theta_n)^2}$$

For $n = \nu$ this makes no sense; we ought to put here instead of

$$\frac{\gamma_\nu\,\Delta i_{\nu\nu}}{(\cos\theta_\nu - \cos\theta_\nu)^2}$$

$$\frac{\gamma_\nu}{2}\frac{d^2 i_\nu}{d\eta^2}(\eta_\nu)$$

if this quantity exists. The advantage of this approximate integration formula consists mainly in the fact that as it is shown in the preceding Appendix IV it still holds if we have

$$\frac{\gamma\,\Delta i_\nu}{(\eta_\nu - \eta)^2} = \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}\frac{\gamma_n\,\Delta i_{\nu n}}{(\eta_\nu - \eta_n)^2}\,g_n(\theta) + \sin(m+1)\theta\sum_{0}^{m-1} a_q\cos q\theta$$

where the a_q represent the m additional degrees of freedom which can be used, *e.g.*, to make the Δi_ν function really independent from the γ_n.

As to the logarithmic singularity of $\Delta i_\nu/(\eta_\nu - \eta)^2$ at $\eta \to \eta_\nu$ we may use another trick to get rid of this difficulty. Since the integration method applied is just a slight modification of Euler's formula by which the integral over a function known at the two terminal points of an interval is given for this interval approximately by half the interval width (here represented by the factor $\sin\theta_n$) multiplied by the sum of the two border values. If we consider the integrand in the interval $\eta_\nu < \eta < \eta_{\nu+1}$ it may be given by

$$\frac{\gamma\,\Delta i_\nu}{(\eta_\nu - \eta')^2} \simeq a + b\ln\frac{\eta' - \eta_\nu}{\eta_{\nu+1} - \eta^\nu}$$

the integral gives

$$\int_{\eta\nu}^{\eta\nu+1}\left(a + b\ln\frac{\eta' - \eta_\nu}{\eta_{\nu+1} - \eta_\nu}\right)d\eta' = (\eta_{\nu+1} - \eta_\nu)(a - b)\,.$$

We may now seek an ersatz station $\bar\eta_\nu$ so that the integral is again given by half the interval width multiplied by the sum of the integrands at the station $\eta_{\nu+1}$ and this ersatz station; this gives:

$$(\eta_{\nu+1} - \eta_\nu)(a - b) = \frac{\eta_{\nu+1} - \eta_\nu}{2}\left[2a + b\ln\frac{\bar\eta_\nu - \eta_\nu}{\eta_{\nu+1} - \eta_\nu}\right]$$

which is valid for

$$\ln\frac{\bar\eta_\nu - \eta_\nu}{\eta_{\nu+1} - \eta_\nu} = -2$$

or

$$\bar\eta_\nu = \eta_\nu + (\eta_{\nu+1} - \eta_\nu)\,.\,\mathrm{e}^{-2} = \eta_\nu + 0\cdot 1353(\eta_{\nu+1} - \eta_\nu)\,.$$

A similar formula may be derived for the interval $\eta_{\nu-1} < \eta < \eta_\nu$.

Now putting the bits together, we have, if we introduce the values for the $b_{\nu n}$ and $b_{\nu\nu}$ of equation (72)

$$\alpha_\nu = \gamma_\nu \left\{ \frac{m+1}{4 \sin \theta_\nu} i_{\nu\nu} - \frac{e^4}{4} \sin \theta_\nu \left[\frac{i_\nu[\eta_\nu + e^{-2}(\eta_{\nu+1} - \eta_\nu)]}{(\eta_{\nu+1} - \eta_\nu)^2} + \frac{i_\nu[\eta_\nu - e^{-2}(\eta_\nu - \eta_{\nu-1})]}{(\eta_\nu - \eta_{\nu-1})^2} \right] \right\}$$

$$- \frac{1}{2(m+1)} \sum_{-\frac{m-1}{2}}^{\frac{m-1}{2}}{}' \frac{\gamma_n \sin \theta_n}{(\eta_n - \eta_\nu)^2} \left[i_{\nu n} - (-1)^{n-\nu} \left(i_{\nu\nu} + (\eta_n - \eta_\nu) \frac{di_\nu}{d\eta} (\eta_\nu) \right) \right] .$$

For the practical application of this formula we need a plotting of $\Delta i/Y^2$ for the neighbourhood of the pivotal points; $di_\nu/d\eta$ may be calculated from the di/dX value at the pivotal station and the amount of sweep of the line connecting the pivotal points.

A practical disadvantage of this formula for α_ν compared with our equation (83) is the appearance of all stations η_n in the sum although those for odd $n - \nu$ are obviously predominant. This means at least twice the work for computing the coefficients of the system of equations. Moreover a solution of these equations might not always be achieved by iteration in which case this part of the calculus becomes rather laborious for more than just a few pivotal points.

If we compare the different methods not on the basis of the same number of pivotal points but as to the computing effort required we may say that with the same amount of work we can have at least 50 per cent more pivotal points with the original method. If is then very doubtful whether any improvement in accuracy can be gained by the alternative method described here.

APPENDIX VI

The Rounding-off Rule for the Middle of Swept Wings

To see how a function $f(\eta)$ with a kink in the middle ($\eta = 0$) is represented by our interpolation polynominals we consider as a typical example

$$f(\eta) = 1 - |\eta| = 1 - |\cos\theta|.$$

Its representation by our interpolation polynominals is

$$F(\theta) = g_0(\theta) + \sum_{1}^{\frac{m-1}{2}} (1 - \cos\theta_n)\,[g_n(\theta) + g_{-n}(\theta)]\,.$$

F meets f at the m stations

$$\eta_n = \cos\theta_n = \sin\frac{n\pi}{m+1}\,, \qquad n = 0, \pm 1, \pm 2 \ldots \pm \frac{m-1}{2}\,.$$

The point which is open to some criticism is whether it is right to lay the interpolation $F(\theta)$ through that rather exposed peak $F = 1$ at $\eta = 0$. The interpolated function $F(\eta)$ is continuous at $\eta = 0$ also in its derivatives it has at $\eta = 0$ a flat maximum instead of a kink. This means that $F(\eta)$ will definitely exceed f in the interval $-\eta_1 < \eta < \eta_1$. We may therefore expect a better general coincidence between $f(\eta)$ and an interpolated curve if we drop the condition that both meet at $\eta = 0$; *i.e.*, we suggest an interpolation function F_1

$$F_1(\theta) = (1 - \Delta)g_0(\theta) + \sum_{1}^{\frac{m-1}{2}} (1 - \cos\theta_n)(g_n + g_{-n})\,.$$

Instead of the condition of coincidence at $\eta = 0$ we need now another condition which defines this correction Δ, the bit clipped off the kink. Usually we would introduce a least-square condition but in this case we may get away with something simpler because in the discrepancy between the given function f and the interpolation F_1 the interval $-\eta_1 < \eta < \eta_1$ dominates. Thus the condition

$$\int_{-1}^{1} F_1(\eta)\,d\eta = \int_{-1}^{1} f(\eta)\,d\eta$$

should be enough to determine the correction Δ. Applying the integration formula for the g_n we have with $\int_{-1}^{1} f(\eta)\,d\eta = 1$:

$$\frac{\pi}{m+1}(1 - \Delta) + \frac{2\pi}{m+1}\sum_{1}^{\frac{m-1}{2}} (1 - \cos\theta_n)\sin\theta_n = 1$$

or

$$\Delta = 1 + 2\sum_{n=1}^{\frac{m-1}{2}}\left(1 - \sin\frac{n\pi}{m+1}\right)\cos\frac{n\pi}{m+1} - \frac{m+1}{\pi}\,.$$

To evaluate the sums, let us consider

$$\begin{aligned} 2(1 - \cos x)\sum_{1}^{\frac{m-1}{2}} \cos nx &= \sum_{1}^{\frac{m-1}{2}} [-\cos(n-1)x + 2\cos nx - \cos(n+1)x] \\ &= -1 + 2\cos x - \cos 2x \\ &\qquad - \cos x + 2\cos 2x - \cos 3x \\ &\qquad\qquad - \cos 2x + \cos 3x - \cos 4x \\ &\qquad\qquad\qquad \text{etc.} \ldots . \end{aligned}$$

Most of these terms cancel each other so that there only remains

$$-1+\cos x+\cos\frac{m-1}{2}x-\cos\frac{m+1}{2}x\,.$$

With $\quad x=\dfrac{\pi}{m+1}$ we have thus

$$2\sum_{1}^{\frac{m-1}{2}}\cos\frac{n\pi}{m+1}=\frac{-1+\cos\dfrac{\pi}{m+1}+\sin\dfrac{\pi}{m+1}}{1-\cos\dfrac{\pi}{m+1}}=\cot\frac{\pi}{2(m+1)}-1=\frac{1+\cos\dfrac{\pi}{m+1}}{\sin\dfrac{\pi}{m+1}}-1\,.$$

Similarly we find

$$2\sum_{n=1}^{\frac{m-1}{2}}\sin\frac{n\pi}{m+1}\cos\frac{n\pi}{m+1}=\sum_{n=1}^{\frac{m-1}{2}}\sin\frac{2n\pi}{m+1}$$

if we consider once more

$$2(1-\cos x)\sum_{1}^{\frac{m-1}{2}}\sin nx=\sum_{1}^{\frac{m-1}{2}}[-\sin(n-1)x+2\sin nx-\sin(n+1)x]$$

most of the terms of this sum cancel each other so that there only remains

$$\sin x+\sin\frac{m-1}{2}x-\sin\frac{m+1}{2}x\,.$$

This gives for $x=\dfrac{2\pi}{m+1}$:

$$\sum_{1}^{\frac{m-1}{2}}\sin\frac{2n\pi}{m+1}=\frac{\sin\dfrac{2\pi}{m+1}}{1-\cos\dfrac{2\pi}{m+1}}=\cot\frac{\pi}{m+1}\,.$$

Summarising we find the correction

$$\Delta=1+\frac{1+\cos\dfrac{\pi}{m+1}}{\sin\dfrac{\pi}{m+1}}-1-\frac{\cos\dfrac{\pi}{m+1}}{\sin\dfrac{\pi}{m+1}}-\frac{m+1}{\pi}$$

$$=\frac{1}{\sin\dfrac{\pi}{m+1}}-\frac{m+1}{\pi}\simeq\frac{1}{6}\cdot\frac{\pi}{m+1}+\frac{7}{360}\left(\frac{\pi}{m+1}\right)^3+\frac{31}{15120}\left(\frac{\pi}{m+1}\right)^5+\ldots$$

It is reasonable to relate this correction to the difference of the f_0 and f_1 values which is $\sin\pi/(m+1)$ because a multiplication of η by any factor does not affect this ratio. We thus get

$$\frac{\Delta}{f_0-f_1}=\frac{\Delta}{\sin\dfrac{\pi}{m+1}}=\frac{1}{6}+\frac{7}{360}\left(\frac{\pi}{m+1}\right)^2+\ldots\simeq\frac{1}{6}$$

and in this form the result applies to a kink of any angle.

In all practical cases we need only use the first term because with too few stations we cannot speak any longer of a decent representation of a kinked curve; since Δ is only a correction it does not pay to have it more accurate than the assumptions on which it is based.

To illustrate the effect of this rounding off we have plotted in Fig. 7 the function $1-|\eta|$ with the two interpolating curves one with and the other without the 1/6-correction. The improvement is quite obvious.

APPENDIX VII

Instruction for Practical Calculations

(This is meant to be a short instruction for those who want to calculate the load distribution for a given wing without bothering about the theoretical background.)

Preliminaries.—A geometrical description of the wing to be calculated must be available. As a rule a number of load cases are to be calculated for one wing according to airworthiness requirements, etc. A load case is determined by a specific arrangement of the local incidences of the wing. The aerodynamic forces are represented by the non-dimensional lift and pitching-moment values at certain spanwise stations

$$\gamma_n = \frac{1}{\rho U^2 b}\frac{dL}{dy}(y_n) , \qquad \mu_n = \frac{1}{\rho U^2 b c(y_n)}\frac{dM}{dy}(y_n)$$

these stations being:

$$y_n = \frac{b}{2}\eta_n = \frac{b}{2}\sin\frac{n\pi}{m+1} , \qquad n = -\frac{m-1}{2} , -\frac{m-3}{2} , \ldots -1, 0, 1 \ldots \frac{m-1}{2} .$$

See Tables 1 to 7.

The γ_n (and μ_n) are calculated from a linear system of equations the coefficients of which are to be computed before, and depend only on, wing plan form and Mach number, whilst the terms independent of γ_n in these equations are also dependent on the distribution of the angles of incidence. Accordingly, the calculation splits up into two processes : the computation of the coefficients and the solution of the system of equations.

Before beginning the calculation we must make up our mind about the number of pivotal points (points at which the integral equation of the lifting surface is satisfied). As to the number of spanwise stations m a reasonable choice is

$$m > 3A\sqrt{(1 - M^2)} .$$

In Tables 1 to 7 some essential constants for $m = 3$, 5, 7, 11, 15, 23 and 31 are collected. Since the work to be done is roughly proportional to the square of the number of pivotal points each higher number represents about twice the work of the preceding one.

One chordwise pivotal point for each spanwise station is sufficient for not too small aspect ratio wings of regular shape or if only the spanwise lift distribution is needed. Wings of low aspect ratio ($A\sqrt{(1 - M^2)} < 3$) or irregular shape as sweepback, cut-outs, etc., need two chordwise pivotal points at each spanwise station. Since there is much more work involved in this case we will deal with both cases separately. As an illustration of these methods compare the examples on Tables 8 to 30.

Computation of coefficients with one chordwise pivotal point (*compare Tables* 8–10).—Firstly we tabulate the quantities needed in the course of this calculation which depend only on one spanwise station, namely:

(1) The number of the spanwise station; for symmetrical wing plan forms we need only $\nu = 0, 1, 2 \ldots (m - 1)/2$, the values of all quantities listed here being either identical or different only in their signs for negative ν's.

(2) The non-dimensional η_ν-co-ordinate from Tables 1 to 7.

(3) The actual spanwise co-ordinate $y_\nu = (b/2)\eta_\nu$.

(4) The chordwise co-ordinate $x_{\nu\,l}$ of the leading edge of the wing section at y_ν. The origin from which it is measured can be arbitrarily chosen. At the middle section of sharply kinked swept wings we take instead of the geometrical co-ordinate $x_{0\,l\,\mathrm{geom}}$

$$x_{0\,l} = \tfrac{5}{6}\,x_{0\,l\,\mathrm{geom}} + \tfrac{1}{6}\,x_{1\,l}.$$

(5) The local wing chord c_ν; again for the middle section of swept wings we take

$$c_0 = \tfrac{5}{6}\,c_{0\,\mathrm{geom}} + \tfrac{1}{6}\,c_1\,.$$

(6) The x-co-ordinate of the pivotal station at $0{\cdot}75c$:

$$x_\nu = x_{\nu\,l} + 0{\cdot}75c_\nu$$

(7) The reciprocal of the non-dimensional chord $b/2c_\nu$.

(8) The correction factors $\dfrac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1}\cos\dfrac{\nu\pi}{m+1}\left(\dfrac{b}{2c_\nu}\right)^2$. The term $\dfrac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1}\cos\dfrac{\nu\pi}{m+1}$ is tabulated in Tables 1 to 7. If we calculate for a Mach number other than zero, these factors are still to be multiplied by $(1 - M^2)$, thus being

$$\frac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1}\cos\frac{\nu\pi}{m+1}\left(\frac{b}{2c_\nu}\right)^2(1 - M^2)\,.$$

(9) The induction factors

$$\bar{i}_{\nu\nu} = 1{\cdot}8847 + 0{\cdot}510_3\,\frac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1}\cos\frac{\nu\pi}{m+1}\left(\frac{b}{2c_\nu}\right)^2(1 - M^2)$$

(10) The coefficients

$$a_\nu = \frac{a_{\nu\nu}}{\bar{i}_{\nu\nu}}$$

with $a_{\nu\nu}$ from Tables 1 to 7.

Next we collect the data which depend on two stations, namely, the inducing section (number n) and the pivotal point (number ν). ν is an even number when n is odd and *vice versa*. We provide a table for every n in which we tabulate:

(11) The number ν of the pivotal stations required, *i.e.*, odd ν for even n and *vice versa*, ν both positive and negative.

(12) The spanwise distances $|\eta_\nu - \eta_n|$, absolute values readily obtained from Tables 1 to 7.

(13) The chordwise distances $(x_\nu - x_{nl})$, the x_ν and x_{nl} being tabulated under (6) and (4).

(14) Non-dimensional spanwise distances $Y_{\nu n} = b/2c_n\,.\,|\eta_\nu - \eta_n|$ from (12) and (7); at Mach numbers other than zero we have to multiply this by $\sqrt{(1 - M^2)}$:

$$Y_{\nu n} = \sqrt{(1 - M^2)}\,\frac{b}{2c_n}\,|\eta_\nu - \eta_n|.$$

(15) Non-dimensional chordwise distances

$$X_{\nu n} = \frac{x_\nu - x_{nl}}{c_n}$$

from (13) and (5).

(16) The influence factors $i_{\nu n}$ taken from Figs. 1, 2 and 3 with the $Y_{\nu n}$ and $X_{\nu n}$ (15) and (16) as entries.

(17) The terms

$$\frac{a_{\nu n}\,.\,i_{\nu n}}{\bar{i}_{\nu\nu}}$$

with the $a_{\nu n}$ from Tables 1 to 7 and the $\bar{i}_{\nu\nu}$ from (9).

(18) The coefficients

$$A_{\nu n} = \frac{a_{\nu n}\dot{i}_{\nu n}}{\bar{i}_{\nu\nu}} + \frac{a_{\nu,-n}\dot{i}_{\nu,-n}}{\bar{i}_{\nu\nu}} \qquad n \neq 0 \text{ for symmetrical load cases}$$

$$A_{\nu 0} = \frac{a_{\nu 0}\dot{i}_{\nu 0}}{\bar{i}_{\nu\nu}}$$

$$A_{\nu n} = \frac{a_{\nu n}\dot{i}_{\nu n}}{\bar{i}_{\nu\nu}} - \frac{a_{\nu,-n}\dot{i}_{\nu,-n}}{\bar{i}_{\nu\nu}} \text{ for anti-symmetrical load cases.}$$

These coefficients may be collected in a new form the values for constant ν in columns those for constant n in horizontal lines to suit the solution of the system of equations. Prepare one form for odd ν and one for even ν.

Solution of the system of equations with one chordwise pivotal point (*compare Tables* 11 *and* 12).—(19) For a given load case we collect first the α_ν values, *i.e.*, the local incidences at 0·75c of the considered spanwise station n_ν. For wings with flaps we take

$$\alpha_\nu = \alpha_{\nu \text{ without flaps}} + \frac{d\alpha}{d\delta}\,\delta_\nu$$

δ_ν being the flap deflection and $d\alpha/d\delta = (dC_L/d\delta)/(dC_L/d\alpha)$. If η_ν is the station next to the beginning of the flap $\eta^* = \cos\theta^*$ we take as a more reasonable value

$$\delta_\nu = \delta^* \left[\frac{1}{2} + \frac{m+1}{\pi}(\theta^* - \theta_n)\right]$$

δ^* being the flap deflection at $\eta^* + 0$. If the flap does not extend to the wing tips we use a similar fairing at the station nearest to the other end of the flap and having $\delta^* = \delta(\eta^* - 0)$

$$\delta_\nu = \delta^* \left[\frac{1}{2} + \frac{m-1}{\pi}(\theta_\nu - \theta^*)\right].$$

(20) Next we estimate very roughly what the γ_n with an even suffix might be. It will not matter very much whether this guess is fairly good or not. Values can be taken, *e.g.*, from a previous calculation with fewer pivotal points or just estimated so as to produce roughly the C_L-value to be expected. These guessed γ_n are marked by the annexed number 0 in square brackets*

$$\gamma_0^{[0]},\ \gamma_2^{[0]},\ \gamma_4^{[0]},\ \text{etc.}$$

The first approximation $\gamma_\nu^{[1]}$ of the $\gamma_1, \gamma_3, \gamma_5 \ldots$ is to be found from:

$$\gamma_\nu^{[1]} = a_\nu\alpha_\nu + \sum_0^{\frac{m-1}{2}}{}' A_{\nu n}\gamma_n^{[0]}, \qquad \nu = 1, 3, 5 \ldots .$$

To work this out we prepare $(m-1)/2$ columns headed by the numbers 1, 3, 5 . . . Here we fill in line after line:

(21) The products $a_\nu \,.\, \alpha_\nu$ with the a_ν from (10).

(22) The products $A_{\nu n}\gamma_n^{[0]}$ with the $A_{\nu n}$ from (18); *i.e.*, we multiply $\gamma_0^{[0]}$ by $A_{1,0}, A_{3,0}, A_{5,0}$, etc. then $\gamma_2^{[0]}$ by $A_{1,2}, A_{3,2}, A_{5,2}$, etc., etc. Slide-rule accuracy is usually sufficient.

(23) By adding up these columns we obtain the $\gamma_\nu^{[1]}$.

Now we calculate the first approximation $\gamma_\nu^{[1]}$ of the $\gamma_0, \gamma_2, \gamma_4 \ldots$ which are found from:

$$\gamma_\nu^{[1]} = a_\nu\alpha_\nu + \sum_0^{\frac{m-1}{2}}{}' A_{\nu n}\gamma_n^{[1]}, \qquad \nu = 0, 2, 4 \ldots .$$

* In the tables shown as circles.

This is done in quite the same way as for the γ_ν with odd suffixes, *i.e.*, in columns headed by the number $\nu = 0, 2, 4 \ldots$ we enter

(24) The products $a_\nu \alpha_\nu$.

(25) The products $A_{\nu n} \gamma_n^{[1]}$ in the way described before.

(26) Adding up these columns we obtain the $\gamma_0^{[1]}, \gamma_2^{[1]}, \gamma_4^{[1]} \ldots$

(27) We compute the first differences

$$\Delta^{[1]} \gamma_\nu = \gamma_\nu^{[1]} - \gamma_\nu^{[0]}, \qquad \nu = 0, 2, 4 \ldots.$$

We calculate further corrections alternatively for the γ with odd suffixes and with even suffixes according to the formulae:

$$\Delta^{[r]} \gamma_\nu = \sum_0^{\frac{m-3}{1}}{}' A_{\nu n} \Delta^{[r-1]} \gamma_n, \qquad \begin{array}{l} \nu = 1, 3, 5 \ldots \\ n = 0, 2, 4 \ldots \end{array}$$

and

$$\Delta^{[r]} \gamma_\nu = \sum_1^{\frac{m-1}{2}}{}' A_{\nu n} \Delta^{[r]} \gamma_n, \qquad \begin{array}{l} \nu = 0, 2, 4, 6 \ldots \\ n = 1, 3, 5 \ldots \end{array}$$

Begin by filling into columns headed by the numbers $\nu = 1, 3, 5 \ldots$ the products

$$A_{1,0} \Delta^{[1]} \gamma_0, \quad A_{3,0} \Delta^{[1]} \gamma_0, \quad A_{5,0} \Delta^{[1]} \gamma_0 \ldots$$

in the next line

$$A_{1,2} \Delta^{[1]} \gamma_2, \quad A_{3,2} \Delta^{[1]} \gamma_2, \quad A_{5,2} \Delta^{[1]} \gamma_2 \ldots$$

and so on; the sums of these columns give $\Delta^{[2]} \gamma_1$, $\Delta^{[2]} \gamma_3$, $\Delta^{[2]} \gamma_5$, which are used to fill into columns headed by $\nu = 0, 2, 4$.

$$A_{0,1} \Delta^{[2]} \gamma_1, \quad A_{2,1} \Delta^{[2]} \gamma_1, \quad A_{4,1} \Delta^{[2]} \gamma_1 \ldots$$

in the next line

$$A_{0,3} \Delta^{[2]} \gamma_3, \quad A_{2,3} \Delta^{[2]} \gamma_3, \quad A_{4,3} \Delta^{[2]} \gamma_3 \ldots \text{ etc.}$$

to sum up for

$$\Delta^{[2]} \gamma_0, \quad \Delta^{[2]} \gamma_2, \quad \Delta^{[2]} \gamma_4 \ldots.$$

This process is continued until the $\Delta\gamma$ are small enough to be ignored.

If we break off earlier we roughly assess the rest omitted after the r-th difference $\Delta^{[r]} \gamma_\nu$ as

$$\frac{(\Delta^{[r]} \gamma_\nu)^2}{\Delta^{[r-1]} \gamma_\nu - \Delta^{[r]} \gamma_\nu}.$$

The γ_ν solving the system of equations are eventually

$$\gamma_\nu = \gamma_\nu^{[1]} + \Delta^{[2]} \gamma_\nu + \Delta^{[3]} \gamma_\nu + \ldots + \Delta^{[r]} \gamma_\nu + \frac{(\Delta^{[r]} \gamma_\nu)^2}{\Delta^{[r-1]} \gamma_\nu - \Delta^{[r]} \gamma_\nu}.$$

A check of these results is always strongly recommended. To do this we do as described under (21) to (26) but with the final γ-values. Within the accuracy tolerances of the whole calculus we must obtain the final $\gamma_1, \gamma_3, \gamma_5$ by inserting the final $\gamma_0, \gamma_2, \gamma_4 \ldots$ in the respective products and *vice versa*. With appreciable discrepancies the differences calculation must be repeated.

Resulting forces are computed with the formulae from section 7. The procedure is obvious, and coefficients are to be found in Tables 1 to 7.

Computation of coefficients with two chordwise pivotal points (*compare Tables* 13 *to* 17).—Again we begin by tabulating those quantities which depend on one spanwise station only. Some of them are the same for one and two pivotal stations; they are repeated here to avoid a muddle.

(1) The number of the spanwise station ν or n; for symmetrical wings we need only $\nu = 0, 1, 2 \ldots (m-1)/2$.

(2) The non-dimensional η_ν co-ordinate from Tables 1 to 7.

(3) The actual spanwise co-ordinate $y_\nu = (b/2)\eta_\nu$.

(4) The x-co-ordinate $x_{\nu\, l}$ of the leading edge of the wing section at y_ν; the origin from which it is measured can be chosen as it is most convenient. At the middle section of sharply kinked swept wings we take instead of the geometrical co-ordinate $x_{0\, l\, \mathrm{geom}}$ the modified value

$$x_{0\, l} = \tfrac{5}{6} x_{0\, \mathrm{geom}} + \tfrac{1}{6} x_{1\, l}\, .$$

(5) The local wing chord c_ν; again for the middle section of swept wings we prefer

$$c_0 = \tfrac{5}{6} c_{0\, \mathrm{geom}} + \tfrac{1}{6} c_1\, .$$

(6) The x-co-ordinate of the rear pivotal point

$$x_\nu' = x_{\nu\, l} + 0 \cdot 9045 c_\nu\, .$$

(7) The x-co-ordinate of the frontal pivotal point

$$x_\nu'' = x_{\nu\, l} + 0 \cdot 3455 c_\nu\, .$$

(8) The non-dimensional reciprocal chord $b/2c_\nu$.

(9) The correction factors

$$\frac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1} \cos \frac{\nu\pi}{m+1} \cdot \left(\frac{b}{2c_\nu}\right)^2 (1 - M^2)\, .$$

The first part of this expression is found in Tables 1 to 7.

(10) The corrected induction factors:

$$\overline{i_{\nu\nu}'} = 1 \cdot 974_2 + 0 \cdot 623_4 \,.\, (9)$$
$$\overline{i_{\nu\nu}''} = 1 \cdot 405_5 + 1 \cdot 009 \,.\, (9)$$
$$\overline{j_{\nu\nu}'} = 0 \cdot 285_9 - 4 \cdot 805 \,.\, (9)$$
$$\overline{j_{\nu\nu}''} = 3 \cdot 170_2 + 5 \cdot 758 \,.\, (9)\, .$$

(11) The determinant $\overline{i_{\nu\nu}'}\, \overline{j_{\nu\nu}''} - \overline{i_{\nu\nu}''}\, \overline{j_{\nu\nu}'}$.

(12) The factors:

$$l_\nu' = \frac{\overline{j_{\nu\nu}''}}{(11)} \qquad m_\nu' = \frac{\overline{i_{\nu\nu}''}}{(11)}$$

$$l_\nu'' = \frac{\overline{j_{\nu\nu}'}}{(11)} \qquad m_\nu'' = \frac{\overline{i_{\nu\nu}'}}{(11)}$$

Next we calculate the coefficients which are dependent on two stations; we prepare a form for each inducing station (marked by the number n) where we tabulate:

(13) The number ν of the pivotal stations required, *i.e.*, odd ν's only for even n and *vice versa*, ν both positive and negative. For every quantity $f_{\nu n}$ collected here $f_{-\nu,-n} = f_{\nu n}$ if the wing planform is symmetrical.

(14) The spanwise distances between pivotal and inducing stations, absolute values only as tabulated in Tables 1 to 7.

$$|\eta_\nu - \eta_n|$$

(15) The chordwise distances between the rear or frontal pivotal points and the leading edge of the inducing wing section

$$x_\nu' - x_{nl}$$
$$x_\nu'' - x_{nl}$$

x_ν', x_ν'' and x_{nl} from (6), (7) and (4).

(16) Spanwise distances measured in chords of the inducing section

$$Y_{\nu n} = \frac{|y_\nu - y_n|}{c_n} \sqrt{(1 - M^2)} = \frac{b}{2c_n} |\eta_\nu - \eta_n| \sqrt{(1 - M^2)}$$

from (8) and (14).

(17) Non-dimensional chordwise distances:

$$X_{\nu n}' = \frac{x_\nu' - x_{nl}}{c_n}$$

$$X_{\nu n}'' = \frac{x_\nu'' - x_{nl}}{c_n}$$

from (15) and (5).

