Calculating the Mass Distribution of the Atmosphere

The function m(Z) is defined to be the total mass contained between the two spheres of radius E and E+Z, where E is the radius of the earth. The integral to be evaluated is

$$m(Z) = \int_0^Z 4\pi(E+r)^2\rho(r)dr$$

It is conceivable that this could be evaluated analytically, being careful to observe that the expression for density changes form at each boundary of the standard atmosphere (0, 11km, 20km, etc.). The expression for density follows from the perfect gas law once the pressure and temperature are known from the equations of hydrostatic equilibrium. Since we have a coded subroutine for density vs. altitude as part of the standard atmosphere package and a routine for numerical integration as part of the numerical methods package, it is simpler to use numerical techniques to get a table of values for m(Z). As with many numerical problems, it helps to have an idea of the answer to the problem when setting up the computational procedure. We know that we routinely launch satellites with orbital radius about 100km. These satellites take quite some time to fall back to earth, so they seem to be well above what we might call the "height" of the atmosphere.

The attached program will calculate m(Z) for values of Z from 0 to 80km in steps of 1 km using the subroutine Quanc8 for integration. Notice that we need a function of one variable for use with Quanc8. The atmosphere procedure is a subroutine that returns temperature, pressure and density. So, we must make a function that calls Atmosphere and returns the value of

$$m(Z) = 4\pi(E+r)^2\rho(r)$$

You may download the source code for the mass distribution program from
http://www.pdas.com/programs/atmmass.f90.
You may download the results from running this program from
http://www.pdas.com/programs/atmmass.out.
atmmass.tex
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