(18) Read from Figs. 1 to 6 with the $Y_{\nu n}$ and $X_{\nu n}$ as entries:
the induction factors:

$$i_{\nu n}' = i(Y_{\nu n}, X_{\nu n}')$$
$$i_{\nu n}'' = i(Y_{\nu n}, X_{\nu n}'')$$
$$j_{\nu n}' = j(Y_{\nu n}, X_{\nu n}')$$
$$j_{\nu n}'' = j(Y_{\nu n}, X_{\nu n}'') \,.$$

(19) Products of these induction factors with the $a_{\nu n}$ from Tables 1 to 7.

$$A_{\nu n} i_{\nu n}'$$
$$a_{\nu n} i_{r n}''$$
$$a_{\nu n} j_{\nu n}'$$
$$a_{\nu n} j_{\nu n}'' \,.$$

(20) With the l_ν and m_ν factors from (12) we calculate the determinants:

$$a_{\nu n}(l_\nu' i_{\nu n}' - l_\nu'' i_{\nu n}'') = l_\nu'(a_{\nu n} i_{\nu n}') - l_\nu''(a_{\nu n} i_{\nu n}'')$$
$$a_{\nu n}(l_\nu' j_{\nu n}' - l_\nu'' j_{\nu n}'') = l_\nu'(a_{\nu n} j_{\nu n}') - l_\nu''(a_{\nu n} j_{\nu n}'')$$
$$a_{\nu n}(m_{\nu n}'' i_{\nu n}'' - m_\nu' i_{\nu n}') = m_\nu''(a_{\nu n} i_{\nu n}'') - m_\nu'(a_{\nu n} i_{\nu n}')$$
$$a_{\nu n}(m_{\nu n}'' j_{\nu n}'' - m_\nu' j_{\nu n}') = m_\nu''(a_{\nu n} j_{\nu n}'') - m_\nu'(a_{\nu n} j_{\nu n}') \,.$$

(21) For symmetrical load cases we calculate the coefficients

$$B_{\nu n} = a_{\nu n}(l_\nu' i_{\nu n}' - l_\nu'' i_{\nu n}'') + a_{\nu, -n}(l_\nu' i_{\nu, -n}' - l_\nu'' i_{\nu, -n}), \qquad \nu \neq 0,\ n \neq 0$$

$$B_{\nu 0} = a_{\nu 0}(l_\nu' i_{\nu 0}' - l_\nu'' i_{\nu 0}'') \qquad B_{0n} = 2a_{0n}(l_0' i_{0n}' - l_0'' i_{0n}'')$$

$$C_{\nu n} = a_{\nu n}(l_\nu' j_{\nu n}' - l_\nu'' j_{\nu n}'') + a_{\nu, -n}(l_\nu' j_{\nu,}'_{-n} - l_\nu' j_{\nu, -n}''), \qquad \nu, n \neq 0$$

$$C_{\nu 0} = a_{\nu 0}(l_\nu' j_{\nu 0}' - l_\nu'' j_{\nu 0}'') \qquad C_{0n} = 2a_{0n}(l_0' j_{0n}' - l_0'' j_{0n}'')$$

$$D_{\nu n} = a_{\nu n}(m_\nu'' i_{\nu n}'' - m_\nu' i_{\nu n}') + a_{\nu, -n}(m_\nu'' i_{\nu, -n}'' - m_\nu' i_{\nu, -n}'), \qquad \nu, n \neq 0$$

$$D_{\nu 0} = a_{\nu 0}(m_\nu'' i_{\nu 0}'' - m_\nu' i_{\nu 0}') \qquad D_{0n} = 2a_{0n}(m_0'' i_{0n}'' - m_0' i_{0n}')$$

$$E_{\nu n} = a_{\nu n}(m_\nu'' j_{\nu n}'' - m_\nu' j_{\nu n}') + a_{\nu, -n}(m_\nu'' j_{\nu, -n}'' - m_\nu' j_{\nu, -n}'), \qquad \nu, n \neq 0$$

$$E_{\nu 0} = a_{\nu 0}(m_\nu'' j_{\nu 0}'' - m_\nu' j_{\nu 0}') \qquad E_{0n} = 2a_{0n}(m_0'' j_{0n}'' - m_0' j_{0n}').$$

We should collect these coefficients in new forms so as best to suit the solution of the system of equations, *i.e.*, the values for $\nu =$ constant in columns, those for $n =$ constant in horizontal lines; coefficients for odd and even ν's should be tabulated separately.

(22) For anti-symmetrical load cases $[a_{-\nu} = - a_\nu]$ the coefficients are written in bold italics:

$$\boldsymbol{B}_{\nu n} = a_{\nu n}(l_\nu' i_{\nu n}' - l_\nu'' i_{\nu n}'') - a_{\nu, -n}(l_\nu' i_{\nu,}'_{-n} - l_\nu'' i_{\nu, -n}'')$$

$$\boldsymbol{C}_{\nu n} = a_{\nu n}(l_\nu' j_{\nu n}' - l_\nu'' j_{\nu n}'') - a_{\nu, -n}(l_\nu' j_{\nu, -n}' - l^{\nu\prime\prime} j_{\nu, -n}'')$$

$$\boldsymbol{D}_{\nu n} = a_{\nu n}(m_\nu'' i_{\nu n}'' - m_\nu' i_{\nu n}') - a_{\nu, -n}(m_\nu'' i_{\nu, -n}'' - m_\nu' i_{\nu, -n}')$$

$$\boldsymbol{E}_{\nu n} = a_{\nu n}(m_\nu'' j_{\nu n}'' - m_\nu' j_{\nu n}') - a_{\nu, -n}(m_\nu'' j_{\nu, -n}'' - m_\nu' j_{\nu, -n}')$$

Solution of the system of equations for 2 *chordwise pivotal points at each station*—(*compare Tables* 18 *to* 22 *for symmetrical load cases, Tables* 23 *to* 27 *for anti-symmetrical load cases*).—(23) For the load case under consideration we collect the α_ν' and α_ν''-values which are for wings with continuous camber and twist

$$\alpha_\nu' = \alpha(0{\cdot}9045 c_\nu, y_\nu)$$

$$\alpha_\nu'' = \alpha(0{\cdot}3455 c_\nu, y_\nu).$$

For a plane wing α_ν is constant. For wings with flaps, ailerons, etc., we take

$$\alpha_\nu = a_{\nu \text{ without flaps}} + \frac{d\alpha}{d\delta} \cdot \delta_\nu$$

δ_ν being the flap deflection at the station η_ν; if this station is next to the beginning of the flap we take as better value

$$\delta_\nu = \delta^*[\tfrac{1}{2} \pm (\theta^* - \theta_\nu)]$$

δ^* being the flap deflection at the beginning or end of the flap $\delta^* = \delta(\eta^* \pm 0)$. $d\alpha_\nu/d\delta$ must be calculated from the flap characteristics in two-dimensional flow:

$$\frac{d\alpha'}{d\delta} = \frac{d\alpha}{d\delta} - 0{\cdot}3934 \frac{\partial C_m}{(\partial\delta)_{0\cdot 25}}$$

with

$$\frac{d\alpha}{d\delta} = \frac{(\partial C_L/\partial\delta)_{\alpha=\text{const}}}{(\partial C_L/\partial\alpha)_{\delta=\text{const}}}$$

$$\frac{d\alpha''}{d\delta} = \frac{d\alpha}{d\delta} + 1{\cdot}030 \frac{\partial C_m}{(\partial\delta)_{0\cdot 25}}$$

(24) We guess roughly what γ_n and μ_n with an even suffix might be. These estimates need not be very near to the final results, we may take them from previous calculations with a smaller number of pivotal stations or so that an estimated $dC_L/d\alpha$ results, etc. These values are marked by the number 0 in square brackets* :

$$\gamma_0{}^{[0]},\ \gamma_2{}^{[0]},\ \gamma_4{}^{[0]}\ \ldots\ \mu_0{}^{[0]},\ \mu_2{}^{[0]}\ \ldots\ .$$

If in doubt $\mu = 0$ will usually be enough for the beginning.

The system of equations to be solved is

$$\gamma_\nu = a_{\nu\nu}(l_\nu'\alpha_\nu' - l_\nu''\alpha_\nu'') + \sum_0^{\frac{m-1}{2}}{}' B_{\nu n}\gamma_n + \sum_0^{\frac{m-1}{2}}{}' C_{\nu n}\mu_n$$

$$\mu_\nu = a_{\nu\nu}(m_\nu''\alpha_\nu'' - m_\nu'\alpha_\nu'') + \sum_0^{\frac{m-1}{2}}{}' D_{\nu n}\gamma_n + \sum_0^{\frac{m-1}{2}}{}' E_{\nu n}\mu_n .$$

The same with the B, C, D, E, in bold italics is to be used for anti-symmetrical load cases.

(25) We calculate the absolute terms

$$a_{\nu\nu}(l_\nu'\alpha_\nu' - l_\nu''\alpha_\nu'')$$
$$a_{\nu\nu}(m_\nu''\alpha_\nu'' - m_\nu'\alpha_\nu')$$

with the data from (23), (12) and the $a_{\nu\nu}$ from Tables 1 to 7.

(26) For every $\nu = 1, 3, 5 \ldots$ we add to these absolute terms the sums of the products $B_{\nu n}\gamma_n{}^{[0]}$ and $C_{\nu n}\mu_n{}^{[0]}$ or $D_{\nu n}\gamma_n{}^{[0]}$ and $E_{\nu n}\mu_n{}^{[0]}$; this gives the first approximation of the γ_ν and μ_ν with odd suffixes:

$$\gamma_\nu{}^{[1]} = a_{\nu\nu}(l_\nu'a_\nu' - l_\nu''\alpha_\nu'') + \sum_{0,2,4\ldots}^{\frac{m-1}{2}}{}' B_{\nu n}\gamma_n{}^{[0]} + \sum_{0,2,4\ldots}^{\frac{m-1}{2}}{}' C_{\nu n}\mu_n{}^{[0]}$$

$$\mu_n{}^{[1]} = a_{\nu\nu}(m_\nu''\alpha_\nu'' - m_\nu'\alpha_\nu') + \sum_{0,2,4\ldots}^{\frac{m-1}{2}}{}' D_{\nu n}\gamma_n{}^{[0]} + \sum_{0,2,4\ldots}^{\frac{m-1}{2}}{}' E_{\nu n}\mu_n{}^{[0]}.$$

The collection of the terms of these sums in columns headed by $\nu = 1, 3, 5 \ldots$ once for γ and once for μ is recommended so that we have in the first horizontal line the absolute terms from (25) in the next $\gamma_0{}^{[0]}$ multiplied by B_{10}, B_{30}, B_{50} . . . and D_{10}, D_{30}, D_{50} . . ., then the $\gamma_2{}^{[0]}$ multiplied by B_{12}, B_{32} . . . and D_{12}, D_{32} . . ., etc., followed by a line for $\mu_0{}^{[0]}$ multiplied by C_{10}, C_{30} . . . and E_{10}, E_{30} . . . and so forth.

(27) In the same way the first approximation of the $\gamma_0, \gamma_2, \gamma_4 \ldots$ and $\mu_0, \mu_2, \mu_4 \ldots$ are calculated from the $\gamma_n{}^{[1]}$ and $\mu_n{}^{[1]}$ with odd suffixes:

$$\gamma_\nu{}^{[1]} = a_{\nu\nu}(l_\nu'\alpha_\nu' - l_\nu''\alpha_\nu'') + \sum_{1,3,5\ldots}^{\frac{m-1}{2}}{}' B_{\nu n}\gamma_n{}^{[1]} + \sum_{1,3,5\ldots}^{\frac{m-1}{2}} C_{\nu n}\mu_n{}^{[1]}, \qquad \nu - 0, 2, 4 \ldots$$

$$\mu_\nu{}^{[1]} = a_{\nu\nu}(m_\nu''\alpha_\nu'' - m_\nu'\alpha_\nu') + \sum_{1,3,5\ldots}^{\frac{m-1}{2}}{}' D_{\nu n}\gamma_n{}^{[1]} + \sum_{1,3,5\ldots}^{\frac{m-1}{2}}{}' E_{\nu n}\mu_n{}^{[1]}.$$

(28) The first differences $\Delta^{[1]}\gamma_n$ and $\Delta^{[1]}\mu_n$ between these values and the initial estimates are calculated:

$$\Delta^{[1]}\gamma_\nu = \gamma_\nu{}^{[1]} - \gamma_\nu{}^{[0]} \qquad \Delta^{[1]}\mu_\nu = \mu_\nu{}^{[1]} - \mu_\nu{}^{[0]}, \qquad \nu = 0, 2, 4 \ldots$$

* In the tables shown as circles.

(29) Second differences of the γ's and μ's with odd suffixes are computed from:

$$\Delta^{[2]}\gamma_\nu = \sum_{0,2,4\ldots}^{\frac{m-1}{2}} B_{\nu n}\Delta^{[1]}\gamma_n + \sum_{0,2,4\ldots}^{\frac{m-1}{2}} C_{\nu n}\Delta^{[1]}\mu_n \qquad \nu = 1, 3, 5 \ldots$$

$$\Delta^{[2]}\mu_\nu = \sum_{0,2,4\ldots}^{\frac{m-1}{2}} D_{\nu n}\Delta^{[1]}\gamma_n + \sum_{0,2,4\ldots}^{\frac{m-1}{2}} E_{\nu n}\Delta^{[1]}\mu_n\,.$$

The arrangement is as under (26) but without the absolute terms in the first line.

(30) And from these the second differences of the even γ's and μ's:

$$\Delta^{[2]}\gamma_\nu = \sum_{1,3,5\ldots}^{\frac{m-1}{2}} B_{\nu n}\Delta^{[2]}\gamma_n + \sum_{1,3,5\ldots}^{\frac{m-1}{2}} C_{\nu n}\Delta^{[2]}\mu_n \qquad \nu = 0, 2, 4 \ldots$$

$$\Delta^{[2]}\mu_\nu = \sum_{1,3,5\ldots}^{\frac{m-1}{2}} D_{\nu n}\Delta^{[2]}\gamma_n + \sum_{1,3,5\ldots}^{\frac{m-1}{2}} E_{\nu n}\Delta^{[2]}\mu_n$$

Arrangement as before.

(31) Third differences of the odd γ's and μ's:

$$\Delta^{[3]}\gamma_\nu = \sum_{0,2,4\ldots}^{\frac{m-1}{2}} B_{\nu n}\Delta^{[2]}\gamma_n + \sum_{0,2,4\ldots}^{\frac{m-1}{2}} C_{\nu n}\Delta^{[2]}\mu_n \qquad \nu = 1, 3, 5 \ldots$$

$$\Delta^{[3]}\mu_\nu = \sum_{0,2,4\ldots}^{\frac{m-1}{2}} D_{\nu n}\Delta^{[2]}\gamma_n + \sum_{0,2,4\ldots}^{\frac{m-1}{2}} E_{\nu n}\Delta^{[2]}\mu_n\,.$$

(32) This to-and-fro calculation is continued until the differences disappear. If one wants to break off after the r-th difference the rest may be estimated as

$$\frac{(\Delta^{[r]}\gamma_\nu)^2}{\Delta^{[r-1]}\gamma_\nu - \Delta^{[r]}\gamma_\nu} \quad \text{or} \quad \frac{(\Delta^{[r]}\mu_\nu)^2}{\Delta^{[r-1]}\mu_\nu - \Delta^{[r]}\mu_\nu}\,.$$

(33) The solutions of our system of equations are then the γ_ν and μ_ν as summed up from the differences:

$$\gamma_\nu = \gamma_\nu^{[1]} + \Delta^{[2]}\gamma_\nu + \Delta^{[3]}\gamma_\nu + \ldots + \Delta^{[r]}\gamma_\nu + \frac{(\Delta^{[r]}\gamma_\nu)^2}{\Delta^{[r-1]}\gamma_\nu - \Delta^{[r]}\gamma_\nu}$$

$$\mu_\nu = \mu_\nu^{[1]} + \Delta^{[2]}\mu_\nu + \Delta^{[3]}\mu_\nu + \ldots.$$

A check of these results by inserting them into the system of equations is very useful. It is mainly a repetition of the process described under (26) and (27) but with the final γ_n- and μ_n- values.

Resulting forces, moments, etc., are calculated according to the formulae in section 7; coefficients required are found in the Tables 1 to 7.

Example.—As an illustration of the calculus all the details for a simple swept wing are written down on Tables 8 to 30. The number of pivotal points are 1×15 and 2×15 for the plane wing of incidence 1, and 2×15 for an aileron case. The work has indeed been overdone a bit in order not to miss any details and in footnotes the main formulae are given.

TABLE 1

$m = 3$

ν or n	-1	0	$+1$
$\eta_\nu = \cos\theta_\nu$	$-0 \cdot 7071$	0	$+0 \cdot 7071$
θ_ν	$135°$	$90°$	$45°$
$\sin\theta_\nu$	$0 \cdot 7071$	1	$0 \cdot 7071$
$\sin 2\theta$	-1	0	1
$a_{\nu\nu}$	$0 \cdot 7071$	1	$0 \cdot 7071$
$\dfrac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1} \cos\dfrac{\nu\pi}{m+1}$	$0 \cdot 1763$	$0 \cdot 3535$	$0 \cdot 1763$

$a_{\nu n}$

ν, n	-1	$+1$
0	$0 \cdot 3536$	$0 \cdot 3536$

$$a_{\nu, n} = a_{n, \nu} = a_{-\nu, -n} = a_{-n, -\nu}$$

TABLE 2

$m = 5$

ν or n	-2	-1	0	$+1$	$+2$
$\eta_\nu = \cos\theta_\nu$	$-0 \cdot 8660$	$-0 \cdot 5$	0	$0 \cdot 5$	$0 \cdot 8660$
θ_ν	$150°$	$120°$	$90°$	$60°$	$30°$
$\sin\theta_\nu$	$0 \cdot 5$	$0 \cdot 8660$	1	$0 \cdot 8660$	$0 \cdot 5$
$\sin 2\theta_\nu$	$-0 \cdot 8660$	$-0 \cdot 8660$	0	$0 \cdot 8660$	$0 \cdot 8660$
$a_{\nu\nu}$	$0 \cdot 3333$	$0 \cdot 5774$	$0 \cdot 6667$	$0 \cdot 5774$	$0 \cdot 3333$
$\dfrac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1} \cos\dfrac{\nu\pi}{m+1}$	$0 \cdot 04167$	$0 \cdot 125$	$0 \cdot 1667$	$0 \cdot 125$	$0 \cdot 04167$

$a_{\nu n}$

ν, n	-1	$+1$
-2	$0 \cdot 3591$	$0 \cdot 0258$
0	$0 \cdot 3849$	$0 \cdot 3849$
2	$0 \cdot 0258$	$0 \cdot 3591$

$|\eta_\nu - \eta_n|$

ν, n	-1	$+1$
-2	$0 \cdot 3660$	$1 \cdot 3660$
0	$0 \cdot 5$	$0 \cdot 5$
$+2$	$1 \cdot 3660$	$0 \cdot 3660$

TABLE 3

$m = 7$

ν or n	-3	-2	-1	0	1	2	3
$\eta_\nu = \cos\theta_\nu$	$-0\cdot9239$	$-0\cdot7071$	$-0\cdot3827$	0	$0\cdot3827$	$0\cdot7071$	$0\cdot9239$
θ_ν	$157\cdot5°$	$135°$	$112\cdot5°$	$90°$	$67\cdot5°$	$45°$	$22\cdot5°$
$\sin\theta_\nu$	$0\cdot3827$	$0\cdot7071$	$0\cdot9239$	1	$0\cdot9239$	$0\cdot7071$	$0\cdot3827$
$\sin 2\theta_\nu$	$-0\cdot7071$	-1	$-0\cdot7071$	0	$0\cdot7071$	1	$0\cdot7071$
$a_{\nu\nu}$	$0\cdot1913$	$0\cdot3536$	$0\cdot4619$	$0\cdot5$	$0\cdot4619$	$0\cdot3536$	$0\cdot1913$
$\dfrac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1} \cos\dfrac{\nu\pi}{m+1}$	$0\cdot0140_1$	$0\cdot0478_4$	$0\cdot0816_6$	$0\cdot0956_7$	$0\cdot0816_6$	$0\cdot0478_4$	$0\cdot0140_1$

$a_{\nu n}$

ν, n	-3	-1	$+1$	$+3$
-2	$0\cdot3599$	$0\cdot3879$	$0\cdot0344$	$0\cdot0064$
0	$0\cdot0280$	$0\cdot3943$	$0\cdot3943$	$0\cdot0280$
$+2$	$0\cdot0064$	$0\cdot0344$	$0\cdot3879$	$0\cdot3599$

$|\eta_\nu - \eta_n|$

ν, n	-3	-1	$+1$	$+3$
-2	$0\cdot2168$	$0\cdot3244$	$1\cdot0898$	$1\cdot6310$
0	$0\cdot9239$	$0\cdot3827$	$0\cdot3827$	$0\cdot9239$
$+2$	$1\cdot6310$	$1\cdot0898$	$0\cdot3244$	$0\cdot2168$

TABLE 4

$m = 11$

ν or n	0	1	2	3	4	5
$\eta_\nu = \cos\theta_\nu$	0	0·2588	0·5	0·7071	0·8660	0·9659
θ_ν	90°	75°	60°	45°	30°	15°
$\sin\theta_\nu$	1	0·9659	0·8660	0·7071	0·5	0·2588
$\sin 2\theta_\nu$	0	0·5	0·8660	1	0·8660	0·5
$a_{\nu\nu}$	0·3333	0·3220	0·2887	0·2357	0·1667	0·0863
$\frac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1}\cos\frac{\nu\pi}{m+1}$	0·04314	0·04025	0·03235	0·02157	0·01078	0·00289

$a_{\nu n}$

ν, n	1	3	5	
−4	0·0106	0·0040	0·0011	4
−2	0·0404	0·0117	0·0029	2
0	0·4005	0·0393	0·0077	0
2	0·3995	0·3966	0·0287	−2
4	0·0364	0·3889	0·3602	−4
	−1	−3	−5	ν, n

$|\eta_\nu - \eta_n|$

ν, n	1	3	5	
−4	1·1249	1·5731	1·8320	4
−2	0·7588	1·2071	1·4659	2
0	0·2588	0·7071	0·9659	0
2	0·2412	0·2071	0·4659	−2
4	0·6072	0·1589	0·0999	−4
	−1	−3	−5	ν, n

TABLE 5

$m = 15$

ν or n	0	1	2	3	4	5	6	7
$\eta_\nu = \cos\theta_\nu$	0	0·1951	0·3827	0·5556	0·7071	0·8315	0·9239	0·9808
θ_ν	90°	78·75°	67·5°	56·25°	45°	33·75°	22·5°	11.25°
$\sin\theta_\nu$	1	0·9808	0·9239	0·8315	0·7071	0·5556	0·3827	0·1951
$\sin 2\theta_\nu$	0	0·3827	0·7071	0·9239	1	0·9239	0·7071	0·3827
$a_{\nu\nu}$	0·25	0·2452	0·2310	0·2079	0·1768	0·1389	0·0957	0·0488
$\dfrac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1}\cos\dfrac{\nu\pi}{m+1}$	0·02439	0·02346	0·02082	0·01686	0·01219	0·00753	$0{\cdot}00357_1$	0·000928

$a_{\nu n}$

ν, n	1	3	5	7	
−6	0·0047	0·0023	0·0011	0·0003	6
−4	0·0133	0·0058	0·0026	0·0008	4
−2	0·0424	0·0136	0·0054	0·0015	2
0	0·4026	0·0421	0·0126	0·0032	0
2	0·4023	0·4016	0·0398	0·0079	−2
4	0·0413	0·4001	0·3969	0·0288	−4
6	0·0110	0·0367	0·3890	0·3602	−6
	−1	−3	−5	−7	ν, n

$|\eta_\nu - \eta_n|$

ν, n	1	3	5	7	
−6	1·1190	1·4795	1·7554	1·9047	6
−4	0·9022	1·2627	1·5386	1·6879	4
−2	0·5778	0·9383	1·2142	1·3635	2
0	0·1951	0·5556	0·8315	0·9808	0
2	0·1876	0·1729	0·4488	0·5981	−2
4	0·5120	0·1515	$0{\cdot}1243_6$	0·2737	−4
6	0·7288	0·3683	$0{\cdot}0924_1$	$0{\cdot}0569_1$	−6
	−1	−3	−5	−7	ν, n

TABLE 6

$m = 23$

ν or n	0	1	2	3	4	5
$\eta_\nu = \cos\theta_\nu$	0	0·1305	0·2588	0·3827	0·5	0·6088
θ_ν	90°	82·5°	75°	67·5°	60°	52·5°
$\sin\theta_\nu$	1	0·9914	0·9659	0·9239	0·8660	0·7934
$\sin 2\theta_\nu$	0	0·2588	0·5	0·7071	0·8660	0·9659
$a_{\nu\nu}$	0·1667	0·1653	0·1610	0·1540	0·1443	0·1322
$\dfrac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1}\cos\dfrac{\nu\pi}{m+1}$	0·01088	0·01069	0·01015	0·00928	0·00816	$0{\cdot}00684_6$

ν or n	6	7	8	9	10	11
$\eta_\nu = \cos\theta_\nu$	0·7071	0·7934	0·8660	0·9239	0·9659	0·9914
θ_ν	45°	37·5°	30°	22·5°	15°	7·5°
$\sin\theta_\nu$	0·7071	0·6088	0·5	0·3827	0·2588	0·1305
$\sin 2\theta_\nu$	1	0·9659	0·8660	0·7071	0·5	0·2588
$a_{\nu\nu}$	0·11785	$0{\cdot}1014_6$	0·08333	0·06378	0·04314	0·02176
$\dfrac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1}\cos\dfrac{\nu\pi}{m+1}$	$0{\cdot}00543_9$	$0{\cdot}00403_1$	$0{\cdot}00271_9$	0·001593	0·000728	0·000185

$a_{\nu n}$

ν, n	1	3	5	7	9	11	
−10	0·0015	0·0009	0·0006	0·0004	0·0002	0·0001	10
− 8	0·0035	0·0021	0·0013	0·0008	0·0004	0·0001	8
− 6	0·0069	0·0038	0·0022	0·0013	0·0007	0·0002	6
− 4	0·0150	0·0071	0·0039	0·0022	0·0011	0·0004	4
− 2	0·0439	0·0151	0·0071	0·0037	0·0018	0·0006	2
0	0·4041	0·0438	0·0149	0·0067	0·0031	0·0009	0
2	0·4041	0·4039	0·0435	0·0143	0·0058	0·0016	− 2
4	0·0437	0·4037	0·4034	0·0425	0·0128	0·0033	− 4
6	0·0146	0·0431	0·4028	0·4019	0·0400	0·0079	− 6
8	0·0064	0·0137	0·0416	0·4002	0·3970	0·0288	− 8
10	0·0026	0·0049	0·0112	0·0367	0·3891	0·3603	−10
	−1	−3	−5	−7	−9	−11	ν, n

TABLE 6—*continued*

$m = 23$

$|\eta_\nu - \eta_n|$

ν, n	1	3	5	7	9	11	
−10	1·0965	1·3486	1·5747	1·7593	1·8898	1·9574	10
− 8	0·9966	1·2487	1·4748	1·6594	1·7899	1·8575	8
− 6	0·8376	1·0898	1·3159	1·5005	1·6310	1·6986	6
− 4	0·6305	0·8827	1·1088	1·2934	1·4239	1·4914	4
− 2	0·3894	0·6415	0·8676	1·0522	1·1827	1·2503	2
0	0·1305	0·3827	0·6088	0·7934	0·9239	0·9914	0
2	0·1283	0·1239	0·3499	0·5345	0·6651	0·7326	− 2
4	0·3695	0·1173	0·1087$_6$	0·2934	0·4239	0·4914	− 4
6	0·5766	0·3244	0·0983$_5$	0·08624	0·2168	0·2843	− 6
8	0·7355	0·4834	0·2573	0·0726$_8$	0·0578$_5$	0·1254$_1$	− 8
10	0·8354	0·5833	0·3572	0·1726	0·0420$_5$	0·0255$_1$	−10
	−1	−3	−5	−7	−9	−11	ν, n

TABLE 7

$m = 31$

ν or n	0	1	2	3	4	5	6	7
$\eta_\nu = \cos\theta_\nu$	0	0·0980	0·1951	0·2903	0·3827	0·4714	0·5556	0·6344
θ_ν	90°	84·375°	78·75°	73·125°	67·5°	61·875°	56·25°	50·625°
$\sin\theta_\nu$	1	0·9952	0·9808	0·9569	0·9239	0·8819	0·8315	0·7730
$\sin 2\theta_\nu$	0	0·1951	0·3827	0·5556	0·7071	0·8315	0·9239	0·9808
$a_{\nu\nu}$	0·125	0·1244	0·1226	0·1196	0·11548	0·11024	0·1039$_3$	0·0966$_3$
$\frac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1} \cos\frac{\nu\pi}{m+1}$	0·00612$_6$	0·00606$_7$	0·00589$_3$	0·00561$_0$	0·00522$_9$	0·00476$_5$	0·00423$_5$	0·00366$_1$

ν or n	8	9	10	11	12	13	14	15
η_ν	0·7071	0·7730	0·8315	0·8819	0·9239	0·9569	0·9808	0·9952
θ_ν	45°	39·375°	33·75°	28·125°	22·5°	16·875°	11·25°	5·625°
$\sin\theta_\nu$	0·7071	0·6344	0·5556	0·4714	0·3827	0·2903	0·1951	0·0980
$\sin 2\theta_\nu$	1	0·9808	0·9239	0·8315	0·7071	0·5556	0·3827	0·1951
$a_{\nu\nu}$	0·0883$_9$	0·0793$_0$	0·0694$_5$	0·0589$_2$	0·0478$_4$	0·03629	0·02439	0·01225
$\frac{\eta_{\nu+1} - \eta_{\nu-1}}{m+1} \cos\frac{\nu\pi}{m+1}$	0·003063	0·002465	0·001891	0·001361	0·000897	0·005162	0·0002332	0·0000588

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TABLE 7—*continued*

$m = 31$

$a_{\nu n}$

ν, n	1	3	5	7	9	11	13	15	
−14	0·0007	0·0005	0·0003	0·0002	0·0002	0·0001	0·0001	—	14
−12	0·0014	0·0010	0·0007	0·0005	0·0003	0·0002	0·0001	—	12
−10	0·0025	0·0017	0·0011	0·0008	0·0005	0·0003	0·0002	0·0001	10
− 8	0·0042	0·0027	0·0018	0·0012	0·0008	0·0005	0·0003	0·0001	8
− 6	0·0076	0·0043	0·0027	0·0018	0·0012	0·0007	0·0004	0·0001	6
− 4	0·0155	0·0076	0·0044	0·0027	0·0017	0·0011	0·0006	0·0002	4
− 2	0·0444	0·0156	0·0076	0·0043	0·0026	0·0016	0·0008	0·0003	2
0	0·4046	0·0444	0·0155	0·0075	0·0041	0·0024	0·0012	0·0004	0
2	0·4046	0·4046	0·0443	0·0153	0·0073	0·0038	0·0019	0·0006	− 2
4	0·0443	0·4045	0·4044	0·0440	0·0150	0·0068	0·0032	0·0009	− 4
6	0·0154	0·0442	0·4043	0·4041	0·0436	0·0144	0·0059	0·0016	− 6
8	0·0074	0·0152	0·0438	0·4038	0·4034	0·0426	0·0128	0·0033	− 8
10	0·0040	0·0071	0·0148	0·0432	0·4029	0·4019	0·0400	0·0079	−10
12	0·0022	0·0036	0·0064	0·0138	0·0417	0·4003	0·3970	0·0288	−12
14	0·0010	0·0015	0·0026	0·0049	0·0112	0·0368	0·3891	0·3603	−14
	−1	−3	−5	−7	−9	−11	−13	−15	ν, n

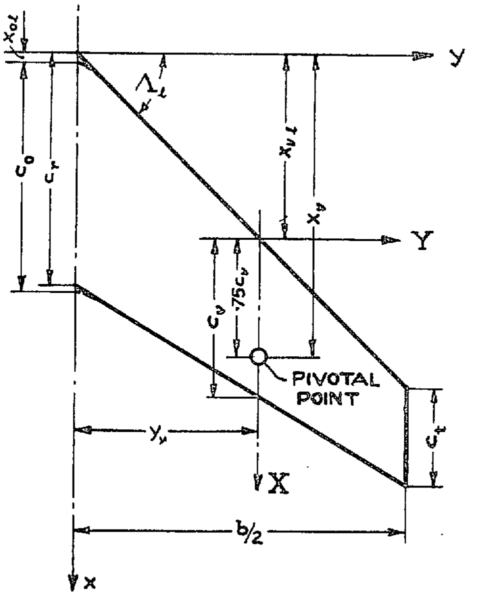
$|\eta_\nu - \eta_n|$

ν, n	1	3	5	7	9	11	13	15	
−14	1·0788	1·2711	1·4522	1·6152	1·7538	1·8627	1·9377	1·9760	14
−12	1·0219	1·2142	1·3953	1·5583	1·6969	1·8058	1·8808	1·9191	12
−10	0·9295	1·1218	1·3029	1·4659	1·6045	1·7134	1·7884	1·8267	10
− 8	0·8051	0·9974	1·1785	1·3415	1·4801	1·5890	1·6640	1·7023	8
− 6	0·6536	0·8459	1·0270	1·1900	1·3286	1·4375	1·5125	1·5508	6
− 4	0·4807	0·6730	0·8541	1·0171	1·1557	1·2646	1·3396	1·3779	4
− 2	0·2931	0·4854	0·6665	0·8295	0·9681	1·0770	1·1520	1·1903	2
0	$0·0980_2$	0·2903	0·4714	0·6344	0·7730	0·8819	0·9569	0·9952	0
2	$0·0970_7$	$0·0952_0$	0·2757	0·4393	0·5779	0·6868	0·7618	0·8001	− 2
4	0·2847	$0·0924_0$	$0·0887_1$	0·2517	0·3903	0·4992	0·5742	0·6125	− 4
6	0·4576	0·2653	$0·0841_7$	$0·0788_2$	0·2174	0·3264	0·4013	0·4396	− 6
8	0·6091	0·4168	0·2357	$0·0727_1$	0·0659	0·1748	0·2498	0·2881	− 8
10	0·7335	0·5412	0·3601	0·1971	$0·0584_6$	$0·0504_5$	$0·1254_7$	0·1637	−10
12	0·8259	0·6336	0·4525	0·2895	0·1509	$0·0419_6$	$0·0330_6$	$0·0713_0$	−12
14	0·8828	0·6905	0·5094	0·3464	0·2078	$0·0988_6$	$0·0238_5$	$0·0144_0$	−14
	−1	−3	−5	−7	−9	−11	−13	−15	ν, n

TABLE 8.

EXAMPLE I.

1 CHORDWISE } PIVOTAL POINTS
15 SPANWISE }



SYMMETRICAL SWEPT BACK WING

SPAN	b	= 20 ft
ASPECT RATIO	A	= 4
ROOT CHORD	c_r	= 7 ft
TIP CHORD	c_t	= 3 ft
L.E. SWEEP BACK	Λ_l	= 45°
TAPER RATIO	c_t/c_r	= ·429

LOW MACH NUMBERS
$\sqrt{1-M^2} \approx 1$

FORM I.

$\lvert n \rvert$ or $\lvert \nu \rvert$	0	1	2	3	4	5	6	7
η_n 1)	0	·1951	·3827	·5556	·7071	·8315	·9239	·9808
y_n	0	1·951	3·827	5·556	7·071	8·315	9·239	9·808
x_{nl} 2)	·3252	1·951	3·827	5·556	7·071	8·315	9·239	9·808
c_n 2)	6·8699	6·2196	5·4692	4·7776	4·1716	3·6740	3·3044	3·0768
x_ν 2)	5·4776	6·6157	7·9289	9·1392	10·1997	11·0705	11·7173	12·1156
$\frac{b}{2c_n}$	1·4556	1·6078	1·8284	2·0931	2·3972	2·7218	3·0263	3·2501
$\tau_{\nu\nu}$ 3)	1·91107	1·9156	1·9202	1·9224	1·9204	1·9132	1·9014	1·8897
A_ν 4)	·1308	·1280	·1203	·1081	·0921	·0726	·0503	0258

1) η_n for $m=15$ see TABLE 5

2) In the plane of symmetry ($n=0$) the kink is rounded off according to the following rules:
$x_{ol} = \frac{1}{6} x_{1l}$ $\quad c_o = c_r - \frac{1}{6}(c_r - c_1)$ $\quad x_o = x_{ol} + \cdot 75 c_o = \cdot 625 c_r + \frac{1}{6} x_1$

3) $\tau_{\nu\nu} = 1{\cdot}8847 + {\cdot}5103 \, \frac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1} \cos\frac{\nu\pi}{m+1} \left(\frac{b}{2c_\nu}\right)^2$

$\frac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1} \cos\frac{\nu\pi}{m+1}$ for $m=15$ see TABLE 5

4) $A_\nu = \frac{a_{\nu\nu}}{\tau_{\nu\nu}}$ $\quad a_{\nu\nu}$ for $m=15$ see TABLE 5

FORM 2. TABLE 9 1 × 15 STATIONS

$n=0$	ν	−7	−5	−3	−1	1	3	5	7
$\lvert\eta_\nu-\eta_n\rvert$		·9808	·8315	·5556	·1951	·1951	·5556	·8315	·9808
$x_\nu - x_{n\ell}$		11·7904	10·7453	8·8140	6·2905	6·2905	8·8140	10·7453	11·7904
$\lvert Y_{\nu n}\rvert$ [1]		1·427$_6$	1·210	·8087	·2840	·2840	·8087	1·210	1·427$_6$
$X_{\nu n}$ [2]		1·716	1·564	1·283	·9156	·915$_6$	1·283	1·564	1·716
$i_{\nu n}$ [3]		1·708	1·726	1·769	1·8628	1·8628	1·769	1·726	1·708
$\frac{a_{\nu n}\, i_{\nu n}}{i_{\nu\nu}}$ [4]		·00289	·0113$_7$	·0387$_4$	·3915	·3915	·0387$_4$	·0113$_7$	·0028$_9$

$n=2$	ν	−7	−5	−3	−1	1	3	5	7
$n=-2$	ν	7	5	3	1	−1	−3	−5	−7
$\lvert\eta_\nu-\eta_n\rvert$		1·3635	1·2142	·9383	·5778	·1876	·1729	·4488	·5981
$x_\nu - x_{n\ell}$		8·2886	7·2435	5·3122	2·7887	2·7887	5·3122	7·2435	8·2886
$\lvert Y_{\nu n}\rvert$		2·492	2·22	1·714	1·056	·3427	·316	·820	1·092
$X_{\nu n}$		1·515	1·323	·970	·510	·510	·970	1·323	1·515
$i_{\nu n}$		1·45$_0$	1·429	1·381	1·228	1·476$_0$	1·8692	1·777	1·744
$\frac{a_{\nu n}\, i_{\nu n}}{i_{\nu\nu}}$		·0011$_5$	·0040$_3$	·0097$_8$	·0272	·3100	·3898	·0370	·0072$_9$

$n=4$	ν	−7	−5	−3	−1	1	3	5	7
$n=-4$	ν	7	5	3	1	−1	−3	−5	−7
$\lvert\eta_\nu-\eta_n\rvert$		1·6879	1·5386	1·2627	·9022	·5120	·1515	·1244	·2737
$x_\nu - x_{n\ell}$		5·0446	3·9995	2·0682	−·4553	−·4553	2·0682	3·9995	5·0446
$\lvert Y_{\nu n}\rvert$		4·047	3·69	3·027	2·16	1·227	·3633	·2983	·656
$X_{\nu n}$		1·209	·958	·496	−·1091	−·1091	·496	·958	1·209
$i_{\nu n}$		1·232	1·188	1·08	·841	·731	1·4460	1·874	1·802
$\frac{a_{\nu n}\, i_{\nu n}}{i_{\nu\nu}}$		·0005$_2$	·0016$_2$	·0032$_6$	·0058$_4$	·0157$_4$	·3011	·3892	·0274$_8$

$n=6$	ν	−7	−5	−3	−1	1	3	5	7
$n=-6$	ν	7	5	3	1	−1	−3	−5	−7
$\lvert\eta_\nu-\eta_n\rvert$		1·904$_7$	1·7554	1·4795	1·1190	·7288	·3683	·0924	·0569
$x_\nu - x_{n\ell}$		2·8766	1·8315	−·0998	−2·6253	−2·6233	−·0998	1·8315	2·8766
$\lvert Y_{\nu n}\rvert$		5·76	5·31	4·47	3·385	2·207	1·114	·2797	·1723
$X_{\nu n}$		·870	·554	−·0302	−·7935	−·7935	−·0302	·554	·870
$i_{\nu n}$		1·107	1·059	·936	·708	·580	·773	1·5705	1·8982
$\frac{a_{\nu n}\, i_{\nu n}}{i_{\nu\nu}}$		·0001$_8$	·0006$_1$	·0011$_2$	·0017$_4$	·0033$_3$	·0147$_5$	·3182$_5$	·3615

1) $\lvert Y_{\nu n}\rvert = \frac{b}{2c_n}\lvert\eta_\nu-\eta_n\rvert$

2) $X_{\nu n} = \frac{1}{c_n}(x_\nu - x_{n\ell})$

3) $i_{\nu n} = f(X_{\nu n}, Y_{\nu n})$ take from CHARTS 1…3

4) $a_{\nu n}$ for $m=15$ see TABLE 5

FORM 2. ctd. TABLE 10 1 × 15 STATIONS

n = 1	ν	−6	−4	−2	0	2	4	6
n = −1	ν	6	4	2	0	−2	−4	−6
$\lvert\eta_\nu - \eta_n\rvert$		1·1190	·9022	·5778	·1951	·1876	·5120	·7288
$x_\nu - x_{n\ell}$		9·7663	8·2487	5·9779	3·5266	5·9779	8·2487	9·7663
$\lvert Y_{\nu n}\rvert$		1·800	1·451	·930	·314	·3018	824	1·173
$X_{\nu n}$		1·571	1·327	·962	·5675	·962	1·327	1·571
$i_{\nu n}$		1·587	1·588	1·578	1·5655	1·8733	1·7765	1·737
$\frac{a_{\nu n} i_{\nu n}}{i_{\nu\nu}}$		$·0039_2$	$·0110_1$	·0349	·3300	·3925	·0382	$·0100_6$

n = 3	ν	−6	−4	−2	0	2	4	6
n = −3	ν	6	4	2	0	−2	−4	−6
$\lvert\eta_\nu - \eta_n\rvert$		1·4795	1·2627	·9383	·5556	·1729	·1515	·3683
$x_\nu - x_{n\ell}$		6·1613	4·6437	2·3729	−·0784	2·3729	4·6437	6·1613
$\lvert Y_{\nu n}\rvert$		3·10	2·645	1·966	1·163	·3620	·3173	·7715
$X_{\nu n}$		1·289	·972	·497	−·0164	·497	·972	1·289
$i_{\nu n}$		1·32	1·26	1·122	·791	1·4473	1·8692	1·785
$\frac{a_{\nu n} i_{\nu n}}{i_{\nu\nu}}$		·0016	·0038	$·0079_4$	$·0174_3$	·3025	·3893	·0345

n = 5	ν	−6	−4	−2	0	2	4	6
n = −5	ν	6	4	2	0	−2	−4	−6
$\lvert\eta_\nu - \eta_n\rvert$		1·7554	1·5386	1·2142	·8315	·4488	·1244	·0924
$x_\nu - x_{n\ell}$		3·3923	1·8847	−·3861	−2·8374	−·3861	1·8847	3·3923
$\lvert Y_{\nu n}\rvert$		4·775	4·185	3·305	2·263	1·222	·3387	·2515
$X_{\nu n}$		·923	·513	−·105	−·772	−·105	·513	·923
$i_{\nu n}$		1·140	1·063	·895	·592	·733	1·4823	1·8832
$\frac{a_{\nu n} i_{\nu n}}{i_{\nu n}}$		$·0006_6$	$·0014_4$	$·0025_2$	$·0039_1$	·01518	·3063	·3852

n = 7	ν	−6	−4	−2	0	2	4	6
n = −7	ν	6	4	2	0	−2	−4	−6
$\lvert\eta_\nu - \eta_n\rvert$		1·9047	1·6879	1·3635	·9808	·5981	2737	·0569
$x_\nu - x_{n\ell}$		1·9093	3917	−1·8791	−4·3304	−1·8791	·3917	1 9093
$\lvert Y_{\nu n}\rvert$		6·19	5·485	4·43	3·188	1 944	890	·185
$X_{\nu n}$		·621	·1274	−·611	−1·407	−·611	·1274	·621
$i_{\nu n}$		1·062	·977	·810	·542	·602	·882	1·7007
$\frac{a_{\nu n} i_{\nu n}}{i_{\nu\nu}}$		$·0001_7$	·0004	$·0006_3$	0009_1	$·0024_8$	$·0132_3$	·3232

FORM 3 ODD ν

TABLE 11

$\alpha = 1$ SYMMETRICAL LOAD

$\alpha_\nu = \alpha_{-\nu}$
$\gamma_\nu = \gamma_{-\nu}$

1 × 15
STATIONS

a) VALUES $A_{\nu n}$

$$A_{\nu n} = \frac{a_{\nu n}\, i_{\nu n}}{i_{\nu\nu}} + \frac{a_{\nu-n}\, i_{\nu-n}}{i_{\nu\nu}} \qquad n \neq 0$$

$$A_{\nu n} = \frac{a_{\nu n}\, i_{\nu n}}{i_{\nu\nu}} \qquad \text{for } n = 0$$

ν	1	3	5	7
n = 0	·3915	·0387$_4$	·0113$_7$	·0028$_9$
2	·3372	·3996	·0410$_3$	·0084$_4$
4	·0215$_8$	·3044	·3908	·0280
6	·0050$_7$	·0158$_7$	·3189	·3617

b) SOLUTION[1] BY ITERATION:

γ_n or $\Delta\gamma_n$ see FORM 3, EVEN ν, except initial values.

INITIAL GUESS:
$\gamma_0^{(0)} = \cdot 47$
$\gamma_2^{(0)} = \cdot 46$
$\gamma_4^{(0)} = \cdot 40$
$\gamma_6^{(0)} = \cdot 23$

INITIAL DIFFERENCES:
$\Delta^{(1)}\gamma_0 = \gamma_0^{(1)} - \gamma_0^{(0)} = -\cdot 0062$
$\Delta^{(1)}\gamma_2 = +\cdot 0056$
$\Delta^{(1)}\gamma_4 = -\cdot 0110$
$\Delta^{(1)}\gamma_6 = +\cdot 0094$

FIRST APPROXIMATION:

ν	1	3	5	7
$A_\nu \alpha_\nu$	·1280	·1081	·0726	·0258
$A_{\nu 0}\gamma_0^{(0)}$	·1840	·0182	·0053$_4$	·0013$_6$
$A_{\nu 2}\gamma_2^{(0)}$	·1551	·1839	·0188$_8$	·0038$_8$
$A_{\nu 4}\gamma_4^{(0)}$	·0086$_4$	·1217	·1563	·0112
$A_{\nu 6}\gamma_6^{(0)}$	·0011$_7$	·0036$_5$	·0734	·0832
$\gamma_\nu^{(1)}$	·4769	·4356	·3265	·1254

FURTHER DIFFERENCES:

ν	1	3	5	7
$A_{\nu 0}\Delta^{(1)}\gamma_0$	−·00243	−·00024	−·00007	−·00002
$A_{\nu 2}\Delta^{(1)}\gamma_2$	+·00189	+·00224	+·00023	+·00005
$A_{\nu 4}\Delta^{(1)}\gamma_4$	−·00024	−·00335	−·00430	−·00031
$A_{\nu 6}\Delta^{(1)}\gamma_6$	+·00005	+·00015	+·00300	+·00340
$\Delta^{(2)}\gamma_\nu$	−·00073	−·00120	−·00110	+·00313

$A_{\nu 0}\Delta^{(2)}\gamma_0$	−·00020	−·00002	−·00001	—
$A_{\nu 2}\Delta^{(2)}\gamma_2$	−·00023	−·00028	−·00003	−·00001
$A_{\nu 4}\Delta^{(2)}\gamma_4$	−·00002	−·00025	−·00032	−·00002
$A_{\nu 6}\Delta^{(2)}\gamma_6$	—	+·00001	+·00017	+·00020
$\Delta^{(3)}\gamma_\nu$	−·00045	−·00054	−·00019	+·00017

$A_{\nu 0}\Delta^{(3)}\gamma_0$	−·00013	−·00001	—	—
$A_{\nu 2}\Delta^{(3)}\gamma_2$	−·00012	−·00014	−·00001	—
$A_{\nu 4}\Delta^{(3)}\gamma_4$	−·00001	−·00009	−·00011	−·00001
$A_{\nu 6}\Delta^{(3)}\gamma_6$	—	—	−·00001	−·00001
$\Delta^{(4)}\gamma_\nu$	−·00026	−·00024	−·00013	−·00002

$A_{\nu 0}\Delta^{(4)}\gamma_0$	−·00007	−·00001	—	—
$A_{\nu 2}\Delta^{(4)}\gamma_2$	−·00006	−·00007	−·00001	—
$A_{\nu 4}\Delta^{(4)}\gamma_4$	—	−·00004	−·00005	—
$A_{\nu 6}\Delta^{(4)}\gamma_6$	—	—	−·00002	−·00003
$\Delta^{(5)}\gamma_\nu$	−·00013	−·00012	−·00008	−·00003

$A_{\nu 0}\Delta^{(5)}\gamma_0$	−·00003	—	—	—
$A_{\nu 2}\Delta^{(5)}\gamma_2$	−·00003	−·00004	—	—
$A_{\nu 4}\Delta^{(5)}\gamma_4$	—	−·00002	−·00003	—
$A_{\nu 6}\Delta^{(5)}\gamma_6$	—	—	−·00001	−·00001
$\Delta^{(6)}\gamma_\nu$	−·00006	−·00006	−·00004	−·00001

SUMMARY:

$$\gamma_\nu = \gamma_\nu^{(1)} + \Delta^{(2)}\gamma_\nu + \Delta^{(3)}\gamma_\nu + \cdots\cdots$$

ν	1	3	5	7
$\gamma_\nu^{(1)}$	·4769	·4356	·3265	·1254
$\Delta^{(2)}\gamma_\nu$	−·0007$_3$	−·0012	−·0011	+·0031$_3$
$\Delta^{(3)}\gamma_\nu$	−·0004$_5$	−·0005$_4$	−·0001$_9$	+·0001$_7$
$\Delta^{(4)}\gamma_\nu$	−·0002$_6$	−·0002$_4$	−·0001$_3$	−·0000$_2$
$\Delta^{(5)}\gamma_\nu$	−·0001$_3$	−·0001$_2$	−·0000$_8$	−·0000$_3$
$\Delta^{(6)}\gamma_\nu$	−·0000$_6$	−·0000$_6$	−·0000$_4$	−·0000$_1$
γ_ν	·4752	·4334	·3249	·1286

CHECK OF RESULTS:

ν	1	3	5	7
$A_\nu \alpha_\nu$	·1280	·1081	·0726	·0258
$A_{\nu 0}\gamma_0$	·1810$_7$	·0179$_2$	·0052$_6$	·0013$_4$
$A_{\nu 2}\gamma_2$	·1565	·1864$_5$	·0190$_5$	·0039$_2$
$A_{\nu 4}\gamma_4$	·0083$_6$	1180$_2$	·1515$_1$	·0108$_5$
$A_{\nu 6}\gamma_6$	·0012$_2$	·0038$_0$	·0764$_4$	·0867$_0$
γ_ν	·4752	·4333	·3249	·1286

[1] Solution for the symmetrical case:

$$\gamma_\nu = A_\nu \alpha_\nu + \sum_0^{\frac{m-1}{2}}{}' A_{\nu n}\gamma_n \qquad\qquad A_{\nu n} = 0 \text{ for } |n-\nu| = 2, 4, \cdots$$

TABLE 12

FORM 3 EVEN ν

$\alpha = 1$ SYMMETRICAL LOAD

$\alpha_\nu = \alpha_{-\nu}$
$\gamma_\nu = \gamma_{-\nu}$

1 × 15
STATIONS

a) VALUES $A_{\nu n}$

$$A_{\nu n} = \frac{a_{\nu n}\, i_{\nu n}}{i_{\nu\nu}} + \frac{a_{\nu -n}\, i_{\nu -n}}{i_{\nu\nu}} \qquad \nu \neq 0$$

$$A_{\nu n} = 2\,\frac{\alpha_{\nu n}\, i_{\nu n}}{i_{\nu\nu}} \qquad \text{for } \nu = 0$$

ν	0	2	4	6
$n =$ 1	·6600	·4274	·0492	$\cdot 0139_8$
3	$\cdot 0348_6$	·3104	·3931	·0361
5	$\cdot 0078_2$	·0177	·3077	·3859
7	$\cdot 0018_2$	$\cdot 0031_1$	$\cdot 0136_3$	·3234

b) SOLUTION BY ITERATION: γ_n or $\Delta\gamma_n$ see FORM 3, ODD ν.

FIRST APPROXIMATION:

ν	0	2	4	6
$A_\nu \alpha_\nu$	·1308	·1203	·0921	·0503
$A_{\nu 1}\gamma_1^{①}$	·3150	·2040	$\cdot 0234_7$	$\cdot 0066_7$
$A_{\nu 3}\gamma_3^{①}$	$\cdot 0151_8$	$\cdot 1351_5$	·1712	$\cdot 0157_3$
$A_{\nu 5}\gamma_5^{①}$	$\cdot 0025_5$	$\cdot 0057_8$	·1005	$\cdot 1260_5$
$A_{\nu 7}\gamma_7^{①}$	$\cdot 0002_3$	$\cdot 0003_9$	$\cdot 0017_1$	·0406
$\gamma_\nu^{①}$	·4638	·4656	·3890	·2394

DIFFERENCES:

ν	0	2	4	6
$A_{\nu 1}\overset{②}{\Delta}\gamma_1$	−·00048	−·00031	−·00004	−·00001
$A_{\nu 3}\overset{②}{\Delta}\gamma_3$	−·00004	−·00037	−·00047	−·00004
$A_{\nu 5}\overset{②}{\Delta}\gamma_5$	−·00001	−·00002	−·00034	−·00042
$A_{\nu 7}\overset{②}{\Delta}\gamma_7$	+·00001	+·00001	+·00004	+·00101
$\overset{②}{\Delta}\gamma_\nu$	−·00052	−·00069	−·00081	+·00054
$A_{\nu 1}\overset{③}{\Delta}\gamma_1$	−·00030	−·00019	−·00002	−·00001
$A_{\nu 1}\overset{③}{\Delta}\gamma_3$	−·00002	−·00017	−·00021	−·00002
$A_{\nu 5}\overset{③}{\Delta}\gamma_5$	—	—	−·00006	−·00007
$A_{\nu 7}\overset{③}{\Delta}\gamma_7$	—	—	—	+·00006
$\overset{③}{\Delta}\gamma_\nu$	−·00032	−·00036	−·00029	−·00004
$A_{\nu 1}\overset{④}{\Delta}\gamma_1$	−·00017	−·00011	−·00001	—
$A_{\nu 3}\overset{④}{\Delta}\gamma_3$	−·00001	−·00007	−·00009	−·00001
$A_{\nu 5}\overset{④}{\Delta}\gamma_5$	—	—	−·00004	−·00005
$A_{\nu 7}\overset{④}{\Delta}\gamma_7$	—	—	—	−·00001
$\overset{④}{\Delta}\gamma_\nu$	−·00018	−·00018	−·00014	−·00007
$A_{\nu 1}\overset{⑤}{\Delta}\gamma_1$	−·00009	−·00006	−·00001	—
$A_{\nu 3}\overset{⑤}{\Delta}\gamma_3$	—	−·00004	−·00005	—
$A_{\nu 5}\overset{⑤}{\Delta}\gamma_5$	—	—	−·00002	−·00003
$A_{\nu 7}\overset{⑤}{\Delta}\gamma_7$	—	—	—	−·00001
$\overset{⑤}{\Delta}\gamma_\nu$	−·00009	−·00009	−·00008	−·00004
$A_{\nu 1}\overset{⑥}{\Delta}\gamma_1$	−·00004	−·00003	—	—
$A_{\nu 3}\overset{⑥}{\Delta}\gamma_3$	—	−·00002	−·00002	—
$A_{\nu 5}\overset{⑥}{\Delta}\gamma_5$	—	—	−·00001	−·00002
$A_{\nu 7}\overset{⑥}{\Delta}\gamma_7$	—	—	—	—
$\overset{⑥}{\Delta}\gamma_\nu$	−·00004	−·00005	−·00003	−·00002

SUMMARY:

$$\gamma_\nu = \gamma_\nu^{①} + \overset{②}{\Delta}\gamma_\nu + \overset{③}{\Delta}\gamma_\nu + \cdots$$

ν	0	2	4	6
$\gamma_\nu^{①}$	·4638	·4656	·3890	·2394
$\overset{②}{\Delta}\gamma_\nu$	$-\cdot 0005_2$	$-\cdot 0006_9$	$-\cdot 0008_1$	$+\cdot 0005_4$
$\overset{③}{\Delta}\gamma_\nu$	$-\cdot 0003_2$	$-\cdot 0003_6$	$-\cdot 0002_9$	$-\cdot 0000_4$
$\overset{④}{\Delta}\gamma_\nu$	$-\cdot 0001_8$	$-\cdot 0001_8$	$-\cdot 0001_4$	$-\cdot 0000_7$
$\overset{⑤}{\Delta}\gamma_\nu$	$-\cdot 0000_9$	$-\cdot 0000_9$	$-\cdot 0000_8$	−·00004
$\overset{⑥}{\Delta}\gamma_\nu$	$-\cdot 0000_4$	$-\cdot 0000_5$	$-\cdot 0000_3$	$-\cdot 0000_2$
γ_ν	·4626	·4642	·3876	·2397

CHECK OF RESULTS:

ν	0	2	4	6
$A_\nu \alpha_\nu$	·1308	·1203	·0921	·0503
$A_{\nu 1}\gamma_1$	$\cdot 3135_6$	$\cdot 2030_6$	$\cdot 0233_8$	$\cdot 0066_4$
$A_{\nu 3}\gamma_3$	$\cdot 0151_0$	$\cdot 1344_7$	$\cdot 1703_0$	$\cdot 0156_4$
$A_{\nu 5}\gamma_5$	·00254	$\cdot 0057_5$	$\cdot 0999_1$	$\cdot 1253_0$
$A_{\nu 7}\gamma_7$	$\cdot 0002_3$	$\cdot 0004_0$	$\cdot 0017_5$	$\cdot 0415_9$
γ_ν	·4622	·4640	·3876	·2395

c) $$\frac{dC_L}{d\alpha} = \frac{2\pi A}{m+1}\left[\cdot 5\,\gamma_0 + \sum_{1}^{\frac{m-1}{2}} \gamma_n \cos\frac{n\pi}{m+1}\right] = \frac{2\pi 4}{16}\left[\begin{array}{l} \cdot 5 \times \cdot 4622 \\ \cdot 9808 \times \cdot 4752 \\ \cdot 9239 \times \cdot 4640 \\ \cdot 8315 \times \cdot 4333 \\ \cdot 7071 \times \cdot 3876 \\ \cdot 5556 \times \cdot 3249 \\ \cdot 3827 \times \cdot 2395 \\ \cdot 1951 \times \cdot 1286 \end{array}\right]$$

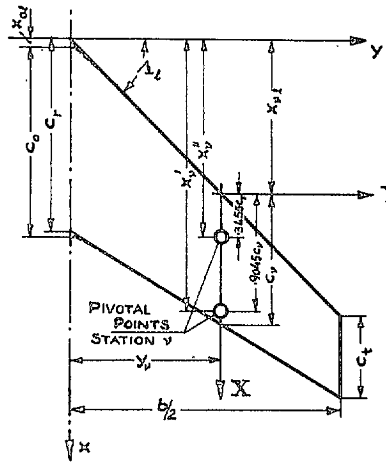
$$= \frac{\pi}{2} \times 2{\cdot}0575$$

$$\frac{dC_L}{d\alpha} = \underline{\underline{3{\cdot}232}}$$

TABLE 13

EXAMPLE II.

2 CHORDWISE } PIVOTAL STATIONS
15 SPANWISE }



SYMMETRICAL SWEPT BACK WING

SPAN	$b = 20$ ft
ASPECT RATIO	$A = 4$
ROOT CHORD	$c_r = 7$ ft
TIP CHORD	$c_t = 3$ ft
L.E. SWEEP BACK	$\Lambda_l = 45°$
TAPER RATIO	$c_t/c_r = \cdot 429$

LOW MACH NUMBER
$\sqrt{1-M^2} \approx 1$

FORM I.

$\lvert\nu\rvert$ or $\lvert n\rvert$	0	1	2	3	4	5	6	7
η_ν 1)	0	·1951	·3827	·5556	·7071	·8315	·9239	·9808
y_ν	0	1·951	3·827	5·556	7·071	8·315	9·239	9·808
x_{nl} 2)	$\cdot 325_2$	1·951	3·827	5·556	7·071	8·315	9·239	9·808
c_n 2)	6·8699	6·2196	5·4692	4·7776	4·1716	3·6740	3·3044	3·0768
x'_ν 2)	$6\cdot 539_0$	7·5766	8·7739	9·8773	10·8442	11·6381	12·2278	12·5910
x''_ν 2)	2·6987	4·0999	5·7166	7·2067	8·5123	9·5844	10·3807	10·8710
$\frac{b}{2c_n}$	1·4556	1·608	1·827	2·093	2·397	2·722	3·027	3·250
$\frac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1}\cos\frac{\nu\pi}{m+1}\left(\frac{b}{2c_\nu}\right)^2$	3) ·05168	·06066	·06950	·07386	·07004	·05579	·03272	·00980
$\bar{i}'_{\nu\nu}$ 4)	2·0064	$2\cdot 012_0$	2·0175	$2\cdot 020_2$	2·0179	$2\cdot 009_0$	$1\cdot 994_6$	$1\cdot 980_3$
$\bar{j}'_{\nu\nu}$ 4)	·0376	−·0056	−·0480	$-\cdot 069_0$	−·0506	·0178	·1287	·2388
$\bar{i}''_{\nu\nu}$ 4)	$1\cdot 457_6$	1·4667	$1\cdot 475_6$	$1\cdot 480_0$	$1\cdot 476_1$	$1\cdot 461_8$	$1\cdot 438_5$	$1\cdot 415_4$
$\bar{j}''_{\nu\nu}$ 4)	$3\cdot 467_8$	$3\cdot 519_5$	$3\cdot 570_4$	$3\cdot 595_5$	$3\cdot 573_5$	$3\cdot 491_5$	$3\cdot 358_6$	$3\cdot 226_6$
$\bar{i}'_{\nu\nu}\bar{j}''_{\nu\nu}-\bar{i}''_{\nu\nu}\bar{j}'_{\nu\nu}$	$6\cdot 903_0$	$7\cdot 089_4$	$7\cdot 274_1$	$7\cdot 365_7$	$7\cdot 285_6$	$6\cdot 988_4$	$6\cdot 513_9$	$6\cdot 051_6$
l'_ν 5)	$\cdot 5023_6$	$\cdot 4964_4$	$\cdot 4908_4$	$\cdot 4881_4$	$\cdot 4904_9$	$\cdot 4996_1$	$\cdot 5156_1$	$\cdot 5331_8$
l''_ν 5)	·00545	−·00079	−·00660	−·00937	−·00695	·00255	·01976	$\cdot 0394_6$
m''_ν 5)	$\cdot 2906_6$	·2838	$\cdot 2773_5$	$\cdot 2742_7$	$\cdot 2769_7$	$\cdot 2874_8$	$\cdot 3062_1$	$\cdot 3272_4$
m'_ν 5)	$\cdot 2111_5$	$\cdot 2068_9$	$\cdot 2028_6$	$\cdot 2009_3$	$\cdot 2026_1$	$\cdot 2091_8$	$\cdot 2208_4$	$\cdot 2338_9$

1) η_n for $m = 15$ see TABLE 5.

2) In the plane of symmetry $(n = 0)$ the kink is rounded off according to the following rules:
$x_{0l} = \frac{1}{6}x_{1l}$; $c_0 = c_r - \frac{1}{6}(c_r - c_1)$; $x'_0 = x_{0l} + \cdot 9045\, c_0$, $x''_0 = x_{0l} + \cdot 3455\, c_0$

3) $\frac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1}\cos\frac{\nu\pi}{m+1}$ for $m = 15$ see TABLE 5.

4) $\bar{i}'_{\nu\nu} = 1\cdot 974_2 + \cdot 623_4 \frac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1}\cos\frac{\nu\pi}{m+1}\left(\frac{b}{2c_\nu}\right)^2$ | $\bar{i}''_{\nu\nu} = 1\cdot 405_5 + 1\cdot 008_7 \frac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1}\cos\frac{\nu\pi}{m+1}\left(\frac{b}{2c_\nu}\right)^2$

$\bar{j}'_{\nu\nu} = \cdot 2859 - 4\cdot 805 \frac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1}\cos\frac{\nu\pi}{m+1}\left(\frac{b}{2c_\nu}\right)^2$ | $\bar{j}''_{\nu\nu} = 3\cdot 170_2 + 5\cdot 758 \frac{\eta_{\nu+1}-\eta_{\nu-1}}{m+1}\cos\frac{\nu\pi}{m+1}\left(\frac{b}{2c_\nu}\right)^2$

5) $l'_\nu = \frac{\bar{j}''_{\nu\nu}}{\bar{i}'_{\nu\nu}\bar{j}''_{\nu\nu}-\bar{i}''_{\nu\nu}\bar{j}'_{\nu\nu}}$ $\qquad m''_\nu = \frac{\bar{i}'_{\nu\nu}}{\bar{i}'_{\nu\nu}\bar{j}''_{\nu\nu}-\bar{i}''_{\nu\nu}\bar{j}'_{\nu\nu}}$

$l''_\nu = \frac{\bar{j}'_{\nu\nu}}{\bar{i}'_{\nu\nu}\bar{j}''_{\nu\nu}-\bar{i}''_{\nu\nu}\bar{j}'_{\nu\nu}}$ $\qquad m'_\nu = \frac{\bar{i}''_{\nu\nu}}{\bar{i}'_{\nu\nu}\bar{j}''_{\nu\nu}-\bar{i}''_{\nu\nu}\bar{j}'_{\nu\nu}}$

FORM 2

TABLE 14

2 × 15 STATIONS

$n = 0$ ν	−7	−5	−3	−1	1	3	5	7
$\lvert\eta_\nu - \eta_n\rvert$	·9808	·8315	·5556	·1951	·1951	·5556	·8315	·9808
$x'_\nu - x_{nl}$	12·2658	11·3129	9·5521	7·2514	7·2514	9·5521	11·3129	12·2658
$x''_\nu - x_{nl}$	10·5458	9·2592	6·8815	3·7747	3·7747	6·8815	9·2592	10·5458
$\lvert Y_{\nu n}\rvert$ [1]	1·428	1·210	·8087	·2840	·2840	·8087	1·210	1·428
$X'_{\nu n}$ [2]	1·786	1·646	1·390	1·056	1·056	1·390	1·646	1·786
$X''_{\nu n}$	1·535	1·347	1·002	·5495	·5495	1·002	1·347	1·535
$i'_{\nu n}$ [3]	1·725	1·747	1·800	1·9037	1·9037	1·800	1·747	1·725
$i''_{\nu n}$	1·659	1·659	1·648	1·5617	1·5617	1·648	1·659	1·659
$j'_{\nu n}$ [3]	·260	·281	·328	·341	·341	·328	·281	·260
$j''_{\nu n}$	·337	·402	·635	1·846	1·846	·635	·402	·337
$a_{\nu n} i'_{\nu n}$ [4]	·00552	·02202	·0758	·7665	·7665	·0758	·02202	·00552
$a_{\nu n} i''_{\nu n}$	·00531	·02088	·06937	·6285	·6285	·06937	·02088	·00531
$a_{\nu n} j'_{\nu n}$	·00083	·00354	·01380	·1374	·1374	·01380	·00354	·00083
$a_{\nu n} j''_{\nu n}$	·00108	·00507	·02673	·7430	·7430	·02673	·00507	·00108
$a_{\nu n} l'_\nu i'_{\nu n}$	·00295	·01099	·0370	·3807	·3807	·0370	·01099	·00295
$a_{\nu n} l''_\nu i''_{\nu n}$	·00021	·00005	−·00065	−·0005$_0$	−·0005$_0$	−·00065	·00005	·00021
$a_{\nu n}(l'_\nu i'_{\nu n} - l''_\nu i''_{\nu n})$	·0027$_4$	·0109$_4$	·0376$_5$	·3812	·3812	·0376$_5$	·0109$_4$	·0027$_4$
$a_{\nu n} l'_\nu j'_{\nu n}$	·00044	·00177	·00674	·06827	·06827	·00674	·00177	·00044
$a_{\nu n} l''_\nu j''_{\nu n}$	·00004	·00001	−·00025	−·00059	−·00059	−·00025	·00001	·00004
$a_{\nu n}(l'_\nu j'_{\nu n} - l''_\nu j''_{\nu n})$	·0004	·0017$_6$	·0070	·0688$_6$	·0688$_6$	·0070	·0017$_6$	·0004
$a_{\nu n} m''_\nu i''_{\nu n}$	·00174	·00601	·01903	·1785	·1785	·01903	·00601	·00174
$a_{\nu n} m'_\nu i'_{\nu n}$	·00124	·00461	·01524	·1587	·1587	·01524	·00461	·00124
$a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n})$	·0005	·0014	·0038	·0198	·0198	·0038	·0014	·0005
$a_{\nu n} m''_\nu j''_{\nu n}$	·00035	·00146	·00733	·2110	·2110	·00733	·00146	·00035
$a_{\nu n} m'_\nu j'_{\nu n}$	·00019	·00074	·00278	·02847	·02847	·00278	·00074	·00019
$a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n})$	·0001$_6$	·0007$_2$	·0045$_5$	·1825	·1825	·0045$_5$	·0007$_2$	·0001$_6$

$n = 2$ ν	−7	−5	−3	−1	1	3	5	7
$n = -2$ ν	7	5	3	1	−1	−3	−5	−7
$\lvert\eta_\nu - \eta_n\rvert$	1·3635	1·2142	·9383	·5778	·1876	·1729	·4488	·5981
$x'_\nu - x_{nl}$	8·764	7·8111	6·0503	3·7496	3·7496	6·0503	7·8111	8·764
$x''_\nu - x_{nl}$	7·044	5·7574	3·3797	·2729	·2729	3·3797	5·7574	7·044
$\lvert Y_{\nu n}\rvert$	2·492	2·220	1·714	1·056	·3427	·316	·820	1·092
$X'_{\nu n}$	1·601	1·426	1·105	·685	·685	1·105	1·426	1·601
$X''_{\nu n}$	1·287	1·051	·6175	·0499	·0499	·6175	1·051	1·287
$i'_{\nu n}$	1·471	1·462	1·439	1·3625	1·6685	1·9127	1·805	1·768
$i''_{\nu n}$	1·382	1·334	1·206	·830	·6535	1·6195	1·671	1·673
$j'_{\nu n}$	·290	·330	·452	·804	1·343	·303	·311	·282
$j''_{\nu n}$	·326	·391	·559	·802	1·409	1·585	·588	·429
$a_{\nu n} i'_{\nu n}$	·00221	·00790	·01957	·0578	·6715	·767	·0718	·01397
$a_{\nu n} i''_{\nu n}$	·00207	·00721	·01639	·03522	·2630	·650	·0665	·01322
$a_{\nu n} j'_{\nu n}$	·00043	·00178	·00615	·03412	·5405	·1216	·01237	·00223
$a_{\nu n} j''_{\nu n}$	·00049	·00211	·00760	·03404	·5670	·636	·02340	·00339
$a_{\nu n} l'_\nu i'_{\nu n}$	·00118	·00395	·00956	·02870	·3333	·3747	·03588	·00745
$a_{\nu n} l''_\nu i''_{\nu n}$	·00008	·00002	−·00015	−·00003	−·00021	−·00609	·00017	·00052
$a_{\nu n}(l'_\nu i'_{\nu n} - l''_\nu i''_{\nu n})$	·0011	·0039$_3$	·0097$_1$	·0287$_3$	·3335	·3808	·0357$_1$	·0069$_3$
$a_{\nu n} l'_\nu j'_{\nu n}$	·00023	·00089	·00301	·01694	·2685	·0594	·00618	·00119
$a_{\nu n} l''_\nu j''_{\nu n}$	·00002	·00001	−·00007	−·00003	−·00045	−·00595	·00006	·00013
$a_{\nu n}(l'_\nu j'_{\nu n} - l''_\nu j''_{\nu n})$	·0002$_1$	·0008$_8$	·0030$_8$	·0169$_7$	·2689	·0653$_5$	·0061$_2$	·0010$_6$
$a_{\nu n} m''_\nu i''_{\nu n}$	·00068	·00207	·00450	·01000	·0747	·1784	·01913	·00433
$a_{\nu n} m'_\nu i'_{\nu n}$	·00052	·00165	·00394	·01197	·1391	·1543	·01503	·00327
$a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n})$	·0001$_6$	·0004$_2$	·0005$_6$	−·0019$_7$	−·0644	·0241	·0041	·0010$_6$
$a_{\nu n} m''_\nu j''_{\nu n}$	·00016	·00061	·00209	·00966	·1611	·1746	·00673	·00111
$a_{\nu n} m'_\nu j'_{\nu n}$	·00010	·00037	·00124	·00706	·1119	·02545	·00259	·00052
$a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n})$	·0000$_6$	·0002$_4$	·0008$_5$	·0026	·0492	·1492	·0041$_4$	·0005$_9$

1) $\lvert Y_{\nu n}\rvert = \frac{b}{2c_n}\lvert\eta_\nu - \eta_n\rvert$ 2) $X_{\nu n} = \frac{1}{c_n}(x_\nu - x_{nl})$ 4) $a_{\nu n}$ for $m = 15$ see TABLE 5.

3) $i_{\nu n}$ and $j_{\nu n}$ as functions of $(X_{\nu n}, Y_{\nu n})$ take from CHARTS 1 ··· 6.

$n=4$ ν	-7	-5	-3	-1	1	3	5	7
$n=-4$ ν	7	5	3	1	-1	-3	-5	-7
$\lvert\eta_\nu - \eta_n\rvert$	1·6879	1·5386	1·2627	·9022	·5120	·1515	·1244	·2737
$x'_\nu - x_{n\ell}$	5·520	4·5671	2·8063	·5056	·5056	2·8063	4·5671	5·520
$x''_\nu - x_{n\ell}$	3·800	2·5134	·1357	−2·9711	−2·9711	·1357	2·5134	3·800
$\lvert Y_{\nu n}\rvert$	4·047	3·69	3·027	2·16	1·227	·3633	·2983	·656
$X'_{\nu n}$	1·323	1·095	·6725	·1212	·1212	·6725	1·095	1·323
$X''_{\nu n}$	·911	·6025	·0325	−·712	−·712	·0325	·6025	·911
$i'_{\nu n}$	1·257	1·224	1·140	·940	·902	1·6445	1·9173	1·836
$i''_{\nu n}$	1·16	1·095	1·025	·600	·395	·630	1·6145	1·666
$j'_{\nu n}$	·228	·256	·320	·446	·740	1·363	·300	·320
$j''_{\nu n}$	·242	·269	·322	·330	·350	1·321	1·655	·757
$a_{\nu n} i'_{\nu n}$	·00101	·00318	·00661	·0125	·0373	·6577	·761	·05285
$a_{\nu n} i''_{\nu n}$	·00093	·00285	·00595	·00798	·01633	·2522	·641	·0480
$a_{\nu n} j'_{\nu n}$	·00018	·00067	·00185	·00592	·0306	·545	·1190	·00922
$a_{\nu n} j''_{\nu n}$	·00019	·00070	·00187	·00439	·0145	·528	·6567	·0218
$a_{\nu n} \ell'_\nu i'_{\nu n}$	·00054	·00159	·00323	·00621	·01853	·3212	·3802	·02817
$a_{\nu n} \ell''_\nu i''_{\nu n}$	·00004	·00001	−·00006	−·00001	−·00001	−·00236	·00163	·00189
$a_{\nu n}(\ell'_\nu i'_{\nu n} - \ell''_\nu i''_{\nu n})$	·0005	·0015$_{8}$	·0032$_{9}$	·0062$_{2}$	·0185$_{4}$	·3235	·3785$_{7}$	·0262$_{8}$
$a_{\nu n} \ell'_\nu j'_{\nu n}$	·00010	·00033	·00090	·00294	·01520	·2662	·05947	·00492
$a_{\nu n} \ell''_\nu j''_{\nu n}$	·00001	—	−·00002	—	−·00001	−·00494	·00167	·00086
$a_{\nu n}(\ell'_\nu j'_{\nu n} - \ell''_\nu j''_{\nu n})$	·0000$_{9}$	·0003$_{3}$	·0009$_{2}$	·0029$_{4}$	·0152$_{1}$	·2711$_{4}$	·0578	·0040$_{6}$
$a_{\nu n} m''_\nu i''_{\nu n}$	·00030	·00082	·00163	·00227	·00464	·06925	·18435	·01570
$a_{\nu n} m'_\nu i'_{\nu n}$	·00024	·00067	·00133	·00259	·00773	·13225	·15925	·01237
$a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n})$	·0000$_{6}$	·0001$_{5}$	·0003	−·0003$_{2}$	−·0031	−·0630	·0251	·0033$_{3}$
$a_{\nu n} m''_\nu j''_{\nu n}$	·00006	·00020	·00051	·00125	·00412	·1448	·1839	·00713
$a_{\nu n} m'_\nu j'_{\nu n}$	·00004	·00014	·00037	·00123	·00634	·1096	·02488	·00216
$a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n})$	·0000$_{2}$	·0000$_{6}$	·0001$_{4}$	·0000$_{2}$	−·0022$_{2}$	·0352	·1640	·0049$_{7}$

$n=6$ ν	-7	-5	-3	-1	1	3	5	7
$n=-6$ ν	7	5	3	1	-1	-3	-5	-7
$\lvert\eta_\nu - \eta_n\rvert$	1·9047	1·7554	1·4795	1·1190	·7288	·3683	·0924	·0569
$x'_\nu - x_{n\ell}$	3·352	2·3991	·6383	−1·6624	−1·6624	·6383	2·3991	3·352
$x''_\nu - x_{n\ell}$	1·632	·3454	−2·0323	−5·1391	−5·1391	−2·0323	·3454	1·632
$\lvert Y_{\nu n}\rvert$	5·76	5·31	4·47	3·385	2·207	1·114	·2797	·1723
$X'_{\nu n}$	1·014	·726	·193	−·503	−·503	·193	·726	1·014
$X''_{\nu n}$	·494	·1045	−·615	−1·554	−1·554	−·615	·1045	·494
$i'_{\nu n}$	1·130	1·089	·988	·782	·683	·956	1·7423	1·9482
$i''_{\nu n}$	1·045	·930	·810	·530	·372	·402	·751	1·557
$j'_{\nu n}$	·170	·191	·223	·268	·361	·826	1·240	·258
$j''_{\nu n}$	·174	·190	·207	·191	·194	·388	1·743	2·286
$a_{\nu n} i'_{\nu n}$	·00034	·00120	·00227	·00368	·00752	·0351	·678	·7015
$a_{\nu n} i''_{\nu n}$	·00031	·00102	·00186	·00249	·00409	·01475	·2921	·561
$a_{\nu n} j'_{\nu n}$	·00005	·00021	·00051	·00126	·00397	·03032	·482	·0929
$a_{\nu n} j''_{\nu n}$	·00005	·00021	·00043	·00090	·00213	·01425	·678	·823
$a_{\nu n} \ell'_\nu i'_{\nu n}$	·00018	·00062	·00111	·00183	·00374	·01713	·3386	·3742
$a_{\nu n} \ell''_\nu i''_{\nu n}$	·00001	—	−·00002	—	—	−·00014	·0007$_{6}$	·0221
$a_{\nu n}(\ell'_\nu i'_{\nu n} - \ell''_\nu i''_{\nu n})$	·0001$_{7}$	·0006$_{2}$	·0011$_{3}$	·0018$_{3}$	·0037$_{4}$	·0172$_{7}$	·3379	·3521
$a_{\nu n} \ell'_\nu j'_{\nu n}$	·00003	·00011	·00025	·00063	·00197	·01480	·2408	·0496
$a_{\nu n} \ell''_\nu j''_{\nu n}$	—	—	—	—	—	−·00013	·0017$_{3}$	·0325
$a_{\nu n}(\ell'_\nu j'_{\nu n} - \ell''_\nu j''_{\nu n})$	·0000$_{3}$	·0001$_{1}$	·00025	·0006$_{3}$	·0019$_{7}$	·0149$_{3}$	·2391	·0171
$a_{\nu n} m''_\nu i''_{\nu n}$	·00010	·00029	·00051	·00071	·00116	·00405	·0840	·1834
$a_{\nu n} m'_\nu i'_{\nu n}$	·00008	·00025	·00046	·00076	·00156	·00706	·1418	·1643
$a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n})$	·0000$_{2}$	·0000$_{4}$	·0000$_{5}$	−·0000$_{5}$	−·0004	−·0030$_{1}$	−·0578	·0191
$a_{\nu n} m''_\nu j''_{\nu n}$	·00002	·00006	·00013	·00026	·00060	·00391	·1950	·2693
$a_{\nu n} m'_\nu j'_{\nu n}$	·00001	·00004	·00010	·00026	·00082	·00609	·1008	·0217$_{6}$
$a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n})$	·0000$_{1}$	·0000$_{2}$	·0000$_{3}$	—	−·0002$_{2}$	−·0021$_{0}$	·0942	·2475

FORM 2 ctd. TABLE 16 2 × 15 STATIONS

$n = 1$	ν	−6	−4	−2	0	2	4	6
$n = -1$	ν	6	4	2	0	−2	−4	−6
$\lvert\eta_\nu - \eta_n\rvert$		1·1190	·9022	·5778	·1951	·1876	·5120	·7288
$x'_\nu - x_{nl}$		10·2768	8·8932	6·8229	4·588	6·8229	8·8932	10·2768
$x''_\nu - x_{nl}$		8·4297	6·5613	3·7656	·7477	3·7656	6·5613	8·4297
$\lvert Y_{\nu n}\rvert$		1·800	1·451	·930	·314	·3018	·824	1·173
$X'_{\nu n}$		1·654	1·430	1·098	·738	1·098	1·430	1·654
$X''_{\nu n}$		1·356	1·055	·606	·1203	·606	1·055	1·356
$i'_{\nu n}$		1·608	1·623	1·650	1·7313	1·917	1·805	1·759
$i''_{\nu n}$		1·516	1·478	1·336	·791	1·616	1·670	1·672
$j'_{\nu n}$		·305	·368	·549	1·186	·300	·308	·275
$j''_{\nu n}$		··377	·511	·928	1·731	1·640	·584	·399
$a_{\nu n} i'_{\nu n}$		·00757	·02157	·0700	·6967	·7717	·0746	·01935
$a_{\nu n} i''_{\nu n}$		·00713	·01966	·05665	·3185	·6500	·0690	·01839
$a_{\nu n} j'_{\nu n}$		·00143	·00490	·02330	·4775	·1207	·01272	·00303
$a_{\nu n} j''_{\nu n}$		·00177	·00680	·03937	·6967	·6595	·02412	·00439
$a_{\nu n} l'_\nu i'_{\nu n}$		·00391	·01058	·03436	·3502	·3787	·03660	·00998
$a_{\nu n} l''_\nu i''_{\nu n}$		·00014	−·00014	−·00037	·00174	−·00429	−·00048	·00036
$a_{\nu n}(l'_\nu i'_{\nu n} - l''_\nu i''_{\nu n})$		·0037$_7$	·0107$_2$	·0347$_2$	·3485	·3830	·0370$_8$	·0096$_2$
$a_{\nu n} l'_\nu j'_{\nu n}$		·00074	·00240	·01143	·2400	·05925	·00624	·00156
$a_{\nu n} l''_\nu j''_{\nu n}$		·00004	−·00005	−·00026	·0038	−·00435	−·00017	·00009
$a_{\nu n}(l'_\nu j'_{\nu n} - l''_\nu j''_{\nu n})$		·0007	·0024$_6$	·0116$_9$	·2362	·0636	·0064$_1$	·0014$_7$
$a_{\nu n} m''_\nu i''_{\nu n}$		·00218	·00544	·01570	·0926	·1803	·01913	·00563
$a_{\nu n} m'_\nu i'_{\nu n}$		·00167	·00437	·01422	·1472	·1566	·01512	·00428
$a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n})$		·0005$_1$	·0010$_7$	·0014$_8$	−·0546	·0237	·0040$_1$	·0013$_5$
$a_{\nu n} m''_\nu j''_{\nu n}$		·00054	·00188	·01091	·2025	·1828	·00667	·00134
$a_{\nu n} m'_\nu j'_{\nu n}$		·00032	·00099	·00473	·1008	·0245	·00257	·00067
$a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n})$		·0002$_2$	·0008$_9$	·0061$_8$	·1017	·1583	·0041	·0006$_7$

$n = 3$	ν	−6	−4	−2	0	2	4	6
$n = -3$	ν	6	4	2	0	−2	−4	−6
$\lvert\eta_\nu - \eta_n\rvert$		1·4795	1·2627	·9383	·5556	·1729	·1515	·3683
$x'_\nu - x_{nl}$		6·6718	5·2882	3·2179	·8030	3·2179	5·2882	6·6718
$x''_\nu - x_{nl}$		4·8247	2·9563	·1606	−2·8573	·1606	2·9563	4·8247
$\lvert Y_{\nu n}\rvert$		3·10	2·645	1·966	1·163	·362	·3173	·7715
$X'_{\nu n}$		1·397	1·108	·675	·168	·675	1·108	1·397
$X''_{\nu n}$		1·010	·619	·0336	−·599	·0336	·619	1·010
$i'_{\nu n}$		1·346	1·306	1·210	·937	1·6478	1·9110	1·815
$i''_{\nu n}$		1·237	1·138	·890	·424	·631	1·6203	1·669
$j'_{\nu n}$		·274	·333	·482	·786	1·356	·302	·311
$j''_{\nu n}$		·304	·369	·479	·399	1·325	1·078	·626
$a_{\nu n} i'_{\nu n}$		·00309	·00757	·01645	·03945	·661	·7645	·0666
$a_{\nu n} i''_{\nu n}$		·00285	·00661	·01210	·01785	·2532	·648	·06125
$a_{\nu n} j'_{\nu n}$		·00063	·00193	·00666	·03307	·544	·04315	·01141
$a_{\nu n} j''_{\nu n}$		·00070	·00214	·00652	·01680	·532	·1208	·02297
$a_{\nu n} l'_\nu i'_{\nu n}$		·00160	·00371	·00808	·01986	·3245	·3750	·03433
$a_{\nu n} l''_\nu i''_{\nu n}$		·00006	−·00005	−·00008	·00012	−·00167	−·0045	·00121
$a_{\nu n}(l'_\nu i'_{\nu n} - l''_\nu i''_{\nu n})$		·0016$_4$	·0037$_6$	·0081$_6$	·0197$_4$	·3262	·3795	·0331$_2$
$a_{\nu n} l'_\nu j'_{\nu n}$		·00032	·00095	·00322	·01666	·2672	·2113	·00589
$a_{\nu n} l''_\nu j''_{\nu n}$		·00001	−·00001	−·00004	·00011	−·0035$_1$	−·0008$_4$	·00045
$a_{\nu n}(l'_\nu j'_{\nu n} - l''_\nu j''_{\nu n})$		·0003$_1$	·0009$_6$	·0032$_6$	·0165$_5$	·2707	·2121	·0054$_4$
$a_{\nu n} m''_\nu i''_{\nu n}$		·00087	·00183	·00336	·00518	·07015	·1794	·01875
$a_{\nu n} m'_\nu i'_{\nu n}$		·00068	·00153	·00334	·00833	·1342	·1547	·01475
$a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n})$		·0001$_9$	·0003	·0000$_2$	−·0031$_5$	−·0640	·0247	·0040
$a_{\nu n} m''_\nu j''_{\nu n}$		·00021	·00059	·00181	·00488	·1474	·03347	·00703
$a_{\nu n} m'_\nu j'_{\nu n}$		·00014	·00039	·00133	·00698	·1105	·00874	·00252
$a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n})$		·0000$_7$	·0002	·0004$_8$	−·0021	·0369	·0247$_3$	·0045$_1$

FORM 2 ctd. TABLE 17 2 × 15 STATIONS

$n = 5$	ν	−6	−4	−2	0	2	4	6
$n = -5$	ν	6	4	2	0	−2	−4	−6
$\lvert\eta_\nu - \eta_n\rvert$		1·7554	1·5386	1·2142	·8315	·4488	·1244	·0924
$x'_\nu - x_{nl}$		3·9128	2·5292	·4589	−1·776	·4589	2·5292	3·9128
$x''_\nu - x_{nl}$		2·0657	·1973	−2·5984	−5·6163	−2·5984	·1973	2·0657
$\lvert Y_{\nu n}\rvert$		4·775	4·185	3·305	2·263	1·222	·3387	·2515
$X'_{\nu n}$		1·064	·688	·125	−·483	·125	·688	1·064
$X''_{\nu n}$		·562	·0537	−·707	−1·527	−·707	·0537	·562
$i'_{\nu n}$		1·167	1·103	·961	·698	·904	1·6743	1·9297
$i''_{\nu n}$		1·066	·950	·722	·388	·395	·660	1·5965
$j'_{\nu n}$		·202	·238	·298	·356	·746	1·336	·291
$j''_{\nu n}$		·210	·238	·260	·199	·351	1·427	1·869
$a_{\nu n} i'_{\nu n}$		·00128	·00287	·00519	·00880	·03597	·665	·751
$a_{\nu n} i''_{\nu n}$		·00117	·00247	·00390	·00488	01572	·2620	·621
$a_{\nu n} j'_{\nu n}$		·00022	·00062	·00161	·00448	·02968	·530	·1132
$a_{\nu n} j''_{\nu n}$		·00023	·00068	·00140	·00251	·01396	·5665	·727
$a_{\nu n} l'_\nu i'_{\nu n}$		·00066	·00141	·00255	·00443	·01766	·3262	·3873
$a_{\nu n} l''_\nu i''_{\nu n}$		·00002	−·00002	·00003	·00003	−·00010	−·0018	·0123
$a_{\nu n}(l'_\nu i'_{\nu n} - l''_\nu i''_{\nu n})$		·0006$_4$	·0014$_3$	·0025$_8$	·0044	·0177$_6$	·3280	·3750
$a_{\nu n} l'_\nu j'_{\nu n}$		·00011	·00030	·00079	·00226	·01456	·2600	·0584
$a_{\nu n} l''_\nu j''_{\nu n}$		—	—	−·00001	·00002	−·00009	−·00394	·0144
$a_{\nu n}(l'_\nu j'_{\nu n} - l''_\nu j''_{\nu n})$		·0001$_1$	·0003	·0008	·0022$_4$	·0146$_5$	·2639$_4$	·0440
$a_{\nu n} m''_\nu i''_{\nu n}$		·00036	·00068	·00108	·00143	·00436	·0726	·1900
$a_{\nu n} m'_\nu i'_{\nu n}$		·00028	·00058	·00105	·00187	·00731	·1346	·1657
$a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n})$		·0000$_8$	·0001	·0000$_3$	−·0004$_4$	−·0029$_5$	−·0620	·0243
$a_{\nu n} m''_\nu j''_{\nu n}$		·00007	·00019	·00039	·00073	·00388	·1568	·2227
$a_{\nu n} m'_\nu j'_{\nu n}$		·00005	·00013	·00033	·00095	·00603	·1073	·0250
$a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n})$		·0000$_2$	·0000$_6$	·0000$_6$	−·0002$_2$	−·0021$_8$	·0495	·1977

$n = 7$	ν	−6	−4	−2	0	2	4	6
$n = -7$	ν	6	4	2	0	−2	−4	−6
$\lvert\eta_\nu - \eta_n\rvert$		1·9047	1·6879	1·3635	·9808	·5981	·2737	·0569
$x'_\nu - x_{nl}$		2·4198	1·0362	−1·0341	−3·269	−1·0341	1·0362	2·4198
$x''_\nu - x_{nl}$		·5727	−1·2957	−4·0914	−7·1093	−4·0914	−1·2957	·5727
$\lvert Y_{\nu n}\rvert$		6·19	5·485	4·43	3·188	1·944	·890	·185
$X'_{\nu n}$		·786	·3368	−·3363	−1·063	−·3363	·3368	·786
$X''_{\nu n}$		·186	−·4213	−1·330	−2·308	−1·330	−·4213	·186
$i'_{\nu n}$		1·088	1·015	·869	·624	·715	1·0955	1·842
$i''_{\nu n}$		·989	·878	·666	·375	·373	·425	·9615
$j'_{\nu n}$		·160	·182	·217	·237	·423	1·037	·999
$j''_{\nu n}$		·160	·177	·185	·140	·222	·490	2·416
$a_{\nu n} i'_{\nu n}$		·00033	·00081	·00130	·00200	·00565	·03155	·6633
$a_{\nu n} i''_{\nu n}$		·00030	·00070	·00100	·00120	·00295	·01224	·3462
$a_{\nu n} j'_{\nu n}$		·00005	·00014	·00033	·00076	·00334	·02987	·3600
$a_{\nu n} j''_{\nu n}$		·00005	·00014	·00028	·00045	·00175	·01411	·8700
$a_{\nu n} l'_\nu i'_{\nu n}$		·00017	·00040	·00064	·00101	·00278	·01547	·3420
$a_{\nu n} l''_\nu i''_{\nu n}$		—	—	−·00001	·00001	−·00002	−·00009	·00684
$a_{\nu n}(l'_\nu i'_{\nu n} - l''_\nu i''_{\nu n})$		·0001$_7$	·0004$_0$	·0006$_5$	·0010	·0028	·0155$_6$	·3351$_6$
$a_{\nu n} l'_\nu j'_{\nu n}$		·00002	·00007	·00016	·00038	·00164	·01464	·1856
$a_{\nu n} l''_\nu j''_{\nu n}$		—	—	—	—	−·00001	−·00010	·0172
$a_{\nu n}(l'_\nu j'_{\nu n} - l''_\nu j''_{\nu n})$		·0000$_2$	·0000$_7$	·0001$_5$	·0003$_8$	·0016$_5$	·0147$_4$	·1684
$a_{\nu n} m''_\nu i''_{\nu n}$		·00009	·00019	·00028	·00035	·00082	·00339	·1060
$a_{\nu n} m'_\nu i'_{\nu n}$		·00007	·00016	·00026	·00042	·00115	·00639	·1465
$a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n})$		·0000$_2$	·0000$_3$	·0000$_2$	−·0000$_7$	−·0003$_3$	−·0030	−·0405
$a_{\nu n} m''_\nu j''_{\nu n}$		·00001	·00004	·00008	·00013	·00049	·00391	·2665
$a_{\nu n} m'_\nu j'_{\nu n}$		·00001	·00003	·00007	·00016	·00068	·00605	·0796
$a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n})$		—	·0000$_1$	·0000$_1$	−·0000$_3$	−·0001$_9$	−·0021$_4$	·1869

TABLE 18

FORM 3 ODD ν

$\alpha_\nu = \alpha_{-\nu}$
$\gamma_\nu = \gamma_{-\nu}$
$\mu_\nu = \mu_{-\nu}$

$\boxed{\alpha'_\nu = \alpha''_\nu = 1}$ **SYMMETRICAL LOAD**

2 × 15 STATIONS.

a) VALUES $B_{\nu n}$, $C_{\nu n}$, $D_{\nu n}$, $E_{\nu n}$.

$$B_{\nu n} = a_{\nu n}(\ell'_\nu i'_{\nu n} - \ell''_\nu i''_{\nu n}) + a_{\nu-n}(\ell'_\nu i'_{\nu-n} - \ell''_\nu i''_{\nu-n}) \quad n \neq 0$$
$$= a_{\nu n}(\ell'_\nu i'_{\nu n} - \ell''_\nu i''_{\nu n}) \quad n = 0$$

ν	1	3	5	7
n = 0	·3812	·0376$_5$	·0109$_4$	·0027$_4$
2	·3622	·3905	·0396$_4$	·0080$_3$
4	·0247$_6$	·3268	·3802	·0267$_8$
6	·0055$_7$	·0184	·3385	·3523

$$D_{\nu n} = a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n}) + a_{\nu-n}(m''_\nu i''_{\nu-n} - m'_\nu i'_{\nu-n}) \quad n \neq 0$$
$$= a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n}) \quad n = 0$$

ν	1	3	5	7
n = 0	·0198	·0038	·0014	·0005
2	–·0664	·0246$_6$	·0045$_2$	·0012$_2$
4	–·0034$_2$	–·0627	·0252$_5$	·0034
6	–·0004$_5$	–·0029$_6$	–·0577$_6$	·0191$_2$

$$C_{\nu n} = a_{\nu n}(\ell'_\nu j'_{\nu n} - \ell''_\nu j''_{\nu n}) + a_{\nu-n}(\ell'_\nu j'_{\nu-n} - \ell''_\nu j''_{\nu-n}) \quad n \neq 0$$
$$= a_{\nu n}(\ell'_\nu j'_{\nu n} - \ell''_\nu j''_{\nu n}) \quad n = 0$$

ν	1	3	5	7
n = 0	·0688$_6$	·0070	·0017$_6$	·0004
2	·2859	·0684	·0070	·0012$_7$
4	·0181$_5$	·2720$_6$	·0581$_3$	·0041$_5$
6	·0026	·0151$_8$	·2392	·0171$_3$

$$E_{\nu n} = a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n}) + a_{\nu-n}(m''_\nu j''_{\nu-n} - m'_\nu j'_{\nu-n}) \quad n \neq 0$$
$$= a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n}) \quad n = 0$$

ν	1	3	5	7
n = 0	·1825	·0045$_5$	·0007$_2$	·0006$_6$
2	·0518	·1500	·0043$_8$	·00065
4	–·0022	·03534	·1640$_6$	·0050
6	–·0002$_2$	–·0021$_5$	·0942	·2475

b) SOLUTION[1] BY ITERATION:

γ_n, μ_n or $\Delta\gamma_n, \Delta\mu_n$ see FORM 3, EVEN ν, except initial values.

INITIAL GUESS: γ-values taken from example I.

$\gamma_0^{(0)} = ·4622$ $\quad \gamma_4^{(0)} = ·3876$ $\quad \mu_0^{(0)} = -·05$

$\gamma_2^{(0)} = ·4640$ $\quad \gamma_6^{(0)} = ·2395$ $\quad \mu_2^{(0)} = \mu_4^{(0)} = \mu_6^{(0)} = 0$

FIRST APPROXIMATION:

ν	1	3	5	7
$a_{\nu\nu}(\ell'_\nu - \ell''_\nu)$	·1219	·1034	·0690	·0241
$B_{\nu 0}\gamma_0^{(0)}$	·1762	·0174	·0051	·0013
$B_{\nu 2}\gamma_2^{(0)}$	·1681	·1812	·0184	·0037
$B_{\nu 4}\gamma_4^{(0)}$	·0096	·1267	·1473	·0104
$B_{\nu 6}\gamma_6^{(0)}$	·0013	·0044	·0811	·0843
$C_{\nu 0}\mu_0^{(0)}$	–·0034	–·0004	–·0001	—
$\gamma_\nu^{(1)}$	·4737	·4327	·3208	·1238

ν	1	3	5	7
$a_{\nu\nu}(m''_\nu - m'_\nu)$	·01886	·01525	·01088	·00456
$D_{\nu 0}\gamma_0^{(0)}$	·00915	·00175	·00065	·00023
$D_{\nu 2}\gamma_2^{(0)}$	–·03081	·01144	·00210	·00057
$D_{\nu 4}\gamma_4^{(0)}$	–·00133	–·02430	·00979	·00131
$D_{\nu 6}\gamma_6^{(0)}$	–·00011	–·00071	–·01383	·00458
$E_{\nu 0}\mu_0^{(0)}$	–·00913	–·00023	–·00004	–·00001
$\mu_\nu^{(1)}$	–·01337	·00320	·00955	·01124

INITIAL DIFFERENCES:

$\Delta^{(1)}\gamma_0 = \gamma_0^{(1)} - \gamma_0^{(0)} = +·0062$ $\quad \Delta^{(1)}\gamma_4 = -·0002$ $\quad \Delta^{(1)}\mu_0 = \mu_0^{(1)} - \mu_0^{(0)} = +·01240$ $\quad \Delta^{(1)}\mu_4 = +·00661$

$\Delta^{(1)}\gamma_2 = +·0005$ $\quad \Delta^{(1)}\gamma_6 = -·0064$ $\quad \Delta^{(1)}\mu_2 = -·00163$ $\quad \Delta^{(1)}\mu_6 = +·01766$

FURTHER DIFFERENCES:

ν	1	3	5	7
$B_{\nu 0}\Delta^{(1)}\gamma_0$	·00235	·00023	·00007	·00002
$B_{\nu 2}\Delta^{(1)}\gamma_2$	·00017	·00018	·00002	—
$B_{\nu 4}\Delta^{(1)}\gamma_4$	–·00001	–·00008	–·00009	–·00001
$B_{\nu 6}\Delta^{(1)}\gamma_6$	–·00004	–·00012	–·00216	–·00225
$C_{\nu 0}\Delta^{(1)}\mu_0$	·00085	·00009	·00002	—
$C_{\nu 2}\Delta^{(1)}\mu_2$	–·00047	–·00011	–·00001	—
$C_{\nu 4}\Delta^{(1)}\mu_4$	·00012	·00180	·00038	·00003
$C_{\nu 6}\Delta^{(1)}\mu_6$	·00005	·00027	·00422	·00030
$\Delta^{(2)}\gamma_\nu$	+·00302	+·00226	+·00245	–·00191

ν	1	3	5	7
$D_{\nu 0}\Delta^{(1)}\gamma_0$	+·00012	+·00002	+·00001	—
$D_{\nu 2}\Delta^{(1)}\gamma_2$	–·00003	–·00001	—	—
$D_{\nu 4}\Delta^{(1)}\gamma_4$	—	+·00001	–·00001	—
$D_{\nu 6}\Delta^{(1)}\gamma_6$	—	+·00002	+·00037	–·00012
$E_{\nu 0}\Delta^{(1)}\mu_0$	+·00226	+·00006	+·00001	—
$E_{\nu 2}\Delta^{(1)}\mu_2$	–·00008	–·00024	–·00001	—
$E_{\nu 4}\Delta^{(1)}\mu_4$	–·00001	+·00023	+·00108	+·00008
$E_{\nu 6}\Delta^{(1)}\mu_6$	—	–·00004	+·00166	+·00437
$\Delta^{(1)}\mu_\nu$	+·00226	+·00005	+·00311	+·00428

$B_{\nu 0}\Delta^{(2)}\gamma_0$	·00125	·00012	·00004	·00001
$B_{\nu 2}\Delta^{(2)}\gamma_2$	·00083	·00090	·00009	·00002
$B_{\nu 4}\Delta^{(2)}\gamma_4$	·00007	00088	·00103	·00007
$B_{\nu 6}\Delta^{(2)}\gamma_6$	·00001	·00002	·00043	·00044
$C_{\nu 0}\Delta^{(2)}\mu_0$	·00001	—	—	—
$C_{\nu 2}\Delta^{(2)}\mu_2$	·00008	·00002	—	—
$C_{\nu 4}\Delta^{(2)}\mu_4$	—	·00002	—	—
$C_{\nu 6}\Delta^{(2)}\mu_6$	—	·00002	·00038	·00003
$\Delta^{(3)}\gamma_\nu$	·00225	·00198	·00197	·00057

$D_{\nu 0}\Delta^{(2)}\gamma_0$	+·00007	+·00001	—	—
$D_{\nu 2}\Delta^{(2)}\gamma_2$	–·00015	+·00006	+·00001	—
$D_{\nu 4}\Delta^{(2)}\gamma_4$	–·00001	–·00017	+·00007	+·00001
$D_{\nu 6}\Delta^{(2)}\gamma_6$	—	—	–·00007	+·00002
$E_{\nu 0}\Delta^{(2)}\mu_0$	+·00002	—	—	—
$E_{\nu 2}\Delta^{(2)}\mu_2$	+·00002	+·00004	—	—
$E_{\nu 4}\Delta^{(2)}\mu_4$	—	—	+·00001	—
$E_{\nu 6}\Delta^{(2)}\mu_6$	—	—	+·00015	+·00039
$\Delta^{(3)}\mu_\nu$	–·00005	–·00006	+·00017	+·00042

[1] Solution for the symmetrical case: (ctd. next page)

$$\gamma_\nu = a_{\nu\nu}(\ell'_\nu\alpha'_\nu - \ell''_\nu\alpha''_\nu) + \sum_{n=0}^{\frac{m-1}{2}}{}' B_{\nu n}\gamma_n + \sum_{n=0}^{\frac{m-1}{2}}{}' C_{\nu n}\mu_n \quad \Big| \quad \mu_\nu = a_{\nu\nu}(m''_\nu\alpha''_\nu - m'_\nu\alpha'_\nu) + \sum_{n=0}^{\frac{m-1}{2}}{}' D_{\nu n}\gamma_n + \sum_{n=0}^{\frac{m-1}{2}}{}' E_{\nu n}\mu_n ,$$

FORM 3 EVEN ν

TABLE 19

$\alpha_\nu = \alpha_{-\nu}$
$\gamma_\nu = \gamma_{-\nu}$
$\mu_\nu = \mu_{-\nu}$

2 × 15 STATIONS.

$\boxed{\alpha'_\nu = \alpha''_\nu = 1}$ SYMMETRICAL LOAD

a) VALUES $B_{\nu n}$, $C_{\nu n}$, $D_{\nu n}$, $E_{\nu n}$.

$$\boxed{B_{\nu n}} = a_{\nu n}(\ell'_\nu i'_{\nu n} - \ell''_\nu i''_{\nu n}) + a_{\nu-n}(\ell'_\nu i'_{\nu-n} - \ell''_\nu i''_{\nu-n}) \qquad \nu \neq 0$$
$$= 2a_{\nu n}(\ell'_\nu i'_{\nu n} - \ell''_\nu i''_{\nu n}) \qquad \nu = 0$$

ν	0	2	4	6
n = 1	·6970	·4177	·0470	$\cdot 0133_9$
3	$\cdot 0394_8$	·3344	·3033	$\cdot 0346_6$
5	·0088	$\cdot 0203_4$	·3294	·3756
7	·0020	$\cdot 0034_5$	$\cdot 0159_6$	·3353

$$\boxed{D_{\nu n}} = a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n}) + a_{\nu-n}(m''_\nu i''_{\nu-n} - m'_\nu i'_{\nu-n}) \qquad \nu \neq 0$$
$$= 2a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n}) \qquad \nu = 0$$

ν	0	2	4	6
n = 1	−·1092	$\cdot 0251_8$	$\cdot 0050_8$	$\cdot 0018_6$
3	−·0063	−·0640	·0250	$\cdot 0041_9$
5	$-\cdot 0008_0$	$-\cdot 0029_2$	−·0619	$\cdot 0243_8$
7	$-\cdot 0001_4$	$-\cdot 0003_1$	$-\cdot 0029_7$	$-\cdot 0404_8$

$$\boxed{C_{\nu n}} = a_{\nu n}(\ell'_\nu j'_{\nu n} - \ell''_\nu j''_{\nu n}) + a_{\nu-n}(\ell'_\nu j'_{\nu-n} - \ell''_\nu j''_{\nu-n}) \qquad \nu \neq 0$$
$$= 2a_{\nu n}(\ell'_\nu j'_{\nu n} - \ell''_\nu j''_{\nu n}) \qquad \nu = 0$$

ν	0	2	4	6
n = 1	·4724	·0753	·00386	·00217
3	·0331	·2740	$\cdot 2130_6$	$\cdot 0057_5$
5	$\cdot 0044_8$	$\cdot 0154_5$	$\cdot 2643_4$	·0441
7	$\cdot 0007_6$	·0018	$\cdot 0148_1$	·1684

$$\boxed{E_{\nu n}} = a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n}) + a_{\nu-n}(m''_\nu j''_{\nu-n} - m'_\nu j'_{\nu-n}) \qquad \nu \neq 0$$
$$= 2a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n}) \qquad \nu = 0$$

ν	0	2	4	6
n = 1	·2034	·1645	·0060	·00088
3	−·0042	$\cdot 0373_8$	$\cdot 0249_3$	·00458
5	$-\cdot 0004_6$	$-\cdot 0020_9$	$\cdot 0495_6$	·1977
7	$-\cdot 0000_6$	$-\cdot 0001_8$	$-\cdot 0021_3$	·1869

b) SOLUTION BY ITERATION:

γ_n, μ_n or $\Delta\gamma_n, \Delta\mu_n$ see FORM 3, ODD ν.

FIRST APPROXIMATION:

ν	0	2	4	6
$a_{\nu\nu}(\ell'_\nu - \ell''_\nu)$	·1242	·1149	·0879	·0474
$B_{\nu 1}\gamma_1^{(1)}$	·3302	·1979	·0226	·0063
$B_{\nu 3}\gamma_3^{(1)}$	·0171	·1447	·1659	·0150
$B_{\nu 5}\gamma_5^{(1)}$	·0028	·0065	·1057	·1206
$B_{\nu 7}\gamma_7^{(1)}$	·0003	·0004	·0020	·0415
$C_{\nu 1}\mu_1^{(1)}$	−·0063	−·0010	−·0001	–
$C_{\nu 3}\mu_3^{(1)}$	·0001	·0009	·0007	–
$C_{\nu 5}\mu_5^{(1)}$	–	·0002	·0025	·0004
$C_{\nu 7}\mu_7^{(1)}$	–	–	·0002	·0019
$\gamma_\nu^{(1)}$	·4684	·4645	·3874	·2331

ν	0	2	4	6
$a_{\nu\nu}(m''_\nu - m'_\nu)$	+·01988	+·01721	+·01315	+·00817
$D_{\nu 1}\gamma_1^{(1)}$	−·05172	+·01193	+·00241	+·00088
$D_{\nu 3}\gamma_3^{(1)}$	−·00273	−·02769	+·01082	+·00181
$D_{\nu 5}\gamma_5^{(1)}$	−·00028	−·00094	−·01986	+·00782
$D_{\nu 7}\gamma_7^{(1)}$	−·00002	−·00004	−·00037	−·00501
$E_{\nu 1}\mu_1^{(1)}$	−·00272	−·00220	−·00007	−·00001
$E_{\nu 3}\mu_3^{(1)}$	−·00001	+·00012	+·00008	+·00001
$E_{\nu 5}\mu_5^{(1)}$	–	−·00002	+·00047	+·00189
$E_{\nu 7}\mu_7^{(1)}$	–	–	−·00002	+·00210
$\mu_\nu^{(1)}$	−·03760	−·00163	+·00661	+·01766

DIFFERENCES:

ν	0	2	4	6
$B_{\nu 1}\Delta^{(2)}\gamma_1$	·00210	·00126	·00014	·00004
$B_{\nu 3}\Delta^{(2)}\gamma_3$	·00009	·00076	·00087	·00008
$B_{\nu 5}\Delta^{(2)}\gamma_5$	·00002	·00005	·00081	·00092
$B_{\nu 7}\Delta^{(2)}\gamma_7$	–	−·00001	−·00003	−·00064
$C_{\nu 1}\Delta^{(2)}\mu_1$	·00107	·00017	·00002	–
$C_{\nu 3}\Delta^{(2)}\mu_3$	–	·00001	·00001	–
$C_{\nu 5}\Delta^{(2)}\mu_5$	·00001	·00005	·00082	·00014
$C_{\nu 7}\Delta^{(2)}\mu_7$	–	·00001	·00006	·00072
$\Delta^{(2)}\gamma_\nu$	+·00329	+·00230	+·00270	+·00126

ν	0	2	4	6
$D_{\nu 1}\Delta^{(2)}\gamma_1$	−·00033	+·00008	+·00002	+·00001
$D_{\nu 3}\Delta^{(2)}\gamma_3$	−·00001	−·00014	+·00006	+·00001
$D_{\nu 5}\Delta^{(2)}\gamma_5$	–	−·00001	−·00015	+·00006
$D_{\nu 7}\Delta^{(2)}\gamma_7$	–	–	+·00001	+·00008
$E_{\nu 1}\Delta^{(2)}\mu_1$	+·00046	+·00037	+·00001	–
$E_{\nu 3}\Delta^{(2)}\mu_3$	–	–	–	–
$E_{\nu 5}\Delta^{(2)}\mu_5$	–	−·00001	+·00015	+·00061
$E_{\nu 7}\Delta^{(2)}\mu_7$	–	–	−·00001	+·00080
$\Delta^{(2)}\mu_\nu$	+·00012	+·00029	+·00009	+·00157

$B_{\nu 1}\Delta^{(3)}\gamma_1$	·00157	·00094	·00011	·00003
$B_{\nu 3}\Delta^{(3)}\gamma_3$	·00008	·00066	·00076	·00007
$B_{\nu 5}\Delta^{(3)}\gamma_5$	·00002	·00004	·00065	·00074
$B_{\nu 7}\Delta^{(3)}\gamma_7$	–	–	·00001	·00019
$C_{\nu 1}\Delta^{(3)}\mu_1$	−·00002	–	–	–
$C_{\nu 3}\Delta^{(3)}\mu_3$	–	−·00002	−·00001	–
$C_{\nu 5}\Delta^{(3)}\mu_5$	–	–	·00004	·00001
$C_{\nu 7}\Delta^{(3)}\mu_7$	–	–	·00001	·00007
$\Delta^{(3)}\gamma_\nu$	+·00165	+·00162	+·00157	+·00111

$D_{\nu 1}\Delta^{(3)}\gamma_1$	−·00025	+·00006	+·00001	–
$D_{\nu 3}\Delta^{(3)}\gamma_3$	−·00001	−·00013	+·00005	+·00001
$D_{\nu 5}\Delta^{(3)}\gamma_5$	–	−·00001	−·00012	+·00005
$D_{\nu 7}\Delta^{(3)}\gamma_7$	–	–	–	−·00002
$E_{\nu 1}\Delta^{(3)}\mu_1$	−·00001	−·00001	–	–
$E_{\nu 3}\Delta^{(3)}\mu_3$	–	–	–	–
$E_{\nu 5}\Delta^{(3)}\mu_5$	–	–	+·00001	+·00003
$E_{\nu 7}\Delta^{(3)}\mu_7$	–	–	–	+·00008
$\Delta^{(3)}\mu_\nu$	−·00027	−·00009	−·00005	+·00015

1) ctd. $B_{\nu n} = C_{\nu n} = D_{\nu n} = E_{\nu n} = 0$ for $|n-\nu| = 2, 4, 6, \cdots$

FORM 3 ODD ν ctd. TABLE 20

2 × 15 STATIONS

DIFFERENCES CTD.

ν	1	3	5	7
$B_{\nu 0}\Delta^{(3)}\gamma_0$	·00063	·00006	·00002	—
$B_{\nu 2}\Delta^{(3)}\gamma_2$	·00059	·00063	·00006	·00001
$B_{\nu 4}\Delta^{(3)}\gamma_4$	·00004	·00051	·00060	·00004
$B_{\nu 6}\Delta^{(3)}\gamma_6$	·00001	·00002	·00038	·00039
$C_{\nu 0}\Delta^{(3)}\mu_0$	−·00002	—	—	—
$C_{\nu 2}\Delta^{(3)}\mu_2$	−·00003	−·00001	—	—
$C_{\nu 4}\Delta^{(3)}\mu_4$	—	−·00001	—	—
$C_{\nu 6}\Delta^{(3)}\mu_6$	—	—	·00004	—
$\Delta^{(4)}\gamma_\nu$	·00122	·00120	·00110	·00044

ν	1	3	5	7
$D_{\nu 0}\Delta^{(3)}\gamma_0$	+·00003	+·00001	—	—
$D_{\nu 2}\Delta^{(3)}\gamma_2$	−·00011	+·00004	+·00001	—
$D_{\nu 4}\Delta^{(3)}\gamma_4$	−·00001	−·00010	+·00004	+·00001
$D_{\nu 6}\Delta^{(3)}\gamma_6$	—	—	−·00006	+·00002
$E_{\nu 0}\Delta^{(3)}\mu_0$	−·00005	—	—	—
$E_{\nu 2}\Delta^{(3)}\mu_2$	—	−·00001	—	—
$E_{\nu 4}\Delta^{(3)}\mu_4$	—	—	−·00001	—
$E_{\nu 6}\Delta^{(3)}\mu_6$	—	—	+·00001	+·00004
$\Delta^{(4)}\mu_\nu$	−·00014	−·00006	−·00001	+·00007

ν	1	3	5	7
$B_{\nu 0}\Delta^{(4)}\gamma_0$	·00032	·00003	·00001	—
$B_{\nu 2}\Delta^{(4)}\gamma_2$	·00033	·00035	·00004	·00001
$B_{\nu 4}\Delta^{(4)}\gamma_4$	·00002	·00029	·00033	−·00002
$B_{\nu 6}\Delta^{(4)}\gamma_6$	—	·00001	·00021	·00022
$C_{\nu 0}\Delta^{(4)}\mu_0$	−·00001	—	—	—
$C_{\nu 2}\Delta^{(4)}\mu_2$	−·00002	—	—	—
$C_{\nu 4}\Delta^{(4)}\mu_4$	—	−·00001	—	—
$C_{\nu 6}\Delta^{(4)}\mu_6$	—	—	·00001	—
$\Delta^{(5)}\gamma_\nu$	·00064	·00067	·00060	·00025

ν	1	3	5	7
$D_{\nu 0}\Delta^{(4)}\gamma_0$	+·00002	—	—	—
$D_{\nu 2}\Delta^{(4)}\gamma_2$	−·00006	+·00002	—	—
$D_{\nu 4}\Delta^{(4)}\gamma_4$	—	−·00006	+·00002	—
$D_{\nu 6}\Delta^{(4)}\gamma_6$	—	—	−·00004	+·00001
$E_{\nu 0}\Delta^{(4)}\mu_0$	−·00003	—	—	—
$E_{\nu 2}\Delta^{(4)}\mu_2$	—	−·00001	—	—
$E_{\nu 4}\Delta^{(4)}\mu_4$	—	—	—	—
$E_{\nu 6}\Delta^{(4)}\mu_6$	—	—	—	+·00001
$\Delta^{(5)}\mu_\nu$	−·00007	−·00005	−·00002	+·00002

ν	1	3	5	7
$B_{\nu 0}\Delta^{(5)}\gamma_0$	·00017	·00002	—	—
$B_{\nu 2}\Delta^{(5)}\gamma_2$	·00017	·00019	·00002	—
$B_{\nu 4}\Delta^{(5)}\gamma_4$	·00001	·00015	·00018	·00001
$B_{\nu 6}\Delta^{(5)}\gamma_6$	—	·00001	·00011	·00012
$C_{\nu 0}\Delta^{(5)}\mu_0$	−·00001	—	—	—
$C_{\nu 2}\Delta^{(5)}\mu_2$	−·00001	—	—	—
$C_{\nu 4}\Delta^{(5)}\mu_4$	—	−·00001	—	—
$C_{\nu 6}\Delta^{(5)}\mu_6$	—	—	—	—
$\Delta^{(6)}\gamma_\nu$	·00033	·00036	·00031	·00013

ν	1	3	5	7
$D_{\nu 0}\Delta^{(5)}\gamma_0$	+·00001	—	—	—
$D_{\nu 2}\Delta^{(5)}\gamma_2$	−·00003	+·00001	—	—
$D_{\nu 4}\Delta^{(5)}\gamma_4$	—	−·00003	+·00001	—
$D_{\nu 6}\Delta^{(5)}\gamma_6$	—	—	−·00002	—
$E_{\nu 0}\Delta^{(5)}\mu_0$	−·00001	—	—	—
$E_{\nu 2}\Delta^{(5)}\mu_2$	—	—	—	—
$E_{\nu 4}\Delta^{(5)}\mu_4$	—	—	—	—
$E_{\nu 6}\Delta^{(5)}\mu_6$	—	—	—	—
$\Delta^{(6)}\mu_\nu$	−·00003	−·00002	−·00001	—

SUMMARY: $\gamma_\nu = \gamma_\nu^{(1)} + \Delta^{(2)}\gamma_\nu + \Delta^{(3)}\gamma_\nu + \cdots$ $\mu_\nu = \mu_\nu^{(1)} + \Delta^{(2)}\mu_\nu + \Delta^{(3)}\mu_\nu + \cdots$

ν	1	3	5	7
$\gamma_\nu^{(1)}$	·4737	·4327	·3208	·1238
$\Delta^{(2)}\gamma_\nu$	$\cdot 0030_2$	$\cdot 0022_6$	$\cdot 0024_6$	$-\cdot 0019_1$
$\Delta^{(3)}\gamma_\nu$	$\cdot 0022_5$	$\cdot 0019_8$	$\cdot 0019_7$	$\cdot 0005_7$
$\Delta^{(4)}\gamma_\nu$	$\cdot 0012_2$	$\cdot 0012_0$	$\cdot 0011_0$	$\cdot 0004_4$
$\Delta^{(5)}\gamma_\nu$	$\cdot 0006_4$	$\cdot 0006_7$	$\cdot 0006_0$	$\cdot 0002_5$
$\Delta^{(6)}\gamma_\nu$	$\cdot 0003_3$	$\cdot 0003_6$	$\cdot 0003_1$	$\cdot 0001_3$
REST [1]	$\cdot 0003_5$	$\cdot 0004_2$	$\cdot 0003_3$	·00014
γ_ν	·4815	·4396	·3276	·1234

ν	1	3	5	7
$\mu_\nu^{(1)}$	−·01337	+·00320	+·00955	+·01124
$\Delta^{(2)}\mu_\nu$	+·00226	+·00005	+·00311	+·00428
$\Delta^{(3)}\mu_\nu$	−·00005	−·00006	+·00017	+·00042
$\Delta^{(4)}\mu_\nu$	−·00014	−·00006	−·00001	+·00007
$\Delta^{(5)}\mu_\nu$	−·00007	−·00005	−·00002	+·00002
$\Delta^{(6)}\mu_\nu$	−·00003	−·00002	−·00001	—
REST	−·00003	−·00002	−·00001	—
μ_ν	−·01143	+·00304	+·01278	+·01603

CHECK OF RESULTS:

ν	1	3	5	7
$a_{\nu\nu}(l_\nu' - l_\nu'')$	·1219	·1034	·0690	·0241
$B_{\nu 0}\,\gamma_0$	·1811	·0179	·0052	·0013
$B_{\nu 2}\,\gamma_2$	·1704	·1837	·0186	·0038
$B_{\nu 4}\,\gamma_4$	·0097	·1286	·1497	·0106
$B_{\nu 6}\,\gamma_6$	·0013	·0044	·0802	·0834
$C_{\nu 0}\,\mu_0$	−·0026	−·0003	−·0001	—
$C_{\nu 2}\,\mu_2$	−·0005	−·0001	—	—
$C_{\nu 4}\,\mu_4$	+·0001	·0018	·0004	—
$C_{\nu 6}\,\mu_6$	+·0001	·0003	·0046	·0003
γ_ν	·4815	·4397	·3276	·1235

ν	1	3	5	7
$a_{\nu\nu}(m_\nu'' - m_\nu')$	·01886	·01525	·01088	·00456
$D_{\nu 0}\,\gamma_0$	·00941	·00181	·00066	·00024
$D_{\nu 2}\,\gamma_2$	−·03123	·01160	·00213	·00057
$D_{\nu 4}\,\gamma_4$	−·00135	−·02468	·00994	·00133
$D_{\nu 6}\,\gamma_6$	−·00011	−·00070	−·01368	·00453
$E_{\nu 0}\,\mu_0$	−·00695	−·00017	−·00003	−·00001
$E_{\nu 2}\,\mu_2$	−·00008	−·00024	−·00001	—
$E_{\nu 4}\,\mu_4$	−·00001	·00023	·00108	·00003
$E_{\nu 6}\,\mu_6$	—	−·00004	·00183	00480
μ_ν	−·01146	+·00306	+·01280	+·01605

[1] ESTIMATION OF REST: REST $\approx \dfrac{(\Delta^{(6)}\gamma_\nu)^2}{\Delta^{(5)}\gamma_\nu - \Delta^{(6)}\gamma_\nu}$ or in μ_ν's respect.

FORM 3 EVEN ν ctd. TABLE 21

2 × 15 STATIONS

DIFFERENCES CTD.

ν	0	2	4	6
$B_{\nu 1}\Delta^{④}\gamma_1$	·00085	·00051	·00006	·00002
$B_{\nu 3}\Delta^{④}\gamma_3$	·00005	·00040	·00046	·00004
$B_{\nu 5}\Delta^{④}\gamma_5$	·00001	·00002	·00036	·00041
$B_{\nu 7}\Delta^{④}\gamma_7$	—	—	·00001	·00015
$C_{\nu 1}\Delta^{④}\mu_1$	−·00007	−·00001	—	—
$C_{\nu 3}\Delta^{④}\mu_3$	—	−·00002	−·00001	—
$C_{\nu 5}\Delta^{④}\mu_5$	—	—	—	—
$C_{\nu 7}\Delta^{④}\mu_7$	—	—	—	·00001
$\Delta^{④}\gamma_\nu$	·00084	·00090	·00088	·00063

ν	0	2	4	6
$D_{\nu 1}\Delta^{④}\gamma_1$	−·00013	+·00003	+·00001	—
$D_{\nu 3}\Delta^{④}\gamma_3$	−·00001	−·00008	+·00003	+·00001
$D_{\nu 5}\Delta^{④}\gamma_5$	—	—	−·00007	+·00003
$D_{\nu 7}\Delta^{④}\gamma_7$	—	—	—	−·00002
$E_{\nu 1}\Delta^{④}\mu_1$	−·00003	−·00002	—	—
$E_{\nu 3}\Delta^{④}\mu_3$	—	—	—	—
$E_{\nu 5}\Delta^{④}\mu_5$	—	—	—	—
$E_{\nu 7}\Delta^{④}\mu_7$	—	—	—	+·00001
$\Delta^{④}\mu_\nu$	−·00017	−·00007	−·00003	+·00003

$B_{\nu 1}\Delta^{⑤}\gamma_1$	·00045	·00027	·00003	·00001
$B_{\nu 3}\Delta^{⑤}\gamma_3$	·00003	·00022	·00026	·00002
$B_{\nu 5}\Delta^{⑤}\gamma_5$	—	·00001	·00020	·00023
$B_{\nu 7}\Delta^{⑤}\gamma_7$	—	—	—	·00008
$C_{\nu 1}\Delta^{⑤}\mu_1$	−·00003	−·00001	—	—
$C_{\nu 3}\Delta^{⑤}\mu_3$	—	−·00001	−·00001	—
$C_{\nu 5}\Delta^{⑤}\mu_5$	—	—	—	—
$C_{\nu 7}\Delta^{⑤}\mu_7$	—	—	—	—
$\Delta^{⑤}\gamma_\nu$	·00045	·00048	·00048	·00034

$D_{\nu 1}\Delta^{⑤}\gamma_1$	−·00007	+·00002	—	—
$D_{\nu 3}\Delta^{⑤}\gamma_3$	—	−·00004	+·00002	—
$D_{\nu 5}\Delta^{⑤}\gamma_5$	—	—	−·00004	+·00001
$D_{\nu 7}\Delta^{⑤}\gamma_7$	—	—	—	−·00001
$E_{\nu 1}\Delta^{⑤}\mu_1$	−·00001	−·00001	—	—
$E_{\nu 3}\Delta^{⑤}\mu_3$	—	—	—	—
$E_{\nu 5}\Delta^{⑤}\mu_5$	—	—	—	—
$E_{\nu 7}\Delta^{⑤}\mu_7$	—	—	—	—
$\Delta^{⑤}\mu_\nu$	−·00008	−·00003	−·00002	—

$B_{\nu 1}\Delta^{⑥}\gamma_1$	·00023	·00014	·00002	—
$B_{\nu 3}\Delta^{⑥}\gamma_3$	·00001	·00012	·00014	·00001
$B_{\nu 5}\Delta^{⑥}\gamma_5$	—	·00001	·00010	·00012
$B_{\nu 7}\Delta^{⑥}\gamma_7$	—	—	—	·00004
$C_{\nu 1}\Delta^{⑥}\mu_1$	−·00001	—	—	—
$C_{\nu 3}\Delta^{⑥}\mu_3$	—	−·00001	—	—
$C_{\nu 5}\Delta^{⑥}\mu_5$	—	—	—	—
$C_{\nu 7}\Delta^{⑥}\mu_7$	—	—	—	—
$\Delta^{⑥}\gamma_\nu$	·00023	·00026	·00026	·00017

$D_{\nu 1}\Delta^{⑥}\gamma_1$	−·00004	+·00001	—	—
$D_{\nu 3}\Delta^{⑥}\gamma_3$	—	−·00002	+·00001	—
$D_{\nu 5}\Delta^{⑥}\gamma_5$	—	—	−·00002	+·00001
$D_{\nu 7}\Delta^{⑥}\gamma_7$	—	—	—	−·00001
$E_{\nu 1}\Delta^{⑥}\mu_1$	−·00001	−·00001	—	—
$E_{\nu 3}\Delta^{⑥}\mu_3$	—	—	—	—
$E_{\nu 5}\Delta^{⑥}\mu_5$	—	—	—	—
$E_{\nu 7}\Delta^{⑥}\mu_7$	—	—	—	—
$\Delta^{⑥}\mu_\nu$	−·00005	−·00002	−·00001	—

SUMMARY: $\gamma_\nu = \gamma_\nu^{①} + \Delta^{②}\gamma_\nu + \Delta^{③}\gamma_\nu + \cdots$ $\mu_\nu = \mu_\nu^{①} + \Delta^{②}\mu_\nu + \Delta^{③}\mu_\nu + \cdots$

ν	0	2	4	6
$\gamma_\nu^{①}$	·4684	·4645	·3874	·2331
$\Delta^{②}\gamma_\nu$	·00329	·0023	·0027	·00126
$\Delta^{③}\gamma_\nu$	·00165	·00162	·00157	·00111
$\Delta^{④}\gamma_\nu$	·00084	·0009	·00088	·00063
$\Delta^{⑤}\gamma_\nu$	·00045	·00048	·00048	·00034
$\Delta^{⑥}\gamma_\nu$	·00023	·00026	·00026	·00017
REST	·00024	·00031	·00031	·00017
γ_ν	·4751	·4704	·3936	·2368

ν	0	2	4	6
$\mu_\nu^{①}$	−·03760	−·00163	+·00661	+·01766
$\Delta^{②}\mu_\nu$	+·00012	+·00029	+·00009	+·00157
$\Delta^{③}\mu_\nu$	−·00027	−·00009	−·00005	+·00015
$\Delta^{④}\mu_\nu$	−·00017	−·00007	−·00003	+·00003
$\Delta^{⑤}\mu_\nu$	−·00008	−·00003	−·00002	—
$\Delta^{⑥}\mu_\nu$	−·00005	−·00002	−·00001	—
REST	−·00007	−·00002	−·00001	—
μ_ν	−·03812	−·00157	+·00658	+·01941

CHECK OF RESULTS:

ν	0	2	4	6
$a_{\nu\nu}(\ell'_\nu - \ell''_\nu)$	·1242	·1149	·0879	·0474
$B_{\nu 1}\ \gamma_1$	·3356	·2011	·0230	·0065
$B_{\nu 3}\ \gamma_3$	·0174	·1470	·1685	·0162
$B_{\nu 5}\ \gamma_5$	·0029	·0067	·1079	·1230
$B_{\nu 7}\ \gamma_7$	·0002	·0004	·0020	·0414
$C_{\nu 1}\ \mu_1$	−·0054	−·0008	−·0001	—
$C_{\nu 3}\ \mu_3$	·0001	·0008	·0007	—
$C_{\nu 5}\ \mu_5$	·0001	·0002	·0034	·0006
$C_{\nu 7}\ \mu_7$	—	—	·0002	·0027
γ_ν	·4751	·4703	·3935	·2368

ν	0	2	4	6
$a_{\nu\nu}(m''_\nu - m'_\nu)$	+·01988	+·01721	+·01315	+·00817
$D_{\nu 1}\ \mu_1$	−·06258	+·01212	+·00245	+·00090
$D_{\nu 3}\ \mu_3$	−·00277	−·02814	+·01099	+·00184
$D_{\nu 5}\ \mu_5$	−·00029	−·00096	−·02028	+·00799
$D_{\nu 7}\ \mu_7$	−·00002	−·00004	−·00037	−·00500
$E_{\nu 1}\ \mu_1$	−·00233	−·00188	−·00006	−·00001
$E_{\nu 3}\ \mu_3$	−·00001	+·00011	+·00008	+·00001
$E_{\nu 5}\ \mu_5$	−·00001	−·00003	+·00063	+·00253
$E_{\nu 7}\ \mu_7$	—	—	−·00003	+·00300
μ_ν	−·03813	−·00161	+·00656	+·01943

TABLE 22

RESULTS $\boxed{\alpha' = \alpha'' = 1 \text{ SYMMETRICAL LOAD}}$ 2 × 15 STATIONS

a) LOCAL LIFT, MOMENT & a.c.

$$\gamma = \frac{C_L c}{2b}\,; \qquad X_{ac.} = \cdot 25 - \frac{\mu_n}{\gamma_n} \qquad \text{for } n \neq 0$$

$$\mu = \frac{C_m c}{2b}\,; \qquad = \frac{1}{c_r}\left[x_{0\ell} + \left(\cdot 25 - \frac{\mu_0}{\gamma_0}\right)c_0\right] \qquad \text{for } n = 0$$

n	0	1	2	3	4	5	6	7
γ_n	·4751	·4815	·4703	·4397	·3935	·3276	·2368	·1235
μ_n	−·03813	−·01146	−·00161	·00306	·00656	·01280	·01943	·01605
$X_{a.c.}$	·3705	·2737	·2533	·2431	·2333	·2110	·1680	·1201

b)
$$\frac{dC_L}{d\alpha} = \frac{2\pi A}{m+1}\left[\cdot 5\gamma_0 + \sum_1^{\frac{m-1}{2}} \gamma_n \cos\frac{n\pi}{m+1}\right] = \frac{\pi}{2}\Big[\begin{array}{l} \cdot 5 \times \cdot 4751 \\ \cdot 9808 \times \cdot 4815 \\ \cdot 9239 \times \cdot 4703 \\ \cdot 8315 \times \cdot 4397 \\ \cdot 7071 \times \cdot 3935 \\ \cdot 5556 \times \cdot 3276 \\ \cdot 3827 \times \cdot 2368 \\ \cdot 1951 \times \cdot 1235 \end{array}\Big] = \frac{\pi}{2} \times 2{\cdot}085$$

$$\frac{dC_L}{d\alpha} = \underline{\underline{3{\cdot}275}}$$

c)
$$C_M = \frac{\pi A^2}{m+1}\left\{\cdot 5\left[\mu_0 \frac{2c_0}{b} - \gamma_0 \xi_{0\,c/4}\right] + \sum_1^{\frac{m-1}{2}}\left[\mu_n \frac{2c_n}{b} - \gamma_n \xi_{n\,c/4}\right]\cos\frac{n\pi}{m+1}\right\}^{1)}$$

$$= \pi\left\{\begin{array}{ll} \cdot 5 & [-\cdot 03813 \times \cdot 687 - \cdot 4751 \times \cdot 2043] \\ \cdot 9808 & [-\cdot 01146 \times \cdot 622 - \cdot 4815 \times \cdot 3506] \\ \cdot 9239 & [-\cdot 00161 \times \cdot 547 - \cdot 4703 \times \cdot 5194] \\ \cdot 8315 & [\cdot 00306 \times \cdot 478 - \cdot 4397 \times \cdot 6750] \\ \cdot 7071 & [\cdot 00656 \times \cdot 417 - \cdot 3935 \times \cdot 8114] \\ \cdot 5556 & [\cdot 01280 \times \cdot 367 - \cdot 3276 \times \cdot 9233] \\ \cdot 3827 & [\cdot 01943 \times \cdot 330 - \cdot 2368 \times 1{\cdot}0065] \\ \cdot 1951 & [\cdot 01605 \times \cdot 3077 - \cdot 1235 \times 1{\cdot}0577] \end{array}\right. = \pi\left\{\begin{array}{l} \cdot 5 \times -\cdot 1232 \\ \cdot 9808 \times -\cdot 1759 \\ \cdot 9239 \times -\cdot 2451 \\ \cdot 8315 \times -\cdot 2953 \\ \cdot 7071 \times -\cdot 3166 \\ \cdot 5556 \times -\cdot 2978 \\ \cdot 3827 \times -\cdot 2319 \\ \cdot 1951 \times -\cdot 1256 \end{array}\right\} = -1{\cdot}2087\,\pi$$

$$C_M = \underline{\underline{-3{\cdot}80}}$$

d)
$$C_{Di} = \frac{\pi A}{4}\left\{\gamma_0^2 + 2\sum_1^{\frac{m-1}{2}} \gamma_n^2 - 4\sum_{\substack{\nu=1 \\ \nu \text{ odd}}}^{\frac{m-1}{2}} \sum_{\substack{-\frac{m-1}{2} \\ n \text{ even}}}^{\frac{m-1}{2}} a_{\nu n}\gamma_\nu \gamma_n\right\}$$

	$a_{\nu n}$ (as in TABLE)				$a_{\nu n}\gamma_\nu$				$\sum_{\nu=1,3..}^{7} a_{\nu n}\gamma_\nu$	$\gamma_n \sum a_{\nu n}\gamma_\nu$
ν	1	3	5	7	1	3	5	7	n = const.	
n = 6	·0110	·0367	·3890	·3602	·0053	·0161	·1274	·0445	·1933	·0458
4	·0413	·4001	·3969	·0288	·0199	·1759	·1301	·0036	·3295	·1297
2	·4023	·4016	·0398	·0079	·1937	·1763	·0130	·0010	·3840	·1807
0	·4026	·0421	·0126	·0032	·1938	·0185	·0041	·0004	·2168	·10[illegible]
−2	·0424	·0136	·0054	·0015	·0204	·0060	·0018	·0002	·0284	·01[illegible]
−4	·0133	·0058	·0026	·0008	·0064	·0025	·0009	·0001	·0099	·0039
−6	·0047	·0023	·0011	·0003	·0023	·0010	·0004	—	·0037	·0009
									$\sum_{\nu=1,3..}^{7} \sum_{n=-6,-4..}^{6} a_{\nu n}\gamma_\nu\gamma_n =$	·4775

$$C_{Di} = \pi\left(\cdot 2257 + 2 \times \cdot 980 - 4 \times \cdot 4775\right) = \pi \times \cdot 2755$$

$$C_{Di} = \underline{\underline{\cdot 8655}}$$

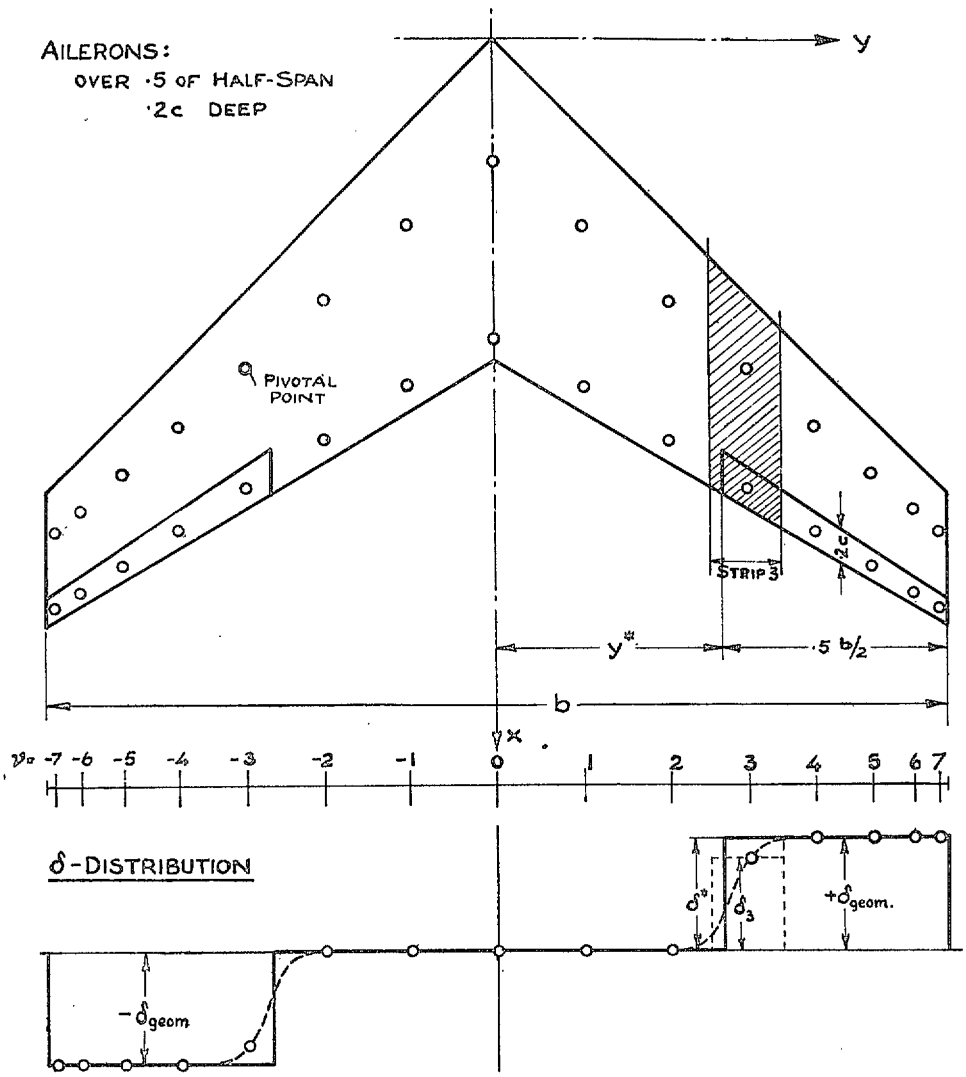
1) $\xi_{n\,c/4} = \dfrac{\cdot 25\, c_n + x_{n\ell}}{b/2}$

(63370) F

TABLE 23

$\delta' = 1$ ANTISYMMETRICAL LOAD

2 × 15
STATIONS



At station 3 which is next to the beginning of the aileron a mean value of δ over the strip coordinated to this station is taken according to

$$\delta_3 = \delta^* \left[\tfrac{1}{2} + \tfrac{m+1}{\pi}(\theta^* - \theta_3)\right] = \cdot 8333$$

With the theoretical value of $\frac{\partial \alpha}{\partial \delta}$ and $\frac{\partial C_m}{\partial \delta}$ for a ·2c hinged flap we obtain

$$\frac{\partial \alpha'}{\partial \delta} = \cdot 802 \quad ; \quad \frac{\partial \alpha''}{\partial \delta} = -\cdot 109 .$$

ν	0	1	2	3	4	5	6	7
α'_ν	0	0	0	·6685	·802	·802	·802	·802
α''_ν	0	0	0	−·0909	−·109	−·109	−·109	−·109
$l'_\nu \alpha'_\nu$	0	0	0	·3260	·3934	·4007	·4135	·4276
$l''_\nu \alpha''_\nu$	0	0	0	·0008$_4$	·0007$_6$	−·0002$_8$	−·00215	−·0043
$a_{\nu\nu}(l'_\nu \alpha'_\nu - l''_\nu \alpha''_\nu)$	0	0	0	·0676	·0694	·0557	·0398	·0211
$m''_\nu \alpha''_\nu$	0	0	0	−·0249	−·0302	−·03133	−·0334	−·0356$_7$
$m'_\nu \alpha'_\nu$	0	0	0	·1343	·1625	·1677$_6$	·1771	·1876
$a_{\nu\nu}(m''_\nu \alpha''_\nu - m'_\nu \alpha'_\nu)$	0	0	0	−·03313	−·03407	−·02765	−·02014	−·01089

FORM 3a ODD ν

TABLE 24

$\alpha_\nu = -\alpha_{-\nu}$
$\gamma_\nu = -\gamma_{-\nu}$
$\mu_\nu = -\mu_{-\nu}$

2 × 15 STATIONS

$\delta = 1$ ANTISYMMETRICAL LOAD

a) VALUES $B_{\nu n}$, $C_{\nu n}$, $D_{\nu n}$, $E_{\nu n}$.

$\boxed{B_{\nu n}} = a_{\nu n}(\ell'_\nu i'_{\nu n} - \ell''_\nu i''_{\nu n}) - a_{\nu,-n}(\ell'_\nu i'_{\nu,-n} - \ell''_\nu i''_{\nu,-n})$

ν	1	3	5	7
n = 2	·3048	·3711	·0317$_8$	·0058$_3$
4	·0123$_2$	·3202	·3770	·0257$_8$
6	·0019$_1$	·0161$_4$	·3373	·3519

$\boxed{D_{\nu n}} = a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n}) - a_{\nu,-n}(m''_\nu i''_{\nu,-n} - m'_\nu i'_{\nu,-n})$

ν	1	3	5	7
n = 2	−·0624$_3$	·0235$_4$	·0036$_8$	·0009
4	−·0027$_8$	−·0633	·0249$_5$	·0032$_7$
6	−·00035	−·0030$_6$	−·0578$_4$	·0190$_8$

$\boxed{C_{\nu n}} = a_{\nu n}(\ell'_\nu j'_{\nu n} - \ell'_\nu j'_{\nu n}) - a_{\nu,-n}(\ell'_\nu j'_{\nu,-n} - \ell''_\nu j''_{\nu,-n})$

ν	1	3	5	7
n = 2	·2519	·0622$_7$	·0052$_4$	·0008$_5$
4	·01227	·2702	·0574$_7$	·0039$_7$
6	·0013$_4$	·0146$_8$	·2390	·0170$_7$

$\boxed{E_{\nu n}} = a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n}) - a_{\nu,-n}(m''_\nu j''_{\nu,-n} - m'_\nu j'_{\nu,-n})$

ν	1	3	5	7
n = 2	·0466	·1484	·0039	·0005$_3$
4	−·0022$_4$	·0350$_6$	·1639	·0049$_5$
6	−·0002$_2$	−·0022$_1$	·0942	·2475

b) SOLUTION BY ITERATION.

INITIAL GUESS: $\gamma_2^{(0)} = \cdot 02$ $\gamma_4^{(0)} = \cdot 12$ $\gamma_6^{(0)} = \cdot 10$

$\mu^{(0)}$-VALUES = 0

FIRST APPROXIMATION:

ν	1	3	5	7
$a_{\nu\nu}(\ell'_\nu\alpha'_\nu - \ell''_\nu\alpha''_\nu)$	0	·0676	·0557	·0211
$B_{\nu 2}\ \gamma_2^{(0)}$	·00610	·00742	·00064	·00012
$B_{\nu 4}\ \gamma_4^{(0)}$	·00148	·03845	·04525	·00309
$B_{\nu 6}\ \gamma_6^{(0)}$	·00019	·00161	·03373	·03520
$\gamma_\nu^{(1)}$	·0078	·1151	·1353	·0595

ν	1	3	5	7
$a_{\nu\nu}(m''_\nu\alpha''_\nu - m'_\nu\alpha'_\nu)$	0	−·03313	−·02765	−·01089
$D_{\nu 2}\ \gamma_2^{(0)}$	−·00125	+·00047	+·00007	+·00002
$D_{\nu 4}\ \gamma_4^{(0)}$	−·00033	−·00760	+·00299	+·00039
$D_{\nu 6}\ \gamma_6^{(0)}$	−·00004	−·00031	−·00578	+·00191
$\mu_\nu^{(1)}$	−·00162	−·04057	−·03037	−·00857

INITIAL DIFFERENCES:

$\Delta^{(1)}\gamma_2 = \gamma_2^{(1)} - \gamma_2^{(0)} = +\cdot 0101$ $\Delta^{(1)}\gamma_4 = +\cdot 0212$ $\Delta^{(1)}\gamma_6 = +\cdot 0111$

$\Delta^{(1)}\mu_2 = \mu_2^{(1)} - 0 = -\cdot 00927$ $\Delta^{(1)}\mu_4 = -\cdot 04231$ $\Delta^{(1)}\mu_6 = -\cdot 02660$

FURTHER DIFFERENCES:

ν	1	3	5	7
$B_{\nu 2}\Delta^{(1)}\gamma_2$	+·00309	+·00375	+·00032	+·00006
$B_{\nu 4}\Delta^{(1)}\gamma_4$	+·00026	+·00680	+·00800	+·00065
$B_{\nu 6}\Delta^{(1)}\gamma_6$	+·00002	+·00018	+·00374	+·00390
$C_{\nu 2}\Delta^{(1)}\mu_2$	−·00233	−·00058	−·00005	−·00001
$C_{\nu 4}\Delta^{(1)}\mu_4$	−·00052	−·01143	−·00243	−·00017
$C_{\nu 6}\Delta^{(1)}\mu_6$	−·00004	−·00039	−·00636	−·00045
$\Delta^{(2)}\gamma_\nu$	+·00048	−·00167	+·00322	+·00388
$B_{\nu 2}\Delta^{(2)}\gamma_2$	−·00047	−·00057	−·00005	−·00001
$B_{\nu 4}\Delta^{(2)}\gamma_4$	−·00004	−·00095	−·00112	−·00008
$B_{\nu 6}\Delta^{(2)}\gamma_6$	—	+·00001	+·00031	+·00032
$C_{\nu 2}\Delta^{(2)}\mu_2$	−·00004	−·00001	—	—
$C_{\nu 4}\Delta^{(2)}\mu_4$	−·00001	−·00022	−·00005	—
$C_{\nu 6}\Delta^{(2)}\mu_6$	—	−·00005	−·00077	−·00005
$\Delta^{(3)}\gamma_\nu$	−·00056	−·00179	−·00168	+·00018
$B_{\nu 2}\Delta^{(3)}\gamma_2$	−·00024	−·00029	−·00002	—
$B_{\nu 4}\Delta^{(3)}\gamma_4$	−·00002	−·00044	−·00052	−·00004
$B_{\nu 6}\Delta^{(3)}\gamma_6$	—	−·00001	−·00026	−·00027
$C_{\nu 2}\Delta^{(3)}\mu_2$	+·00003	+·00001	—	—
$C_{\nu 4}\Delta^{(3)}\mu_4$	—	+·00001	—	—
$C_{\nu 6}\Delta^{(3)}\mu_6$	—	—	−·00008	−·00001
$\Delta^{(4)}\gamma_\nu$	−·00023	−·00072	−·00088	−·00032

ν	1	3	5	7
$D_{\nu 2}\Delta^{(1)}\gamma_2$	−·00063	+·00024	+·00004	+·00001
$D_{\nu 4}\Delta^{(1)}\gamma_4$	−·00006	−·00134	+·00053	+·00007
$D_{\nu 6}\Delta^{(1)}\gamma_6$	—	−·00003	−·00064	+·00021
$E_{\nu 2}\Delta^{(1)}\mu_2$	−·00043	−·00137	−·00004	—
$E_{\nu 4}\Delta^{(1)}\mu_4$	+·00009	−·00148	−·00694	−·00021
$E_{\nu 6}\Delta^{(1)}\mu_6$	+·00001	+·00006	−·00251	−·00659
$\Delta^{(2)}\mu_\nu$	−·00102	−·00392	−·00956	−·00651
$D_{\nu 2}\Delta^{(2)}\gamma_2$	+·00010	−·00004	−·00001	—
$D_{\nu 4}\Delta^{(2)}\gamma_4$	+·00001	+·00019	−·00007	−·00001
$D_{\nu 6}\Delta^{(2)}\gamma_6$	—	—	−·00005	+·00002
$E_{\nu 2}\Delta^{(2)}\gamma_2$	−·00001	−·00003	—	—
$E_{\nu 4}\Delta^{(2)}\gamma_4$	—	−·00003	−·00013	—
$E_{\nu 6}\Delta^{(2)}\gamma_6$	—	+·00001	−·00030	−·00080
$\Delta^{(3)}\mu_\nu$	+·00010	+·00010	−·00056	−·00079
$D_{\nu 2}\Delta^{(3)}\gamma_2$	+·00005	−·00002	—	—
$D_{\nu 4}\Delta^{(3)}\gamma_4$	—	+·00009	−·00003	—
$D_{\nu 6}\Delta^{(3)}\gamma_6$	—	—	+·00005	−·00001
$E_{\nu 2}\Delta^{(3)}\mu_2$	+·00001	+·00002	—	—
$E_{\nu 4}\Delta^{(3)}\mu_4$	—	—	—	—
$E_{\nu 6}\Delta^{(3)}\mu_6$	—	—	−·00003	−·00008
$\Delta^{(4)}\mu_\nu$	+·00006	+·00009	−·00001	−·00009

(63370) E 2

FORM 3a EVEN ν

TABLE 25

$\alpha_\nu = -\alpha_{-\nu}$
$\gamma_\nu = -\gamma_{-\nu}$
$\mu_\nu = -\mu_{-\nu}$

2 × 15 STATIONS

$\delta = 1$ ANTISYMMETRICAL LOAD

a) VALUES $B_{\nu n}, C_{\nu n}, D_{\nu n}, E_{\nu n}$

$B_{\nu n} = a_{\nu n}(l'_\nu i'_{\nu n} - l''_\nu i''_{\nu n}) - a_{\nu,-n}(l'_\nu i'_{\nu,-n} - l''_\nu i''_{\nu,-n})$

ν	2	4	6
n = 1	·3483	·0263$_6$	·0058$_5$
3	·3180	·3757	·0315$_8$
5	·0151$_8$	·3266	·3744
7	·0021$_5$	·0151$_6$	·3350

$D_{\nu n} = a_{\nu n}(m''_\nu i''_{\nu n} - m'_\nu i'_{\nu n}) - a_{\nu,-n}(m''_\nu i''_{\nu,-n} - m'_\nu i'_{\nu,-n})$

ν	2	4	6
n = 1	·0222$_2$	·0029$_4$	·0008$_4$
3	–·0640	·0244	·0038$_1$
5	–·0029$_8$	–·0621	·0242$_2$
7	–·0003$_5$	–·0030$_3$	–·0405

$C_{\nu n} = a_{\nu n}(l'_\nu j'_{\nu n} - l''_\nu j''_{\nu n}) - a_{\nu,-n}(l'_\nu j'_{\nu,-n} - l''_\nu j''_{\nu,-n})$

ν	2	4	6
n = 1	·0519$_1$	·0039$_6$	·0007$_7$
3	·2674	·2111	·0051$_3$
5	·01385	·2636	·0439
7	·0015	·0146$_7$	·1684

$E_{\nu n} = a_{\nu n}(m''_\nu j''_{\nu n} - m'_\nu j'_{\nu n}) - a_{\nu,-n}(m''_\nu j''_{\nu,-n} - m'_\nu j'_{\nu,-n})$

ν	2	4	6
n = 1	·1521	·0032$_1$	·0004$_5$
3	·0364$_2$	·0245$_3$	·0044$_4$
5	–·0022$_1$	·0494$_4$	·1977
7	–·0002	–·0021$_5$	·1869

b) SOLUTION BY ITERATION:

FIRST APPROXIMATION:

ν	2	4	6
$a_{\nu\nu}(l'_\nu\alpha'_\nu - l''_\nu\alpha''_\nu)$	0	·0694	·0398
$B_{\nu 1}\ \gamma_1^{(1)}$	·00271	·00020	·00005
$B_{\nu 3}\ \gamma_3^{(1)}$	·03660	·04323	·00364
$B_{\nu 5}\ \gamma_5^{(1)}$	·00205	·04420	·05065
$B_{\nu 7}\ \gamma_7^{(1)}$	·00013	·00090	·01993
$C_{\nu 1}\ \mu_1^{(1)}$	–·00008	–·00001	—
$C_{\nu 3}\ \mu_3^{(1)}$	–·01086	–·00857	–·00021
$C_{\nu 5}\ \mu_5^{(1)}$	–·00042	–·00801	–·00133
$C_{\nu 7}\ \mu_7^{(1)}$	–·00001	–·00013	–·00144
$\gamma_\nu^{(1)}$	·0301	·1412	·1111

ν	2	4	6
$a_{\nu\nu}(m''_\nu\alpha''_\nu - m'_\nu\alpha'_\nu)$	0	–·03407	–·02014
$D_{\nu 1}\ \gamma_1^{(1)}$	+·00017	+·00002	+·00001
$D_{\nu 3}\ \gamma_3^{(1)}$	–·00736	+·00281	+·00044
$D_{\nu 5}\ \gamma_5^{(1)}$	–·00040	–·00840	+·00328
$D_{\nu 7}\ \gamma_7^{(1)}$	–·00002	–·00018	–·00241
$E_{\nu 1}\ \mu_1^{(1)}$	–·00025	–·00001	—
$E_{\nu 3}\ \mu_3^{(1)}$	–·00148	–·00100	–·00018
$E_{\nu 5}\ \mu_5^{(1)}$	+·00007	–·00150	–·00600
$E_{\nu 7}\ \mu_7^{(1)}$	—	+·00002	–·00160
$\mu_\nu^{(1)}$	–·00927	–·04231	–·02660

DIFFERENCES:

ν	2	4	6
$B_{\nu 1}\Delta^{(2)}\gamma_1$	+·00017	+·00001	—
$B_{\nu 3}\Delta^{(2)}\gamma_3$	–·00053	–·00063	–·00005
$B_{\nu 5}\Delta^{(2)}\gamma_5$	+·00005	+·00105	+·00120
$B_{\nu 7}\Delta^{(2)}\gamma_7$	+·00001	+·00006	+·00130
$C_{\nu 1}\Delta^{(2)}\mu_1$	–·00005	—	—
$C_{\nu 3}\Delta^{(2)}\mu_3$	–·00105	–·00083	–·00002
$C_{\nu 5}\Delta^{(2)}\mu_5$	–·00013	–·00252	–·00042
$C_{\nu 7}\Delta^{(2)}\mu_7$	–·00001	–·00010	–·00110
$\Delta^{(2)}\gamma_\nu$	–·00154	–·00296	+·00091
$B_{\nu 1}\Delta^{(3)}\gamma_1$	–·00020	–·00001	—
$B_{\nu 3}\Delta^{(3)}\gamma_3$	–·00057	–·00067	–·00006
$B_{\nu 5}\Delta^{(3)}\gamma_5$	–·00003	–·00055	–·00063
$B_{\nu 7}\Delta^{(3)}\gamma_7$	—	—	+·00006
$C_{\nu 1}\Delta^{(3)}\mu_1$	—	—	—
$C_{\nu 3}\Delta^{(3)}\mu_3$	+·00003	+·00002	—
$C_{\nu 5}\Delta^{(3)}\mu_5$	–·00001	–·00015	–·00002
$C_{\nu 7}\Delta^{(3)}\mu_7$	—	–·00001	–·00013
$\Delta^{(3)}\gamma_\nu$	–·00078	–·00137	–·00078
$B_{\nu 1}\Delta^{(4)}\gamma_1$	–·00008	–·00001	—
$B_{\nu 3}\Delta^{(4)}\gamma_3$	–·00023	–·00027	–·00002
$B_{\nu 5}\Delta^{(4)}\gamma_5$	–·00001	–·00029	–·00033
$B_{\nu 7}\Delta^{(4)}\gamma_7$	—	—	–·00011
$C_{\nu 1}\Delta^{(4)}\mu_1$	—	—	—
$C_{\nu 3}\Delta^{(4)}\mu_3$	+·00002	+·00002	—
$C_{\nu 5}\Delta^{(4)}\mu_5$	—	—	—
$C_{\nu 7}\Delta^{(4)}\mu_7$	—	—	–·00002
$\Delta^{(4)}\gamma_\nu$	–·00030	–·00055	–·00048

ν	2	4	6
$D_{\nu 1}\Delta^{(2)}\gamma_1$	+·00001	—	—
$D_{\nu 3}\Delta^{(2)}\gamma_3$	+·00011	–·00004	–·00001
$D_{\nu 5}\Delta^{(2)}\gamma_5$	–·00001	–·00020	+·00008
$D_{\nu 7}\Delta^{(2)}\gamma_7$	—	–·00001	–·00016
$E_{\nu 1}\Delta^{(2)}\mu_1$	–·00016	—	—
$E_{\nu 3}\Delta^{(2)}\mu_3$	–·00014	–·00010	–·00002
$E_{\nu 5}\Delta^{(2)}\mu_5$	+·00002	–·00047	–·00189
$E_{\nu 7}\Delta^{(2)}\mu_7$	—	+·00001	–·00122
$\Delta^{(2)}\mu_\nu$	–·00017	–·00081	–·00322
$D_{\nu 1}\Delta^{(3)}\gamma_1$	–·00001	—	—
$D_{\nu 3}\Delta^{(3)}\gamma_3$	+·00011	–·00004	–·00001
$D_{\nu 5}\Delta^{(3)}\gamma_5$	—	+·00010	–·00004
$D_{\nu 7}\Delta^{(3)}\gamma_7$	—	—	–·00001
$E_{\nu 1}\Delta^{(3)}\mu_1$	+·00001	—	—
$E_{\nu 3}\Delta^{(3)}\mu_3$	—	—	—
$E_{\nu 5}\Delta^{(3)}\mu_5$	—	–·00003	–·00011
$E_{\nu 7}\Delta^{(3)}\mu_7$	—	—	–·00015
$\Delta^{(3)}\mu_\nu$	+·00011	+·00003	–·00032
$D_{\nu 1}\Delta^{(4)}\gamma_1$	–·00001	—	—
$D_{\nu 3}\Delta^{(4)}\gamma_3$	+·00005	–·00002	—
$D_{\nu 5}\Delta^{(4)}\gamma_5$	—	+·00005	–·00002
$D_{\nu 7}\Delta^{(4)}\gamma_7$	—	—	–·00001
$E_{\nu 1}\Delta^{(4)}\mu_1$	+·00001	—	—
$E_{\nu 3}\Delta^{(4)}\mu_3$	—	—	—
$E_{\nu 5}\Delta^{(4)}\mu_5$	—	—	—
$E_{\nu 7}\Delta^{(4)}\mu_7$	—	—	–·00002
$\Delta^{(4)}\mu_\nu$	+·00005	+·00003	–·00005

FORM 3a ODD ν ctd. TABLE 26

2 × 15 STATIONS

DIFFERENCES CTD.

ν	1	3	5	7
$B_{\nu 2}\Delta^{(4)}\gamma_2$	−·00009	−·00011	−·00001	—
$B_{\nu 4}\Delta^{(4)}\gamma_4$	−·00001	−·00018	−·00021	−·00001
$B_{\nu 6}\Delta^{(4)}\gamma_6$	—	−·00001	−·00016	−·00017
$C_{\nu 2}\Delta^{(4)}\mu_2$	+·00001	—	—	—
$C_{\nu 4}\Delta^{(4)}\mu_4$	—	+·00001	—	—
$C_{\nu 6}\Delta^{(4)}\mu_6$	—	—	−·00001	—
$\Delta^{(5)}\gamma_\nu$	−·00009	−·00029	−·00039	−·00018

ν	1	3	5	7
$D_{\nu 2}\Delta^{(4)}\gamma_2$	+·00002	−·00001	—	—
$D_{\nu 4}\Delta^{(4)}\gamma_4$	—	+·00003	−·00001	—
$D_{\nu 6}\Delta^{(4)}\gamma_6$	—	—	+·00003	−·00001
$E_{\nu 2}\Delta^{(4)}\mu_2$	—	+·00001	—	—
$E_{\nu 4}\Delta^{(4)}\mu_4$	—	—	—	—
$E_{\nu 6}\Delta^{(4)}\mu_6$	—	—	—	−·00001
$\Delta^{(5)}\mu_\nu$	+·00002	+·00003	+·00002	−·00002

ν	1	3	5	7
$B_{\nu 2}\Delta^{(5)}\gamma_2$	−·00004	−·00004	—	—
$B_{\nu 4}\Delta^{(5)}\gamma_4$	—	−·00007	−·00008	—
$B_{\nu 6}\Delta^{(5)}\gamma_6$	—	—	−·00007	−·00008
$C_{\nu 2}\Delta^{(5)}\mu_2$	+·00001	—	—	—
$C_{\nu 4}\Delta^{(5)}\mu_4$	—	—	—	—
$C_{\nu 6}\Delta^{(5)}\mu_6$	—	—	—	—
$\Delta^{(6)}\gamma_\nu$	−·00003	−·00011	−·00015	−·00008

ν	1	3	5	7
$D_{\nu 2}\Delta^{(5)}\gamma_2$	+·00001	—	—	—
$D_{\nu 4}\Delta^{(5)}\gamma_4$	—	+·00001	−·00001	—
$D_{\nu 6}\Delta^{(5)}\gamma_6$	—	—	+·00001	—
$E_{\nu 2}\Delta^{(5)}\mu_2$	—	—	—	—
$E_{\nu 4}\Delta^{(5)}\mu_4$	—	—	—	—
$E_{\nu 6}\Delta^{(5)}\mu_6$	—	—	—	—
$\Delta^{(6)}\mu_\nu$	+·00001	+·00001	—	—

SUMMARY: $\gamma_\nu = \gamma_\nu^{(1)} + \Delta^{(2)}\gamma_\nu + \Delta^{(3)}\gamma_\nu + \cdots$ $\mu_\nu = \mu_\nu^{(1)} + \Delta^{(2)}\mu_\nu + \Delta^{(3)}\mu_\nu + \cdots$

ν	1	3	5	7
$\gamma_\nu^{(1)}$	·0078	·1151	·1353	·0595
$\Delta^{(2)}\gamma_\nu$	+·00048	−·00167	+·00322	+·00388
$\Delta^{(3)}\gamma_\nu$	−·00056	−·00179	−·00168	+·00018
$\Delta^{(4)}\gamma_\nu$	−·00023	−·00072	−·00088	−·00032
$\Delta^{(5)}\gamma_\nu$	−·00009	−·00029	−·00039	−·00018
$\Delta^{(6)}\gamma_\nu$	−·00003	−·00011	−·00015	−·00008
REST	−·00002	−·00007	−·00009	−·00006
γ_ν	·0073	·1104	·1353	·0630

ν	1	3	5	7
$\mu_\nu^{(1)}$	−·00162	−·04057	−·03037	−·00857
$\Delta^{(2)}\mu_\nu$	−·00102	−·00392	−·00956	−·00651
$\Delta^{(3)}\mu_\nu$	+·00010	+·00010	−·00056	−·00079
$\Delta^{(4)}\mu_\nu$	+·00006	+·00009	−·00001	−·00009
$\Delta^{(5)}\mu_\nu$	+·00002	+·00003	+·00002	−·00002
$\Delta^{(6)}\mu_\nu$	+·00001	+·00001	—	—
REST	+·00001	+·00001	—	—
μ_ν	−·00244	−·04425	−·04048	−·01598

CHECK OF RESULTS:

ν	1	3	5	7
$a_{\nu\nu}(l'\alpha' - l'_\nu\alpha'_\nu)$	0	·0676	·0597	·0211
$B_{\nu 2}\,\gamma_2$	·00835	·01015	·00087	·00016
$B_{\nu 4}\,\gamma_4$	·00167	·04353	·05127	·00351
$B_{\nu 6}\,\gamma_6$	·00021	·00178	·03722	·03885
$C_{\nu 2}\,\mu_2$	−·00233	−·00058	−·00005	−·00001
$C_{\nu 4}\,\mu_4$	−·00053	−·01163	−·00247	−·00017
$C_{\nu 6}\,\mu_6$	−·00004	−·00044	−·00722	−·00052
γ_ν	·0073	·1104	·1353	·0629

ν	1	3	5	7
$a_{\nu\nu}(m''\alpha'' - m'_\nu\alpha'_\nu)$	0	−·03313	−·02765	−·01089
$D_{\nu 2}\,\gamma_2$	−·00171	+·00064	+·00010	+·00002
$D_{\nu 4}\,\gamma_4$	−·00038	−·00861	+·00339	+·00044
$D_{\nu 6}\,\gamma_6$	−·00004	−·00034	−·00639	+·00211
$E_{\nu 2}\,\mu_2$	−·00043	−·00137	−·00004	—
$E_{\nu 4}\,\mu_4$	+·00010	−·00151	−·00705	−·00021
$E_{\nu 6}\,\mu_6$	+·00001	+·00007	−·00285	−·00747
μ_ν	−·00245	−·04425	−·04049	−·01600

RESULTS:

a) LOCAL LIFT, MOMENT & a.c.

ν	0	1	2	3	4	5	6	7
γ_ν	0	·0073	·0273	·1104	·1360	·1353	·1103	·0629
μ_ν	0	−·00245	−·00923	−·04425	−·04301	−·04049	−·03020	−·01600
$X_{\nu a.c.}$	—	·586	·588	·651	·566	·549	·524	·505

FORM 3a EVEN ν ctd. TABLE 27

2 × 15 STATIONS

DIFFERENCES CTD.

ν	2	4	6
$B_{\nu 1}\Delta^{⑤}\gamma_1$	$-\cdot 00003$	—	—
$B_{\nu 3}\Delta^{⑤}\gamma_3$	$-\cdot 00009$	$-\cdot 00011$	$-\cdot 00001$
$B_{\nu 5}\Delta^{⑤}\gamma_5$	$-\cdot 00001$	$-\cdot 00013$	$-\cdot 00015$
$B_{\nu 7}\Delta^{⑤}\gamma_7$	—	—	$-\cdot 00006$
$C_{\nu 1}\Delta^{⑤}\mu_1$	—	—	—
$C_{\nu 3}\Delta^{⑤}\mu_3$	$+\cdot 00001$	$+\cdot 00001$	—
$C_{\nu 5}\Delta^{⑤}\mu_5$	—	$+\cdot 00001$	—
$C_{\nu 7}\Delta^{⑤}\mu_7$	—	—	—
$\Delta^{⑤}\gamma_\nu$	$-\cdot 00012$	$-\cdot 00022$	$-\cdot 00022$
$B_{\nu 1}\Delta^{⑥}\gamma_1$	$-\cdot 00001$	—	—
$B_{\nu 3}\Delta^{⑥}\gamma_3$	$-\cdot 00003$	$-\cdot 00004$	—
$B_{\nu 5}\Delta^{⑥}\gamma_5$	—	$-\cdot 00005$	$-\cdot 00006$
$B_{\nu 7}\Delta^{⑥}\gamma_7$	—	—	$-\cdot 00003$
$C_{\nu 1}\Delta^{⑥}\mu_1$	—	—	—
$C_{\nu 3}\Delta^{⑥}\mu_3$	—	—	—
$C_{\nu 5}\Delta^{⑥}\mu_5$	—	—	—
$C_{\nu 7}\Delta^{⑥}\mu_7$	—	—	—
$\Delta^{⑥}\gamma_\nu$	$-\cdot 00004$	$-\cdot 00009$	$-\cdot 00009$

ν	2	4	6
$D_{\nu 1}\Delta^{⑤}\gamma_1$	—	—	—
$D_{\nu 3}\Delta^{⑤}\gamma_3$	$+\cdot 00002$	$-\cdot 00001$	—
$D_{\nu 5}\Delta^{⑤}\gamma_5$	—	$+\cdot 00002$	$-\cdot 00001$
$D_{\nu 7}\Delta^{⑤}\gamma_7$	—	—	$-\cdot 00001$
$E_{\nu 1}\Delta^{⑤}\mu_1$	—	—	—
$E_{\nu 3}\Delta^{⑤}\mu_3$	—	—	—
$E_{\nu 5}\Delta^{⑤}\mu_5$	—	—	—
$E_{\nu 7}\Delta^{⑤}\mu_7$	—	—	—
$\Delta^{⑤}\mu_\nu$	$+\cdot 00002$	$+\cdot 00001$	$-\cdot 00002$
$D_{\nu 1}\Delta^{⑥}\gamma_1$	—	—	—
$D_{\nu 3}\Delta^{⑥}\gamma_3$	$+\cdot 00001$	—	—
$D_{\nu 5}\Delta^{⑥}\gamma_5$	—	$+\cdot 00001$	—
$D_{\nu 7}\Delta^{⑥}\gamma_7$	—	—	—
$E_{\nu 1}\Delta^{⑥}\mu_1$	—	—	—
$E_{\nu 3}\Delta^{⑥}\mu_3$	—	—	—
$E_{\nu 5}\Delta^{⑥}\mu_5$	—	—	—
$E_{\nu 7}\Delta^{⑥}\mu_7$	—	—	—
$\Delta^{⑥}\mu_\nu$	$+\cdot 00001$	$+\cdot 00001$	—

SUMMARY: $\gamma_\nu = \gamma_\nu^{①} + \Delta^{②}\gamma_\nu + \Delta^{③}\gamma_\nu + \cdots$ $\qquad \mu_\nu = \mu_\nu^{①} + \Delta^{②}\mu_\nu + \Delta^{③}\mu_\nu + \cdots$

ν	2	4	6
$\gamma_\nu^{①}$	$\cdot 0301$	$\cdot 1412$	$\cdot 1111$
$\Delta^{②}\gamma_\nu$	$-\cdot 00151$	$-\cdot 00296$	$+\cdot 00091$
$\Delta^{③}\gamma_\nu$	$-\cdot 00078$	$-\cdot 00137$	$-\cdot 00078$
$\Delta^{④}\gamma_\nu$	$-\cdot 00030$	$-\cdot 00055$	$-\cdot 00048$
$\Delta^{⑤}\gamma_\nu$	$-\cdot 00012$	$-\cdot 00022$	$-\cdot 00022$
$\Delta^{⑥}\gamma_\nu$	$-\cdot 00004$	$-\cdot 00009$	$-\cdot 00009$
REST	$-\cdot 00002$	$-\cdot 00006$	$-\cdot 00006$
γ_ν	$\cdot 0273$	$\cdot 1360$	$\cdot 1104$

ν	2	4	6
$\mu_\nu^{①}$	$-\cdot 00927$	$-\cdot 04231$	$-\cdot 02660$
$\Delta^{②}\mu_\nu$	$-\cdot 00017$	$-\cdot 00081$	$-\cdot 00322$
$\Delta^{③}\mu_\nu$	$+\cdot 00011$	$+\cdot 00063$	$-\cdot 00032$
$\Delta^{④}\mu_\nu$	$+\cdot 00005$	$+\cdot 00003$	$-\cdot 00005$
$\Delta^{⑤}\mu_\nu$	$+\cdot 00002$	$+\cdot 00001$	$-\cdot 00002$
$\Delta^{⑥}\mu_\nu$	$+\cdot 00001$	$+\cdot 00001$	—
REST	$+\cdot 00001$	$+\cdot 00001$	—
μ_ν	$-\cdot 00924$	$-\cdot 04303$	$-\cdot 03021$

CHECK OF RESULTS:

ν	2	4	6
$a_{\nu\nu}(l_\nu'\alpha_\nu' - l_\nu''\alpha_\nu'')$	0	$\cdot 0694$	$\cdot 0398$
$B_{\nu 1}\ \gamma_1$	$\cdot 00255$	$\cdot 00019$	$\cdot 00004$
$B_{\nu 3}\ \gamma_3$	$\cdot 03515$	$\cdot 04153$	$\cdot 00349$
$B_{\nu 5}\ \gamma_5$	$\cdot 00205$	$\cdot 04420$	$\cdot 05063$
$B_{\nu 7}\ \gamma_7$	$\cdot 00014$	$\cdot 00095$	$\cdot 02108$
$C_{\nu 1}\ \mu_1$	$-\cdot 00013$	$-\cdot 00001$	—
$C_{\nu 3}\ \mu_3$	$-\cdot 01184$	$-\cdot 00934$	$-\cdot 00023$
$C_{\nu 5}\ \mu_5$	$-\cdot 00056$	$-\cdot 01067$	$-\cdot 00178$
$C_{\nu 7}\ \mu_7$	$-\cdot 00002$	$-\cdot 00023$	$-\cdot 00270$
γ_ν	$\cdot 0273$	$\cdot 1360$	$\cdot 1103$

ν	2	4	6
$a_{\nu\nu}(m_\nu''\alpha_\nu'' - m_\nu'\alpha_\nu')$	0	$-\cdot 03407$	$-\cdot 02014$
$D_{\nu 1}\ \gamma_1$	$+\cdot 00016$	$+\cdot 00002$	$+\cdot 00001$
$D_{\nu 3}\ \gamma_3$	$-\cdot 00707$	$+\cdot 00270$	$+\cdot 00042$
$D_{\nu 5}\ \gamma_5$	$-\cdot 00040$	$-\cdot 00840$	$+\cdot 00328$
$D_{\nu 7}\ \gamma_7$	$-\cdot 00002$	$-\cdot 00019$	$-\cdot 00255$
$E_{\nu 1}\ \mu_1$	$-\cdot 00037$	$-\cdot 00001$	—
$E_{\nu 3}\ \mu_3$	$-\cdot 00161$	$-\cdot 00109$	$-\cdot 00020$
$E_{\nu 5}\ \mu_5$	$+\cdot 00009$	$-\cdot 00200$	$-\cdot 00801$
$E_{\nu 7}\ \mu_7$	—	$+\cdot 00003$	$-\cdot 00299$
μ_ν	$-\cdot 00922$	$-\cdot 04301$	$-\cdot 03019$

RESULTS CTD.

b) ROLLING MOMENT DUE TO FLAP DEFLECTION.

$$\frac{\partial C_\ell}{\partial \delta} = \frac{\pi A}{2(m+1)} \sum_{n=1}^{\frac{m-1}{2}} \gamma_n \sin\frac{2n\pi}{m+1} = \frac{\pi}{8}\left[\begin{array}{l} \cdot 3827 \times \cdot 0073 \\ \cdot 7071 \times \cdot 0273 \\ \cdot 9239 \times \cdot 1104 \\ 1\cdot 0 \times \cdot 1360 \\ \cdot 9239 \times \cdot 1353 \\ \cdot 7071 \times \cdot 1103 \\ \cdot 3827 \times \cdot 0629 \end{array}\right. = \frac{\pi}{8} \times \cdot 4827$$

$$\frac{\partial C_\ell}{\partial \delta} = \underline{\cdot 1913}$$

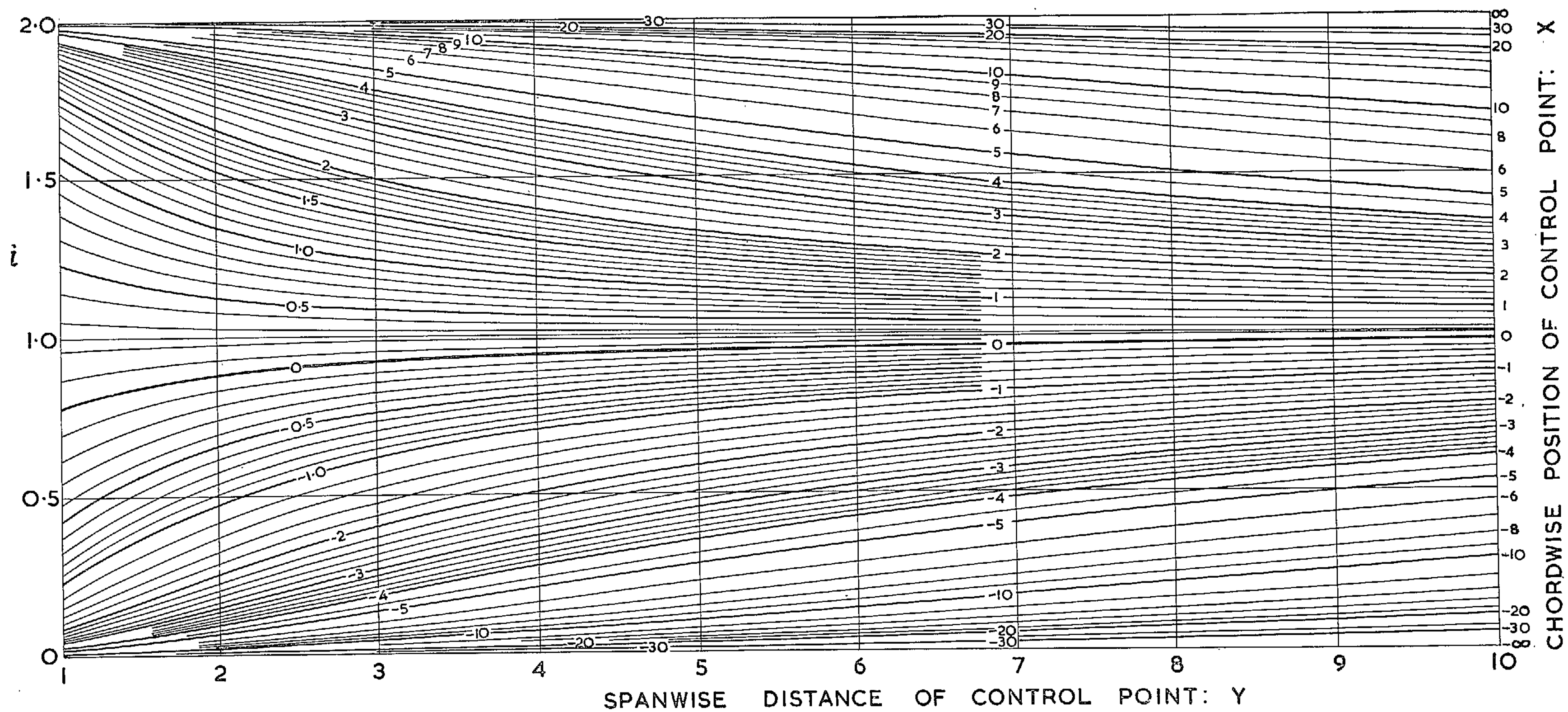


FIG. 1. Influence function $i(X,Y)$.

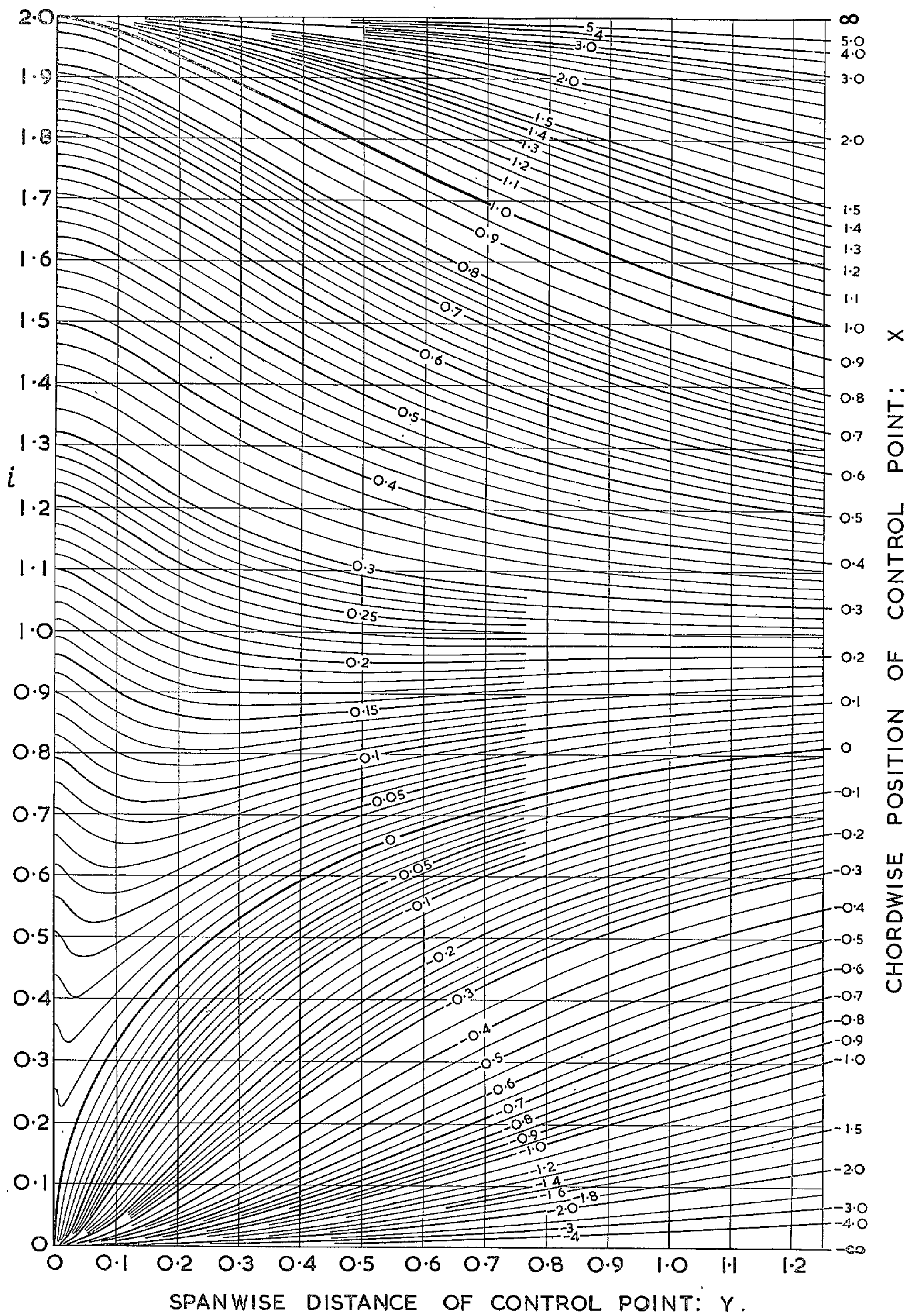


FIG. 2. Influence function $i(X,Y)$.

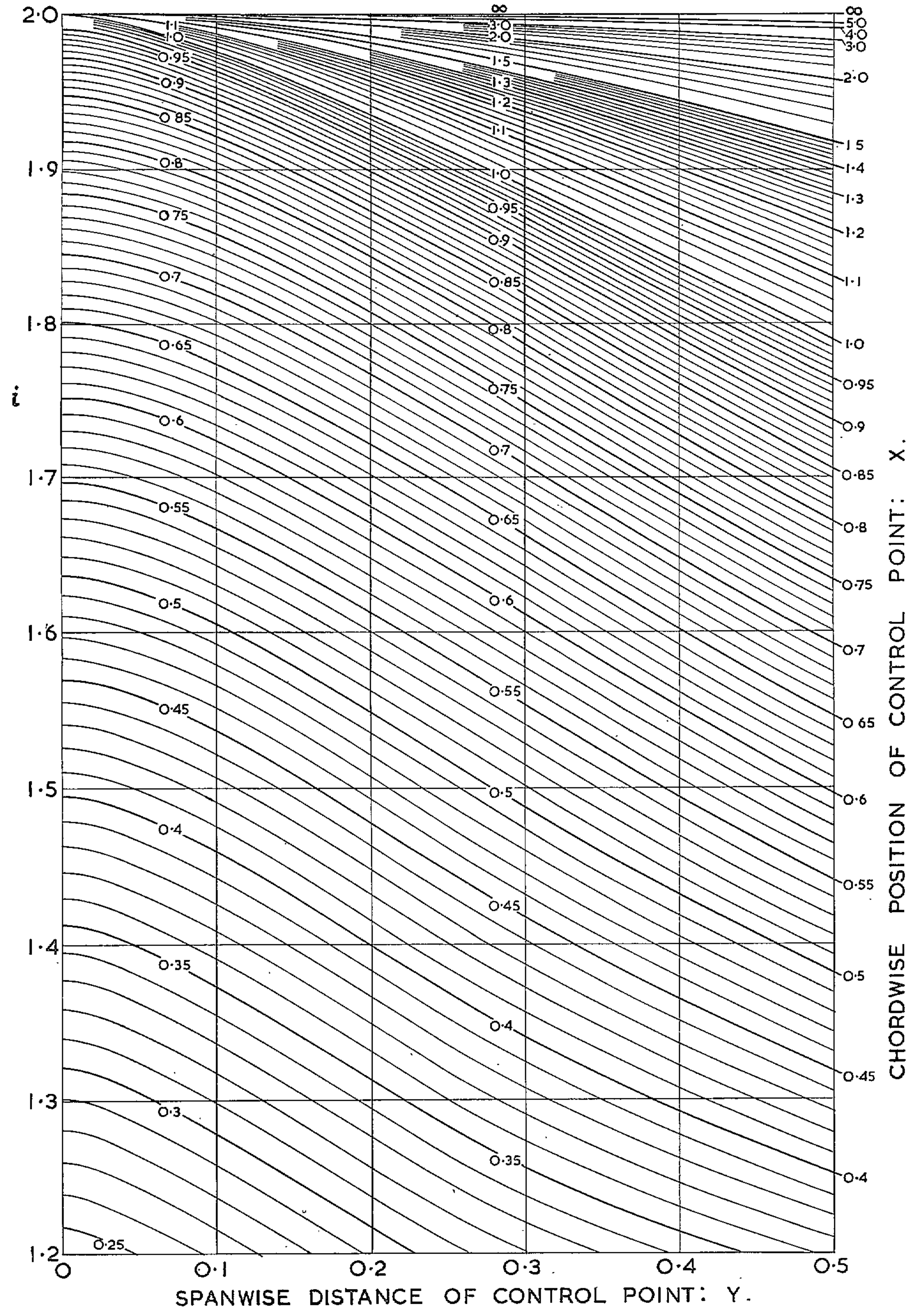


FIG. 3. Influence function $i(X,Y.)$

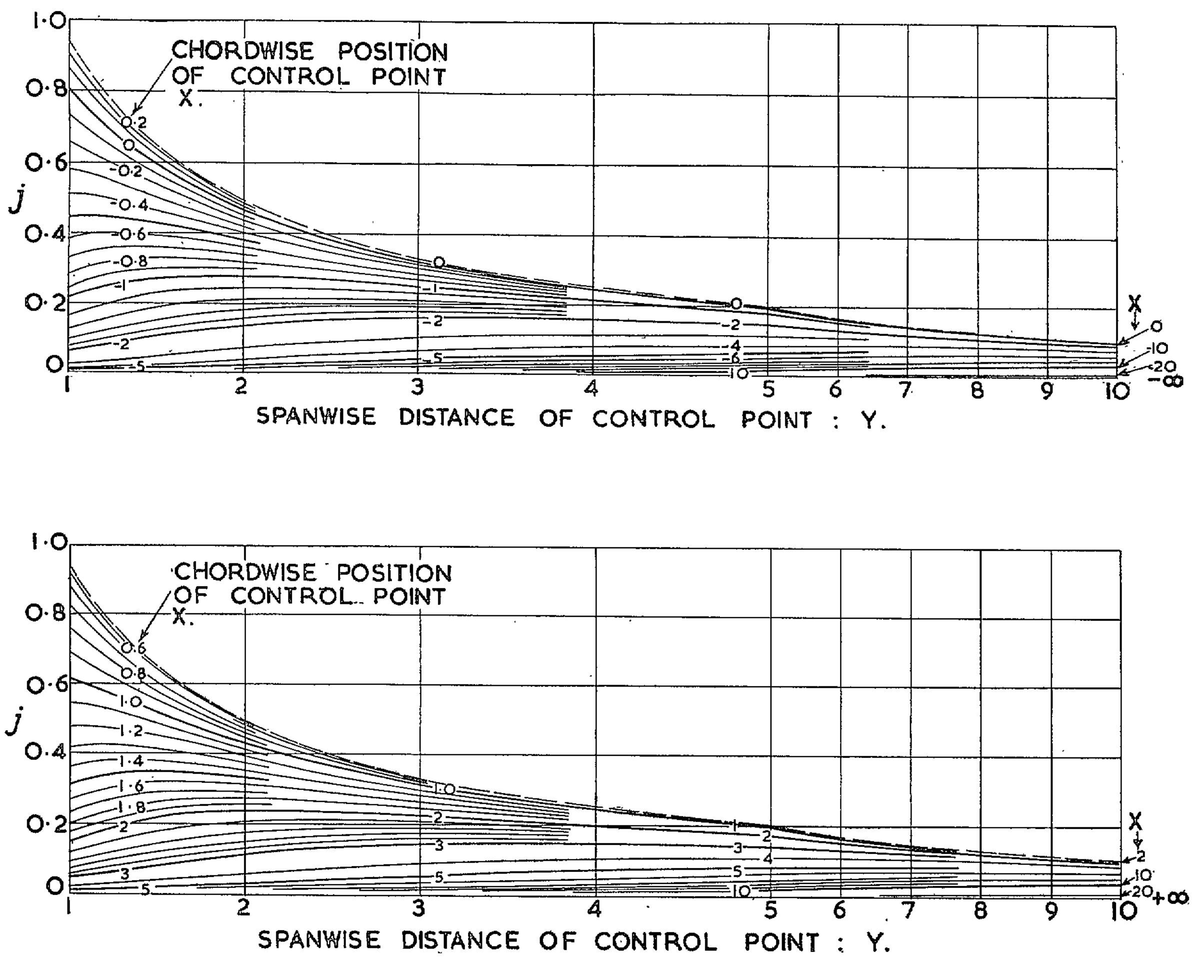


FIG. 4. Influence function $j(X,Y)$.

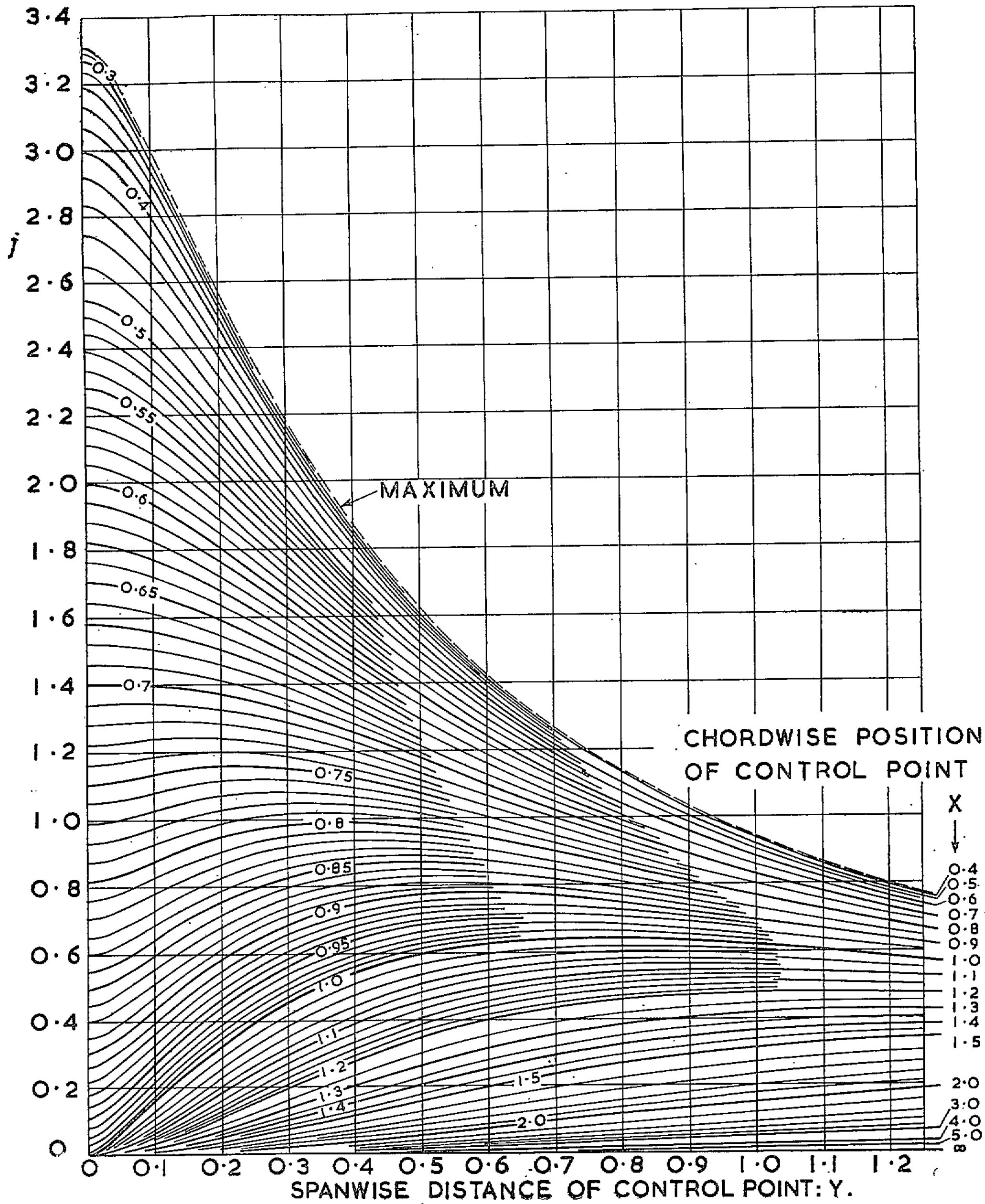


FIG. 5. Influence function $j(X,Y)$.

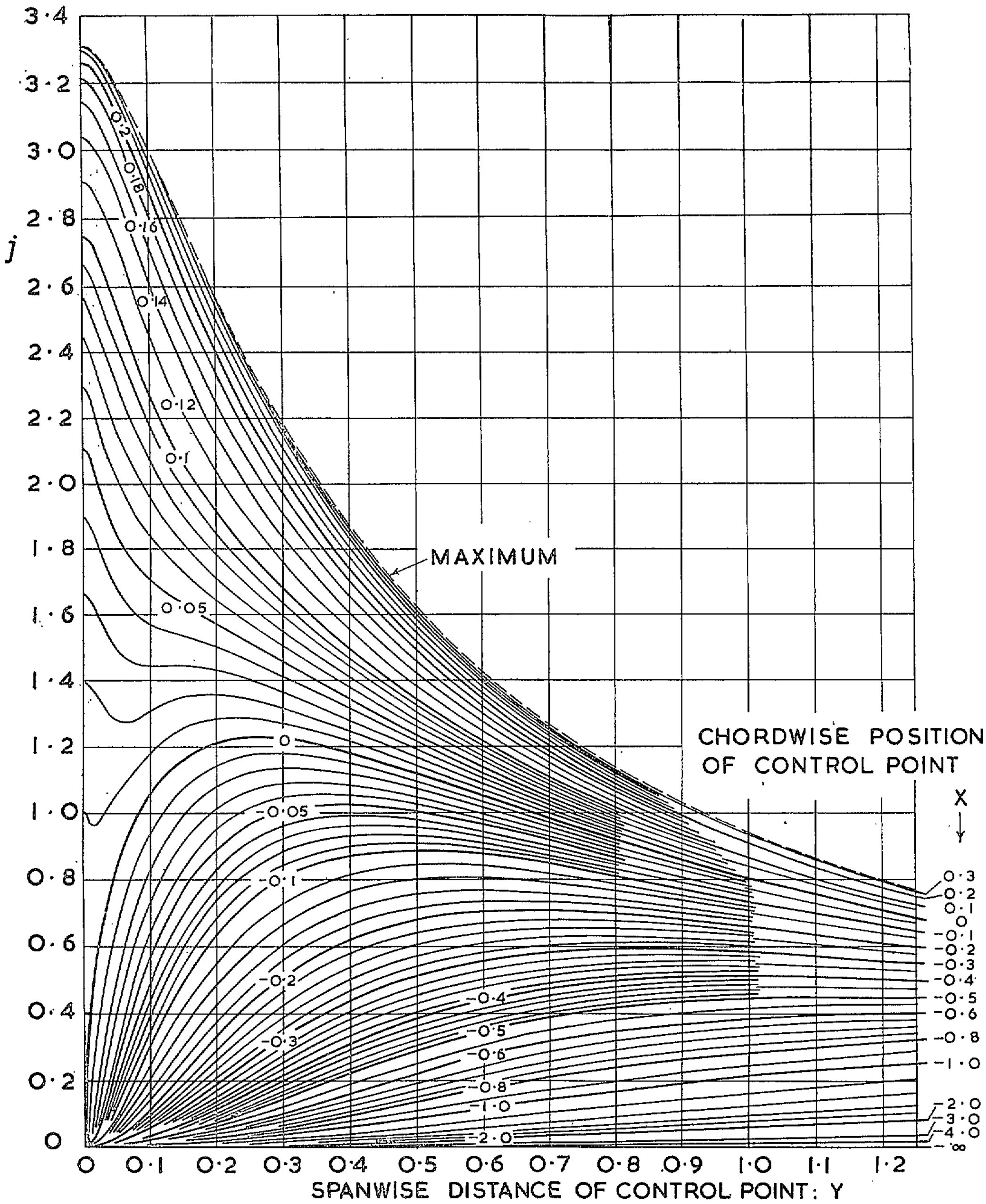


FIG. 6. Influence function $j(X,Y)$.

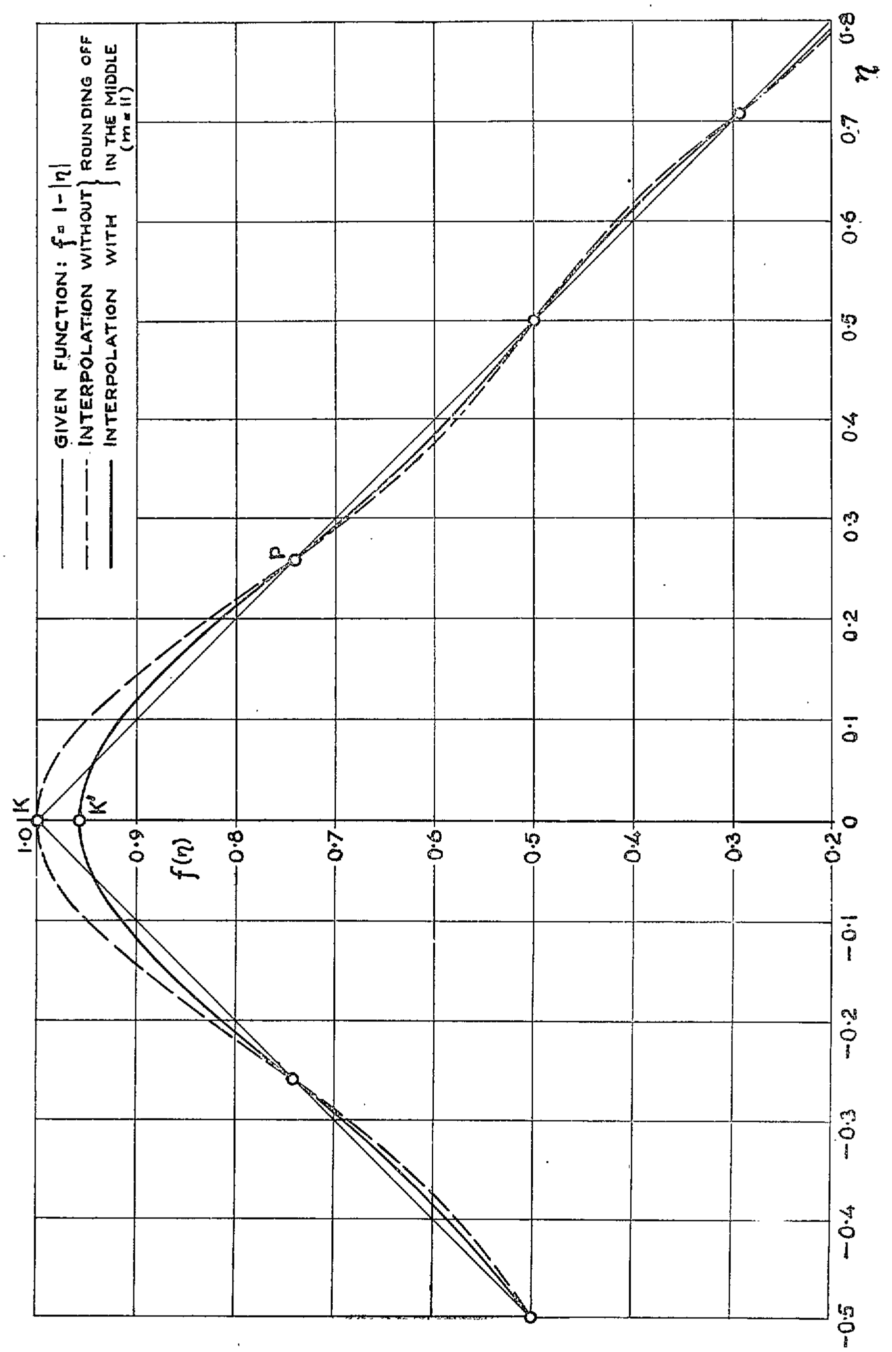
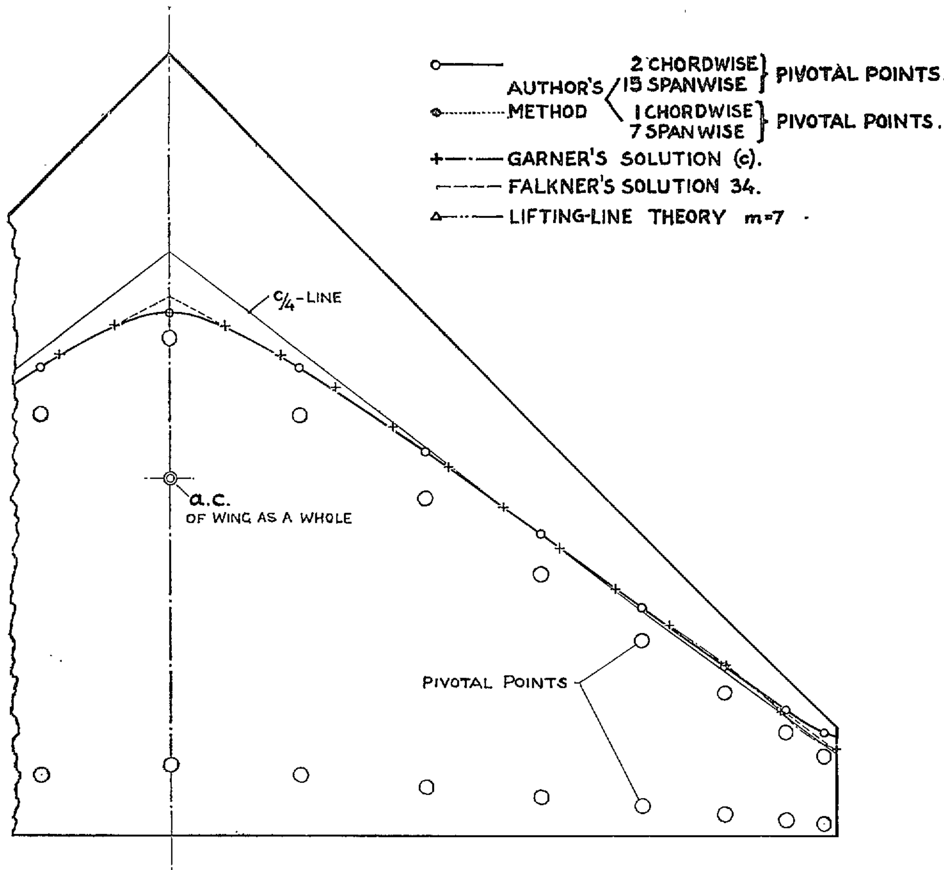
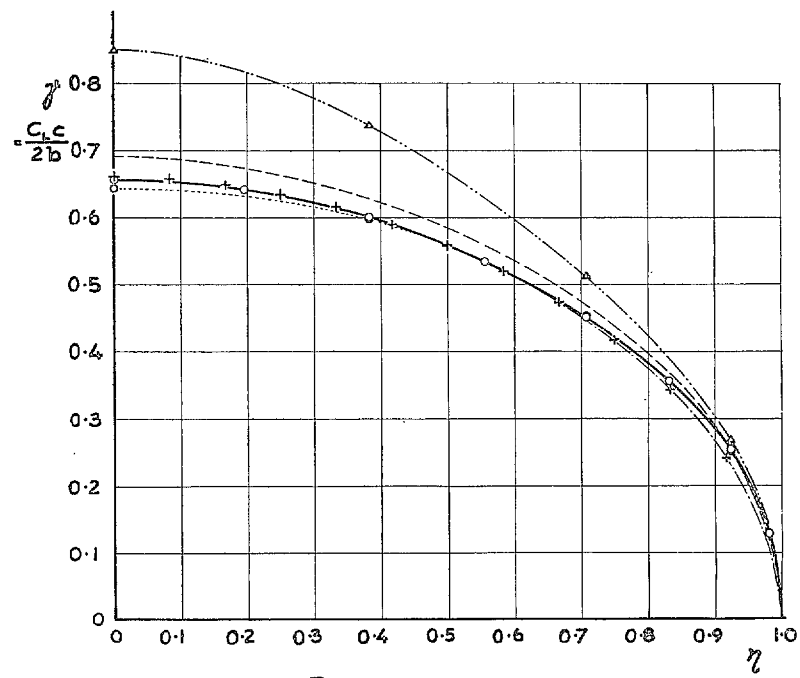


FIG. 7. Representation of a kinked function by interpolation polynomials.

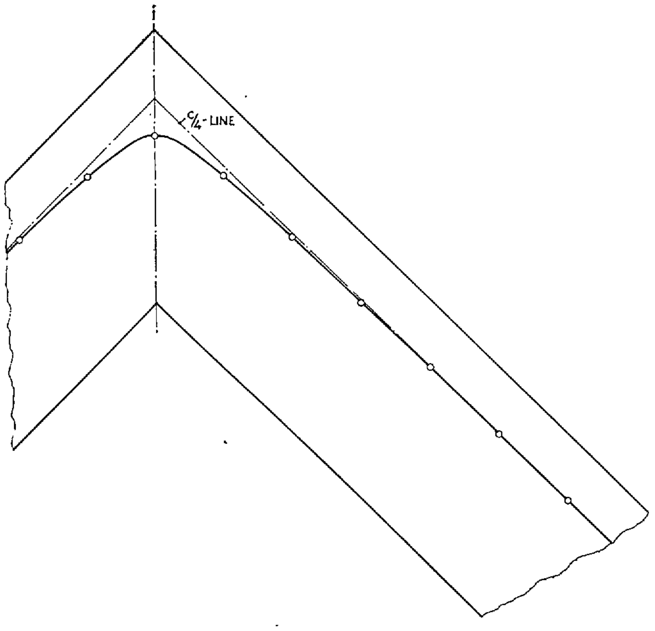


a) PLAN FORM & SECTIONAL AERODYNAMIC CENTRES.

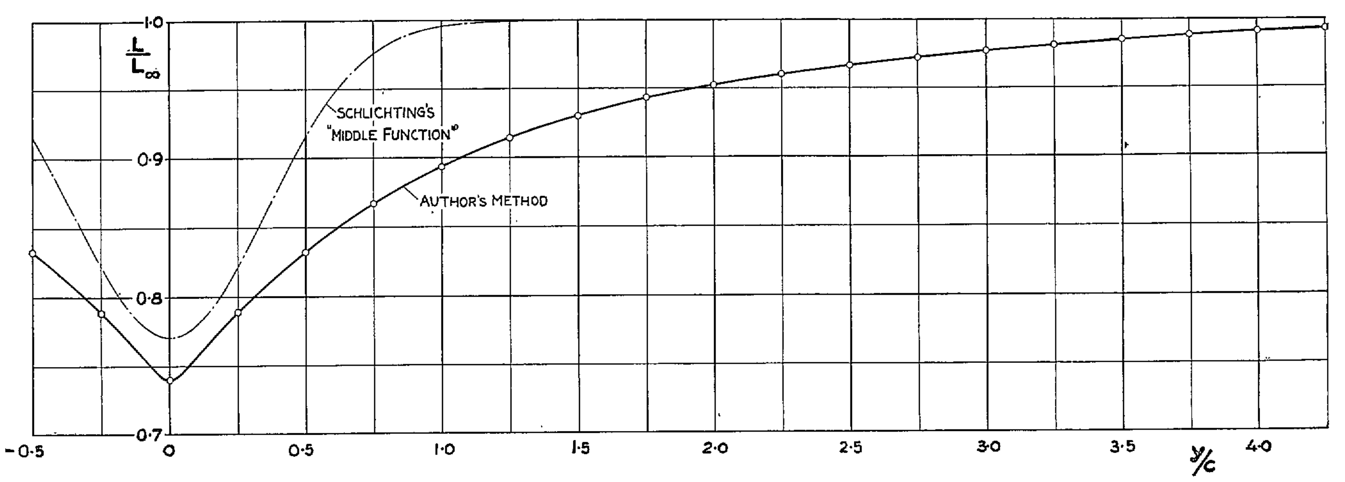


b) SPANWISE LIFT DISTRIBUTION.

FIG. 8. Delta wing. $A = 3$.



a) PLAN FORM & SECTIONAL AERODYNAMIC CENTRES.



b) SPANWISE LOAD DISTRIBUTION.

FIG. 9. Swept constant-chord wing of infinite aspect ratio. $\Lambda = 45$ deg.

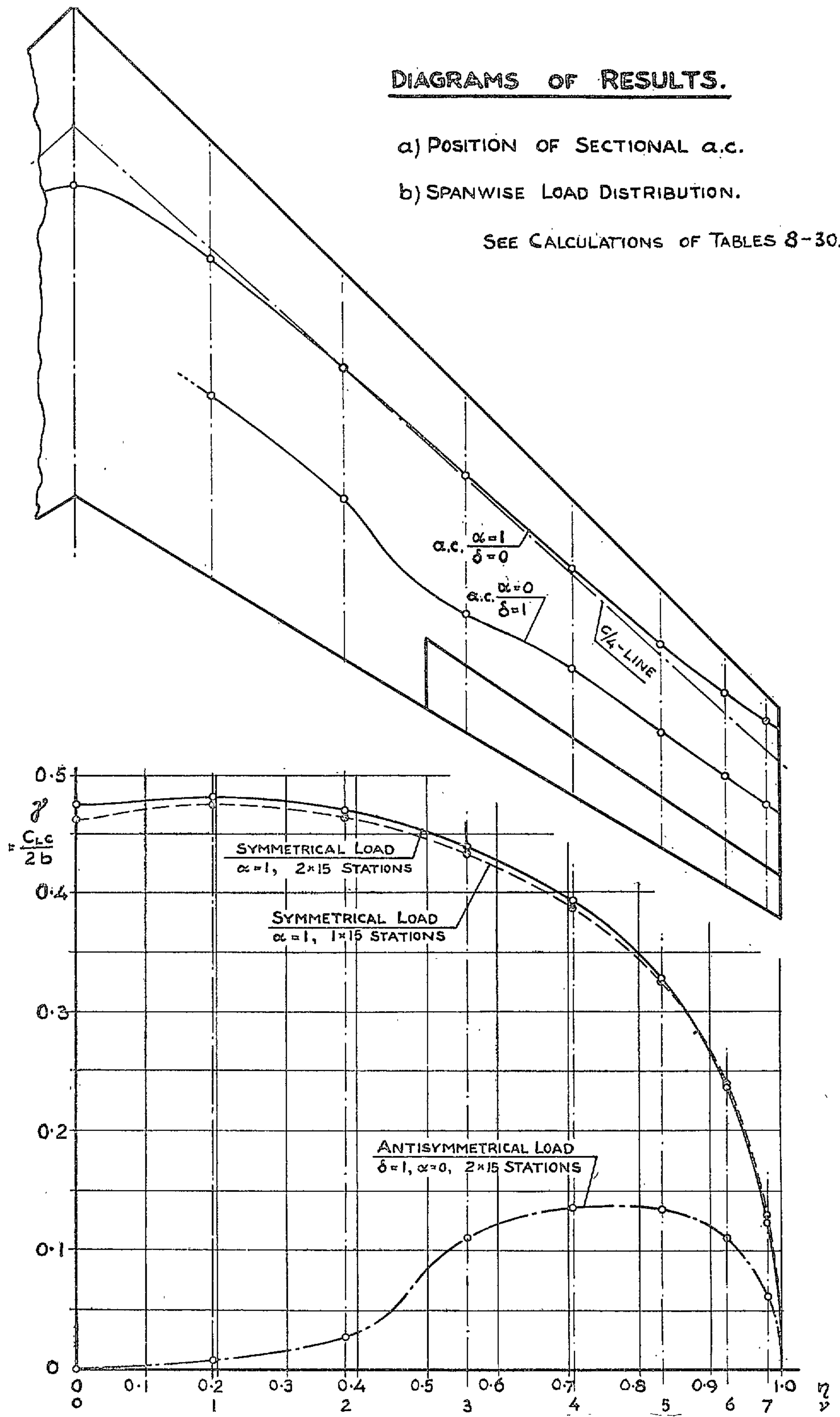


FIG. 10. Results for example worked in Tables 8 to 30.

(63370) Wt. 18/9296 K.9 5/55 Hw.

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